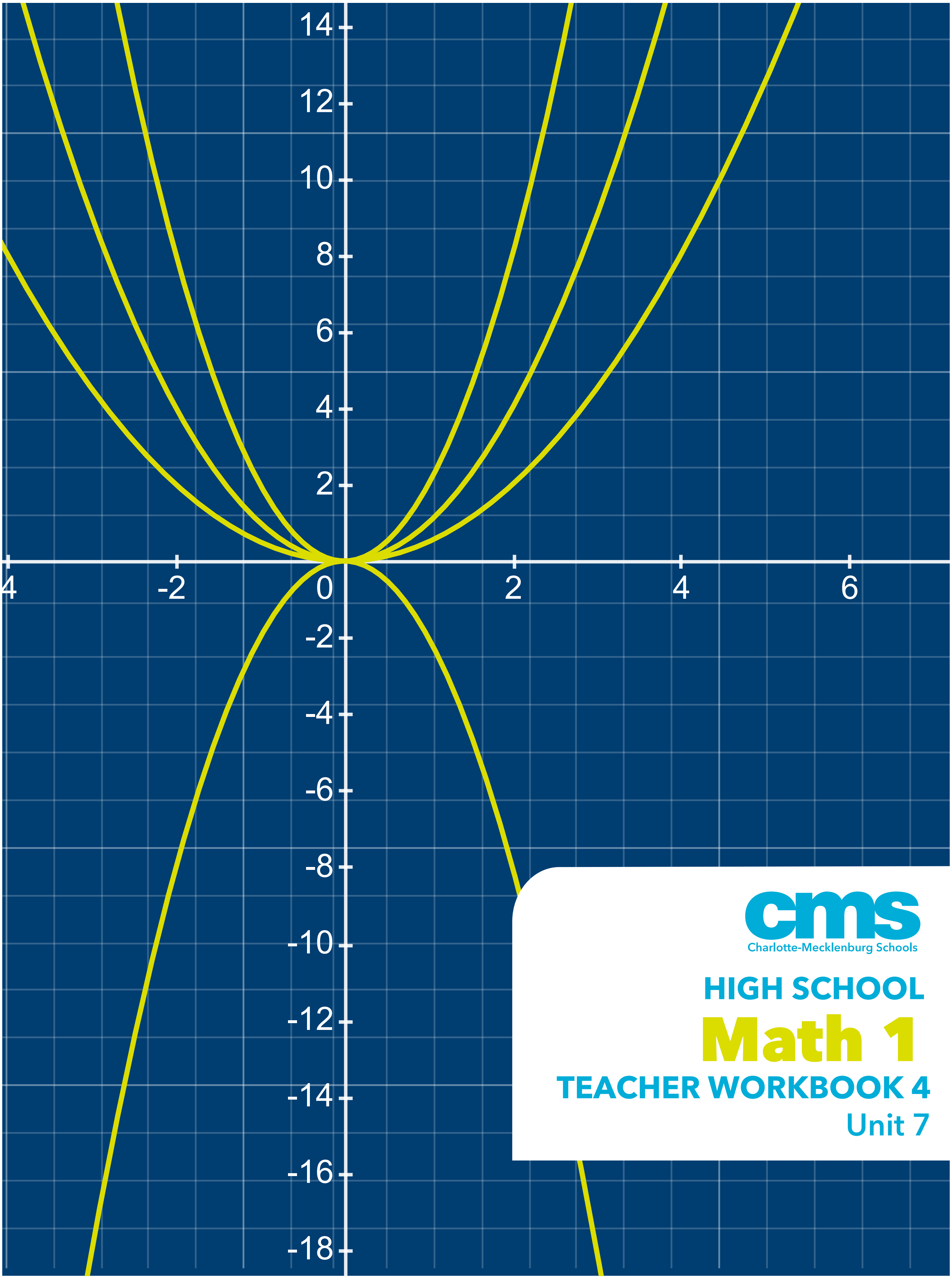


Charlotte-Mecklenburg Schools

HIGH SCHOOL

Math 1

TEACHER WORKBOOK 4

Unit 7

Table of Contents

Unit 7: Quadratic Functions and Equations

Prior to this unit, students have studied what it means for a relationship to be a function, used function notation, and investigated linear and exponential functions. In Lessons 1 and 2 of this unit, they begin by exploring some situations that can be modeled by quadratic functions, using tables, graphs, and equations. In Lesson 3, they compare quadratic growth with linear and exponential growth. They further observe that, eventually, quadratic functions grow more quickly than linear functions but more slowly than exponential functions.

In Lessons 4 and 5, students examine the important example of free-falling objects whose height over time can be modeled with quadratic functions. This may be an opportunity to collaborate with a science teacher around the demonstration of gravity to support students' understanding across subjects. They use tables, graphs, and equations to describe the movement of these objects, eventually looking at the situation where a projectile is launched upward. This leads to the important interpretation that in a quadratic function such as $f(t) = 5 + 30t - 16t^2$, representing the vertical position of an object after t seconds, 5 represents the initial height of the object, $30t$ represents its initial upward path, and $-16t^2$ represents the effect of gravity. Through this investigation, students also begin to appreciate how the different coefficients in a quadratic function influence the shape of the graph. In Lesson 6, students examine other situations represented by quadratic functions, including area and revenue. Additionally, Lesson 6 re-introduces function notation as well as the terms "domain" and "range." Students find connections between the domain and range of quadratic functions and the zeros and vertex of their graphs.

Lesson 7 is a transitional lesson in which students learn to use a version of the area diagrams they learned in middle school to rewrite expressions like $(x+a)(x+b)$ using the distributive property. They see that these diagrams can still be useful to find the expanded form when a or b is negative, despite the fact that length and area cannot be negative. This prepares students for Lessons 8–11, in which they examine the standard and factored forms of quadratic expressions. They investigate how each form is useful for understanding the graph of the function defined by these equivalent forms. The factored form is helpful for finding when the quadratic function takes the value 0 to obtain the x -intercept(s) of its graph, while the constant term in the standard form shows the y-intercept. Students also find that the factored form is useful for finding the vertex of the graph because its x-coordinate is halfway between the points where the graph intersects the x-axis (if it has two x-intercepts). As for the standard form, students investigate the coefficients of the quadratic and linear terms further, noticing that the coefficient of the quadratic term determines if it opens upward or downward. The effect of the coefficient of the linear term is somewhat mysterious and more complicated. Students explore how it shifts the graph both vertically and horizontally in Lesson 12, and they conjecture a formula for the x-coordinate of the vertex. In Lesson 13, they apply what they have learned in Lessons 8–11 to answer questions about real-life contexts that can be described by quadratic models.

Lessons 14 and 15 are Checkpoint lessons. Station B, a Desmos activity in which students are introduced to adding and subtracting quadratic expressions, is required. Optional stations include puzzles about sums and products of factors (to prepare students for the work later in this unit), an analysis of wages versus housing costs in Charlotte (which can be adapted to other cities), and micro-modeling problems. The first micro-modeling problem harkens back to systems of equations, though students will find there is more wiggle room in the problem. Likewise, the second micro-modeling problem looks at first like a familiar topic in this unit: maximizing area given a fixed perimeter. However, students have more room for exploration when the requirement that the shape be a rectangle is relaxed.

The Mid-Unit Assessment should be administered after Lesson 15. This assessment assesses topics from Lessons 1–13. (Note: The End-of-Unit Assessment will assess topics from Lessons 14–26.)

In the second half of this unit, students interpret, write, and solve quadratic equations. Lessons 16 and 17 both motivate the need to find input values that produce certain output values. For example, students consider the revenue of a theater as a function of the ticket price for a performance and determine what ticket price would earn the theater a certain amount. Previously, students were only able to solve such problems by observing graphs and estimating, or by guessing and checking. Writing and solving quadratic equations will enable students to answer such questions algebraically.

In Lesson 18, students begin solving quadratic equations by reasoning. For instance, to solve $x^2 + 9 = 25$, they think: “Adding 9 to a squared number makes 25. That squared number must be 16, so must be 4 or -4.” Along the way, students see that quadratic equations can have 2, 1, or 0 real solutions.

In Lesson 19, students learn that equations of the form $(x - m)(x - n) = 0$ can be easily solved by applying the zero product property, which says that when two factors have a product of 0, one of the factors must be 0. In Lesson 20, students look at quadratic equations that do not have one side equal to 0. They learn to solve these equations by first rearranging the terms so that one side is 0 and then solving by graphing and finding the x-intercepts. In doing so, they make the connection between zeros of a function, x-intercepts of a graph, and solutions to an equation for which one side is 0.

Lessons 21–23 further explore the technique of rewriting expressions of the form $x^2 + bx + c$ in factored form so that students are ready to use this technique to solve quadratic equations in Lesson 24. In Lesson 25, students learn a technique for factoring expressions of the form $ax^2 + bx + c$, where $a \neq 1$. In Lesson 26, students bring together several ideas from the unit, using factoring to solve applied problems and to find key features of graphs. Lesson 27 occurs after administering the End-of-Unit 7 Assessment and includes post-assessment activities.

Instructional Routines

Aspects of Mathematical Modeling: Lessons 2, 14 & 15, 16

Card Sort: Lesson 11

Co-Craft Questions (MLR5): Lessons 4, 6, 13, 26

Collect and Display (MLR2): Lessons 1, 3, 4, 9, 10, 11, 12, 13, 17

Compare and Connect (MLR7): Lessons 3, 4, 6, 16, 18, 24, 26

Critique, Correct, Clarify (MLR3): Lessons 7, 18, 23

Discussion Supports (MLR8): Lessons 1, 2, 3, 4, 5, 6, 8, 11, 12, 17, 19, 20, 21, 22, 23, 24, 25

Graph It: Lessons 2, 5, 10, 11, 12, 13, 20, 24

Math Talk: Lessons 8, 19, 20, 23

Notice and Wonder: Lessons 1, 4, 22, 24, 25

Poll the Class: Lessons 3, 10, 17

Round Robin: Lesson 26

Stronger and Clearer Each Time (MLR1): Lessons 8, 20, 22, 27

Take Turns: Lessons 2, 7, 9, 17, 21

Three Reads: Lessons 16, 25

Which One Doesn't Belong?: Lessons 6, 9, 25

Lesson 1: A Different Kind of Change

PREPARATION

Lesson Goals	Learning Targets
• Describe (orally and in writing) a quadratic relationship when represented by a graph. • Given an interesting context, create drawings, tables, and graphs that represent a quadratic relationship.	• I can describe a pattern of change represented by a graph that is not linear or exponential. • I can create drawings, tables, and graphs that represent the area of a garden.

Lesson Narrative

In this lesson, students encounter a situation where a quantity increases and then decreases. They don't yet have a name for this new pattern of change, but they recognize that it is neither linear nor exponential, and that the graph is unlike the graph of an exponential function.

Students make sense of this new kind of relationship in a geometric context and describe it in concrete and qualitative ways (MP2). Though some students may choose to represent the relationships with calculations or with expressions, these are not required or emphasized in the lesson. Students will have many opportunities to reason symbolically about quadratic patterns in upcoming lessons.

Technology isn't required for this lesson, but there are opportunities for students to choose to use appropriate tools to solve problems, so consider making technology available.

In what ways might this lesson give students opportunities to surprise you with their thinking or reasoning?

Focus and Coherence

Building On	Addressing	Building Towards
NC.4.MD.3: Solve problems with area and perimeter. • Find areas of rectilinear figures with known side lengths. • Solve problems involving a fixed area and varying perimeters and a fixed perimeter and varying areas. • Apply the area and perimeter formulas for rectangles in real world and mathematical problems. **NC.M1.F-LE.1:** Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model for a situation based on the rate of change over equal intervals.	**NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.	**NC.M1.F-BF.1b:** Write a function that describes a relationship between two quantities. **b.** Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication.

Agenda, Materials, and Preparation

Technology is optional for this lesson: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
  - Graph paper
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L1 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

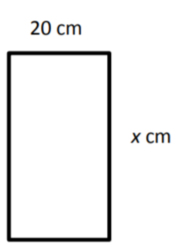
Building On: NC.4.MD.3

The purpose of this bridge is to revisit the concept of perimeter and area of a rectangle in preparation for Activity 1. Students determine the missing side length given a set perimeter and the length of one side. Next, they calculate the area of the rectangle.

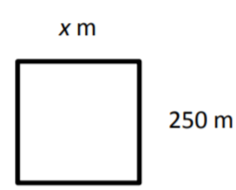
Student Task Statement

Given the rectangle's perimeter, find the unknown side length and the area.[1]

a. P = 120 cm



b. P = 1,000 m



DO THE MATH

PLANNING NOTES

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

Warm-up: Three Tables (*5 minutes*)

Instructional Routine: Notice and Wonder	
Building On: NC.M1.F-LE.1	

This warm-up encourages students to notice a new pattern of change (quadratic) by contrasting it to two familiar patterns (linear and exponential) through the *Notice and Wonder* routine. In the table showing a quadratic relationship, students are not expected to recognize how the input and output values are related. This prompt gives students opportunities to see and make use of structure (MP7). The specific structure they might notice is that the output values don't change by equal amounts or equal factors over equal intervals, and that the output values increase and then decrease.

Step 1

- Display the three tables.
- Ask students to think of things they notice and wonder.
- Give students 1 minute of think time.
- Give students 1 minute to discuss the things they notice and wonder with a partner.

Student Task Statement

Look at the patterns in the three tables. What do you notice? What do you wonder?

x	y
1	0
2	5
3	10
4	15
5	20

x	y
1	3
2	6
3	12
4	24
5	48

x	y
1	8
2	11
3	10
4	5
5	-4

Step 2

- Facilitate a whole-class discussion by asking students to share what they noticed and wondered.
- Record and display their responses for all to see.
- After all responses have been recorded without commentary or editing, ask students, "Is there anything on these lists that you are wondering about?" Encourage students to respectfully disagree, ask for clarification, point out contradicting information, etc.
- If no one mentioned it, ask students to consider the rule for the relationship in the third table or how the y-values are changing. Tell students that in this unit they will investigate relationships such as shown in the third table.

DO THE MATH	PLANNING NOTES

Activity 1: Measuring a Garden (*15 minutes*)

Instructional Routines: Collect and Display (MLR2); Discussion Supports (MLR8) - Responsive Strategy
Building Towards: NC.M1.F-BF.1b

This activity gives students a concrete experience with a quadratic relationship in a familiar geometric context. Given a rectangle with a fixed perimeter, students experiment with how changes to one side length of the rectangle affect its area. Along the way, they notice that as one length increases, the area does not continue to increase. Instead, at some point it begins to decrease.

Students will determine length and width pairs that satisfy the fixed perimeter constraint and calculate the area of each rectangle. Students are not expected to write an equation such as $A = l \cdot (25 - l)$ for the area of the rectangle. For this activity, informal observations on how the values are changing are sufficient. Students will have ample opportunities throughout this unit to examine the formal structure of quadratic relationships in depth.

As students work, encourage them to consider side lengths that are not whole numbers. Look for students who organize their work in different systematic ways and select them to share their work later.

Considering the relationship between the dimensions and area of a rectangle given a fixed perimeter requires students to reason abstractly and quantitatively (MP2). Making technology available gives students an opportunity to choose appropriate tools strategically (MP5).

Step 1

- Display a rectangle for all to see and label the sides with some lengths. Ask students to find the perimeter of the rectangle and the area of the region enclosed by this rectangle.
- Ask for the definitions of perimeter and area. Before beginning the activity, make sure students are clear about the distinction between the two measures—that the perimeter is the distance all the way around a region, and the area is the number of unit squares that cover a region without gaps or overlaps. Once students have reviewed the meanings for perimeter and area, give them 1–2 minutes to begin question 1 individually.

RESPONSIVE STRATEGIES

Provide students with graph paper and an example that illustrates how to use graph paper to determine the length and width.

Supports accessibility for: Conceptual processing

- Ask students to arrange themselves into groups of three or use visibly random grouping. Prompt groups to pool their initial responses for question 1 and add more if needed to recognize a pattern to help with question 2.
- Give students access to graph paper and tell students that they can use graph paper for the first question if they wish. Also provide access to calculators and/or technology such as Desmos. Some students may benefit from using them to work on the task.

Monitoring Tip: Look for students who organize the lengths, widths, and areas in a table or in other systematic ways. Let them know they may be asked to share later.

Advancing Student Thinking: Some students may exclude a rectangle with side lengths 12.5 and 12.5 from their diagrams of Noah's garden, possibly because they think a square is not a rectangle, or possibly because they only generate whole numbers. Emphasize that a square is a type of rectangle that happens to have four sides of equal length. Prompt them to think of the definition of a rectangle and explain why a square meets all the criteria for a rectangle.

Student Task Statement

Noah has 50 meters of fencing to completely enclose a rectangular garden in the backyard.

1. Draw some possible diagrams of Noah's garden. Label the length and width of each rectangle.
2. Find the length and width of such a rectangle that would produce the largest possible area. Explain or show why you think that pair of length and width gives the largest possible area.

Step 2

- Display the work of a student who organized lengths, widths, and areas in a table. If needed, ask students to reorganize the table with the length increasing to help see the patterns of change. If no students created a table, generate one as a class by asking groups to contribute at least a set of values. An example is shown here. (Keep posted as pairs of length and area values will be needed in the next activity.)

Length (meters)	Width (meters)	Area (square meters)
5	20	100
10	15	150
12	13	156
12.5	12.5	156.25
18	7	126
20	5	100
24	1	24

- Use the *Collect and Display* routine while facilitating a discussion about the following prompts:
  - “What do you notice about the relationship between the length and the width?” (As the length increases, the width decreases. The relationship is linear.)
  - “What do you notice about the relationship between the length and the area?” (As the length increases from 1 to 24, the area increases and then decreases. The relationship is not linear.)
  - Listen for, and scribe, the variety of ways that students express the following observations: the length and width add up to 25; each time the length increases by 1, the width decreases by 1; the relationship between the length and width is linear; as the length increases from 1 to 24, the area increases and then decreases; the relationship between the length and the area is not linear. Prompt students for these ideas if needed.

RESPONSIVE STRATEGY

Use this routine to support whole-class discussion. For each observation that is shared, ask students to restate what they heard using precise mathematical language. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others.

Discussion Supports (MLR8)

- Ask students to describe the rectangle they found to have the largest area and how they went about finding it. It is likely that many students will say that it has side lengths of 12 and 13 since these are the whole-number values that produce the greatest area. If no students tried 12.5 and 12.5, ask them to compute this area.
- Solicit some ideas from students on how the area is related to the length. Allow students to make some hypotheses for this set of questions, but it's not necessary to confirm one way or another. Ask questions such as:
  - "How do we know if 12.5 and 12.5 would indeed produce the greatest area?"
  - "Do you think going from 5 to 8 meters in length would produce a rectangle with a greater area? What about going from 15 to 18? Why or why not?"
- Tell students that we'll now try to get a better idea of what's happening between the side lengths and the area of the rectangle by plotting some points.

DO THE MATH	PLANNING NOTES

Activity 2: Plotting the Measurements of the Garden (*10 minutes*)

Building Towards: NC.M1.F-IF.4

In this activity, which builds from Activity 1, students plot the points that represent the relationship between a side length and the area of a rectangle with a perimeter of 50 meters. They encounter a graph where as one quantity increases, a second quantity increases and then decreases. Making sense of the graph in context helps them see why it is reasonable to expect the second quantity to decrease after a certain point.

Making graphing technology available gives students an opportunity to choose appropriate tools strategically (MP5).

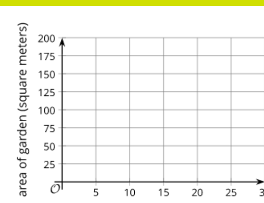
Step 1

- Keep a table from the previous activity displayed. If students created their own table, encourage them to use the values in their table.

Monitoring Tip: As students work, look for those whose graphs include enough points to hint at a quadratic shape. Let them know they may be asked to share later.

Student Task Statement

1. Plot some values for the length and area of the garden on the coordinate plane.
2. What do you notice about the plotted points?
3. The points $(3, 66)$ and $(22, 66)$ each represent the length and area of the garden. Plot these two points on coordinate plane, if you haven't already done so. What do these points mean in this situation?



Are You Ready For More?

1. Find a few other pairs of points representing (length, area), like $(3, 66)$ and $(22, 66)$, that have different x-coordinates but the same y-coordinate.
2. What do you notice about all these pairs of points? How would you explain to a friend how to find more?

Step 2

- Select a student to present their graph, or use Desmos to display a graph with some points already plotted and amend it with additional points students provide. Refer back to any relevant student language collected and displayed during Activity 1 while interpreting the graph.
- Discuss with students:
  - "Is the graph linear? Is it exponential?" (It is neither.)
  - "If we plot a bunch more points—say for every whole-number length between 0 and 25—what do you think the graph would look like?" (Possible predictions: A stretched out upside-down U, or an arch. A line with a positive slope that then turns into one with a negative slope. A curve. Some students might even recall the term "parabola.")
  - Which whole-number lengths between 0 and 25 are reasonable in this situation? (Not 0 or 25, because then either the length or width is 0; the other whole-number lengths are possible, though, for instance, a length of 1 meter and width of 24 meters may not be desirable.)
  - "For what lengths is the area increasing? Decreasing?" (The area increases when the length is between 0 m and 12.5 m and decreases between 12.5 m and 25 m.)
  - "What appears to be the maximum and what does it mean in this situation?" (When the length is 12.5 m is when there is the largest possible area.)
- Tell students that this unit will focus on functions like that relating the length and area of the garden. The output of the function may both increase and decrease, so we know they are neither linear nor exponential.

RESPONSIVE STRATEGY

Provide a graphic organizer that focuses on increasing the length of on-task orientation in the face of distractions. For example, during the whole-class discussion create a graphic organizer that includes the questions that will be asked in one column and a blank space to record the answers in the other column.

Supports accessibility for: Attention; Social-emotional skills

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to recognize how the relationship between the side lengths and the area of a rectangle differs from other relationships they've seen. It is not essential that students frame their observations in precise ways at this point. Their capacity to do so will be developed in the coming lessons.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

Consider asking students to record their observations about:

- The values in the table relating the length and the area of the rectangle. (As the length increased, the area increased, reached a maximum, and then decreased.)
- The graph representing the length-area relationship. (The graph looks like an upside-down U.)

PLANNING NOTES

Student Lesson Summary and Glossary

In this lesson, we looked at the relationship between the side length and the area of a rectangle when the perimeter is unchanged.

If a rectangle has a perimeter of 40 inches, we can represent possible lengths and widths as shown in the table.

- We could also consider lengths and widths that are decimal values, such as 6.5 inches and 13.5 inches. For the purpose of looking at the relationship between length, width, and area, we have chosen to look at the whole-number lengths and widths.
- We know that twice the length and twice the width must equal 40, which means that the length plus width must equal 20.

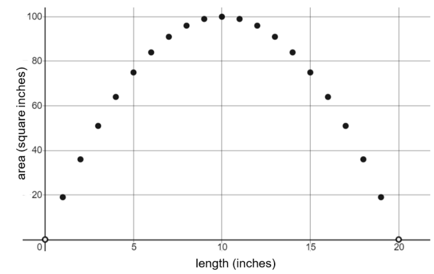
What about the relationship between the side lengths and the area of rectangles with a perimeter of 40 inches?

- Here are the areas of some different rectangles whose perimeter are 40 inches.

Length (inches)	Width (inches)	Area (square inches)
1	19	19
2	18	36
3	17	51
4	16	64
5	15	75
6	14	84
7	13	91
8	12	96
9	11	99
10	10	100

Length (inches)	Width (inches)	Area (square inches)
11	9	99
12	8	96
13	7	91
14	6	84
15	5	75
16	4	64
17	3	51
18	2	36
19	1	19

- Here is a graph of the lengths and areas represented in the table:



A few things to notice about the relationship shown in the table and the graph:

- The length cannot be 0 inches because a rectangle cannot have zero length. The length also cannot be 20 inches because that would make the width of the rectangle zero. (These points have been plotted with an open circle.) The length must be more than 0 inches and less than 20 inches.
- Initially, as the length of the rectangle increases from 0 inches to 10 inches, the area also increases. Later, however, as the length increases from 10 inches to 20 inches, the area decreases.
- The highest point on the graph is (10,100). This means the maximum area of the rectangle occurs when the length of the rectangle is 10 inches and the area is 100 square inches.

We have not studied relationships like this yet and will investigate them further in this unit.

Cool-down: 100 Meters of Fencing (*5 minutes*)

Building Towards: NC.M1.F-BF.1b

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

A rectangular yard is enclosed by 100 meters of fencing. The table shows some possible values for the length and width of the yard.

1. Complete the table with the missing values.
2. If the values for length and area are plotted, what would the graph look like?
3. How is the relationship between the length and the area of the rectangle different from other kinds of relationships we've seen before?

Length (meters)	Width (meters)	Area (square meters)
10	40	400
20	30	
25	25	625
35	15	525
40		

Student Reflection:

Consider what you learned today in this lesson. What is one thing you already knew, and what is one thing you look forward to learning more about in upcoming lessons?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

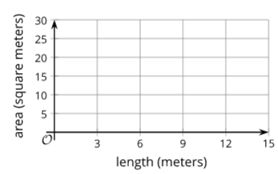
Identify ways the math community you are working to foster is going well. What aspects would you like to work on? What actions can you take to improve those areas?

Practice Problems

1. Here are a few pairs of positive numbers whose sums are 50.
    a. Find the product of each pair of numbers.
    b. Find a pair of numbers that has a sum of 50 and will produce the largest possible product.
    c. Explain how you determined which pair of numbers has the largest product.

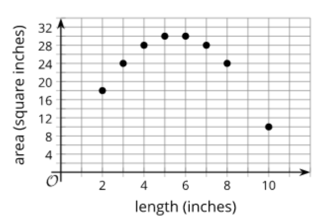
First number	Second number	Product
1	49	
2	48	
10	40	

2. Here are some lengths and widths of several rectangles whose perimeters are 20 meters.
    a. Complete the table. What do you notice about the areas?
    b. Without calculating, predict whether the area of the rectangle will be greater or less than 25 square meters if the length is 5.25 meters.
    c. On the coordinate plane, plot the points for length and area from your table.
    d. Do the values change in a linear way? Do they change in an exponential way?

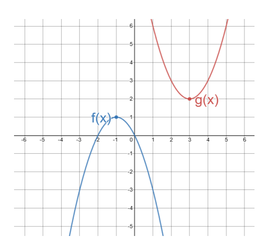


Length (meters)	Width (meters)	Area (square meters)
1	9	
3	7	
5		
7		
9		

3. Which statement best describes the relationship between a rectangle's side length and area as represented by the graph?
    a. As the side length increases by 1, the area increases and then decreases by an equal amount.
    b. As the side length increases by 1, the area increases and then decreases by an equal factor.
    c. As the side length increases by 1, the area does not increase or decrease by an equal amount.
    d. As the side length increases by 1, the area does not change.



4. The graph shows two functions, $f(x)$ and $g(x)$. For each function:
    a. On what interval is the value increasing for each function?
    b. On what interval is the value decreasing for each function?



5. Copies of a book are arranged in a stack. Each copy of a book is 2.1 cm thick.

   a. Complete the table.

   b. What do you notice about the differences in the height of the stack of books when a new copy of the book is added?

   c. What do you notice about the factor by which the height of the stack of books changes when a new copy is added?

   d. How high is a stack of b books?

   (From Unit 6)

Copies of book	Stack height in cm
0	
1	
2	
3	
4	

6. The value of a phone when it was purchased was \$500. It loses $\frac{1}{5}$ of its value a year.

   a. What is the value of the phone after 1 year? What about after 2 years? 3 years?

   b. Tyler says that the value of the phone decreases by \$100 each year since $\frac{1}{5}$ of 500 is 100. Do you agree with Tyler? Explain your reasoning.

   (From Unit 6)

7. (*Technology required.*) The data in the table represent the price of one gallon of milk in different years.

   Use graphing technology to create a scatter plot of the data.

   a. Does a linear model seem appropriate for the data? Why or why not?

   b. If the linear model seems appropriate, create the line of best fit. Round to two decimal places.

   c. What is the slope of the line of best fit, and what does it mean in this context? Is it realistic?

   d. What is the y-intercept of the line of best fit, and what does it mean in this context? Is it realistic?

   (From Unit 4)

x, time (years)	Price per gallon of milk (dollars)
1930	0.26
1935	0.47
1940	0.52
1940	0.50
1945	0.63
1950	0.83
1955	0.93
1960	1.00
1965	1.05
1970	1.32
1970	1.25
1975	1.57
1985	2.20
1995	2.50
2005	3.20
2018	2.90
2018	3.25

8. Give a value for r, the correlation coefficient, that indicates that a line of best fit has a negative slope and the scatterplot shows a strong linear relationship.

   (From Unit 4)

9. Match each inequality to the graph of its solution.

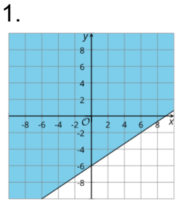
   a. $3x + 4y \leq 36$

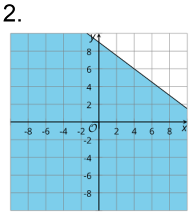
   b. $12x + 3y \leq 36$

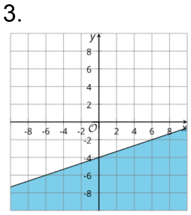
   c. $6x + 4y \geq 36$

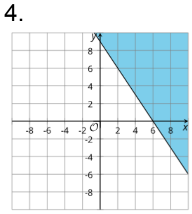
   d. $3x - 9y \geq 36$

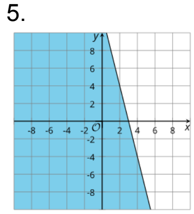
   e. $4x - 6y \leq 36$

   1. 

   2. 

   3. 

   4. 

   5. 

   (From Unit 3)

Lesson 2: Expressing Revenue

PREPARATION

Lesson Goals	Learning Targets
• Know that a quadratic relationship can be expressed as $ax^2 + bx + c$ when $a \neq 0$. • Write and graph expressions that define quadratic functions.	• I can identify a quadratic expression. • I can write and graph a quadratic function to model economic situations.

Lesson Narrative

In this lesson, students encounter a quadratic relationship in a business context. They explore multiple representations of the relationship between price and revenue. Students begin by creating a table of values. After calculating several related values, students look for and express repeated reasoning to build a quadratic expression (MP8).

In the second activity, students recognize the equivalency of $x(180 - x)$ and $180x - x^2$ and are introduced to the term **quadratic expression**. They use the expression to define a **quadratic function**. Students use technology to graph the function and identify key features relevant to the context. In building a quadratic function to solve a pricing problem, students engage in aspects of modeling (MP4).

What do you hope to learn about your students during this lesson?

Focus and Coherence

Building On	Addressing	Building Towards
NC.7.EE.1: Apply properties of operations as strategies to: • Add, subtract, and expand linear expressions with rational coefficients. • Factor linear expression with an integer GCF.	**NC.M1.A-SSE.1b:** Interpret expressions that represent a quantity in terms of its context. **b.** Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression. **NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities. **NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation. (*continued*)	**NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

	NC.M1.F-IF.4: Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums. **NC.M1.F-BF.1b:** Write a function that describes a relationship between two quantities. **b.** Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication.	

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
  - Graphing technology is required. Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L2 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.7.EE.1

In this bridge, students revisit using the distributive property to write equivalent expressions. This task is aligned to Check Your Readiness question 2.

Student Task Statement

1. Select all expressions that are equivalent to $10-2x$.

   $2(5-x)$ $\quad$ $10(1-2x)$ $\quad$ $-2(5-x)$ $\quad$ $-2(x-5)$

2. Write an equivalent expression for one of the expressions above that is not equivalent to $10-2x$.

DO THE MATH	PLANNING NOTES

Warm-up: A Flying Arrow (*5 minutes*)

Addressing: NC.M1.F-IF.4

The purpose of this warm-up is to revisit interpreting key features of graphs from Unit 5. Students are not expected to know that the graph is of a quadratic equation.

Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.
- Give students 1 minute of individual think time and then ask them to discuss with their partner.

Student Task Statement

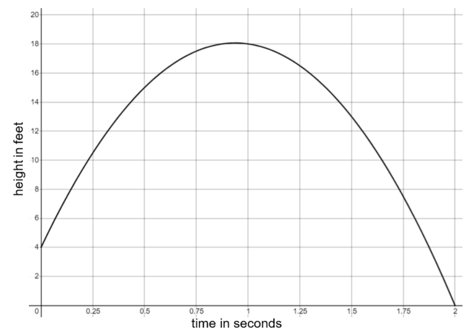
Clare was learning how to use a bow and arrow. The graph shows the height of the arrow as a function of the time since she released it.

Use the graph to identify each of the following. Be prepared to share your reasoning.

a. At what height did Clare release the arrow?

b. What was the maximum height of the arrow?

c. When did the arrow hit the ground?



Step 2

- Display the graph for all to see.
- Ask students to share their responses. As students share their reasoning, annotate the graph to show where the answers to the questions can be found.

DO THE MATH

PLANNING NOTES

Activity 1: What Price to Charge? *(15 minutes)*

Instructional Routines: Take Turns; Discussion Supports (MLR8) - Responsive Strategy	
Addressing: NC.M1.A-SSE.1b; NC.M1.A-CED.2; NC.M1.A-REI.10; NC.M1.F-BF.1b	

In this activity, students encounter a quadratic relationship in an economic context. They study the relationship between the price of downloading a movie and the number of downloads, and they see that the relationship can be described with a quadratic expression. Students use the expression to write a quadratic equation.

Students practice looking for and expressing regularity in repeated reasoning (MP8) as they calculate the number of downloads and the expected revenue at various prices.

Step 1

- Have students remain with their partner from the previous activity.
- Ask students to read the opening paragraph of the activity statement. Then, ask students to make some predictions:
  - "Suppose the price per download increases. What do you think would happen to the number of downloads as the price goes up?" (Students are likely to predict that as the price increases, the number of downloads decreases, as fewer people are willing to pay the higher price.)
  - "What happens to the number of downloads as the price decreases?" (As the price decreases, the number of downloads increases. Lower prices tend to encourage more people to purchase a product.)
  - "Would a business make more money when it sells a product at a higher price or when it sells at a lower price?" (Allow students to make some hypotheses, but it's not necessary to confirm one way or another.)

RESPONSIVE STRATEGY

Read the opening task statement and questions aloud. Students who both listen to and read the information will benefit from extra processing time.

Supports accessibility for: Language; Conceptual processing

Explain that price and the number of sales affect the revenue of a business, and that the term "revenue" means the money collected when someone sells something. For example, if the price of a movie download is \$3 and there are 10 downloads, the revenue is \$30. Tell students to *Take Turns* with their partner to fill in the numerical rows of the table and then work together to answer questions 2 and 3.

Monitoring Tip: Most students will write the equation as $r = x(18 - x)$. Identify any students that write the equation as $r = 18x - x^2$. Tell them they may be asked to share their equation during the class discussion.

Advancing Student Thinking: If students struggle with writing the equation, ask them to describe how to calculate the revenue. They might also benefit from writing each calculation as an expression such as $3(18 - 3)$ until they recognize the repeated reasoning.

Student Task Statement

A company that sells movies online is deciding how much to charge customers to download a new movie.[1] Based on data from previous sales, the company predicts that if they charge x dollars for each download, then the number of downloads, in thousands, is $18 - x$.

1. Complete the table to show the predicted number of downloads at each listed price. Then, find the revenue at each price. The first row has been completed for you.
2. Write an equation for the revenue, r, as a function of the price per download, x.
3. Verify that $(14, 56)$ is a solution to the equation. Show your work and explain what this tells us about the situation.

Price (dollars per download)	Number of downloads (thousands)	Revenue (thousands of dollars)
3	15	45
5		
10		
12		
15		
18		
x		

Step 2

RESPONSIVE STRATEGY

Use this routine to support whole-class discussion of the prompts listed in Step 2. After a student shares their thinking with the class, provide the class with the following sentence frames to help them respond: "I agree because…." or "I disagree because…." If necessary, revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.

Discussion Supports (MLR8)

- Display the completed table for all to see. Ask students to share their equations. Include any previously selected students.
- Facilitate a discussion that focuses on the structure and interpretation of the parts of the equation.
  - "In the equation $r = x(18 - x)$ what does the x represent? What does the $(18 - x)$ represent?" (The x is the price of the download in dollars, and $18 - x$ is the number of thousands of downloads if the price is x dollars.)
  - "How is the revenue calculated given the price and number of downloads?" (You multiply the price and the number of downloads.)
  - "How does the expression $x(18 - x)$ indicate multiplication?" (Writing the x beside the parentheses that includes $18 - x$ is how you can show multiplication of x and $18 - x$.)
- Discuss the equivalency of the two equations.
  - If a student wrote the equation as $r = 18x - x^2$, ask students, "Is the expression $18x - x^2$ equivalent to $x(18 - x)$?"
  - If a student did not write the equation as $r = 18x - x^2$, ask students, "How can the expression $x(18 - x)$ be written differently?"
  - Give students individual think time and ask them to discuss with a partner before discussing as a whole class.
  - Ask, "What is different about the expression $18x - x^2$ than what we have seen in linear and exponential expressions?" (It has a squared term.)
- Tell students the expression $18x - x^2$ is an example of a quadratic expression. A quadratic expression is an expression that can be written in the form of $ax^2 + bx + c$ where $a \neq 0$. Ask, "How can $18x - x^2$ be written in this form?" $(-x^2 + 18x)$

[1] Adapted from IM 9–12 Math Algebra 1, Unit 6, Lesson 7 https://curriculum.illustrativemathematics.org/HS/teachers/index.html,

- Tell students they will investigate these expressions in more depth in future lessons. For now, they will continue exploring the relationship between price and revenue.
- A quadratic expression can be used to write the equation that defines a quadratic function.
- Tell students they will now graph the equation defined by the quadratic expression and explore what information the graph reveals about the situation.

Activity 2: Analyzing Price and Revenue (*10 minutes*)

Instructional Routines: Graph It; Aspects of Mathematical Modeling		
Building On: NC.M1.F-IF.4	**Addressing:** NC.M1.A-CED.2	**Building Towards:** NC.M1.F-IF.7

In Unit 5, students interpreted key features of graphs to describe functions that arise in applications. Those key features included intercepts and maximum or minimum values. In this *Graph It* activity that continues from Activity 1, students analyze a quadratic function by graphing the function using technology. They identify key features and use them to answer questions regarding the situation.

Students engage in *Aspects of Mathematical Modeling* as they create a graph to further understand the relationship and use their model to make a business recommendation (MP4).

Step 1

- Keep students in pairs.
- Give students access to graphing technology such as Desmos.

Advancing Student Thinking: Students may need to experiment with the graphing window. Remind students they can use the wrench icon to manually adjust the settings. Encourage them to use the table to help determine minimum and maximum values for each axis.

Student Task Statement[2]

1. Graph your equation for revenue based on the price for a download using graphing technology.
2. Identify the horizontal intercepts and explain what they mean in this situation.
3. What price would you recommend the company charge for a new movie? Explain your reasoning.

Are You Ready For More?

A function that predicts how much of a product will sell given its price is called a "demand function." An example is the function that uses the price (in dollars per download), x, to determine the number of downloads (in thousands), $18 - x$. Economists are interested in factors that can affect the demand function and therefore the price suppliers wish to set.[3]

1. What are some things that could increase the number of downloads predicted for a given price?
2. If the demand shifted so that we predicted $20 - x$ thousand downloads at a price of x dollars per download, what do you think will happen to the price that gives the maximum revenue? Check what actually happens.

[2] Adapted from IM 9–12 Math Algebra 1, Unit 6, Lesson 7 https://curriculum.illustrativemathematics.org/HS/teachers/index.html,
[3] Adapted from IM 9–12 Math Algebra 1, Unit 6, Lesson 7 https://curriculum.illustrativemathematics.org/HS/teachers/index.html,

Step 2

- Display the graph for all to see.
- Ask students to share their responses to problems 2 and 3. Discuss questions such as:
  - "Why would the revenue be \$0 when the price per download is \$18?" (The price is too high, and no one will download the movie.)
  - "Is it possible for the company to lose money?" (It depends. If it costs the company money to buy the rights to the movie from the producer, then not collecting revenue could be seen as losing money.)
  - "The company needs a revenue of at least 40 thousand dollars. They are also interested in having as many downloads as possible. How might we use the graph to help determine the price per download?" (Trace the graph for where the revenue is 40, or graph the horizontal line $y = 40$ and find the x-value where the line and the graph of the quadratic first intersect. The price would be approximately \$2.60.)

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to explore the different representations of a quadratic relationship. In this lesson, students generated a table of values, wrote a quadratic expression which defined the quadratic function, and graphed the function. In this debrief, help students consolidate the ideas of the lesson.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- "How do you know if the relationship between two quantities represents a quadratic function?" (The expression has a squared term. The outputs of the function will increase and then decrease. The graph is shaped like a U.)
- "How is a quadratic relationship different than linear or exponential?" (A quadratic relationship will change direction. A linear or an exponential relationship is either always increasing or always decreasing.)
- "Is $x(3x + 2)$ a quadratic expression? Explain why or why not." (Yes, the expression can be written as $3x^2 + 6x$.)

PLANNING NOTES

Student Lesson Summary and Glossary

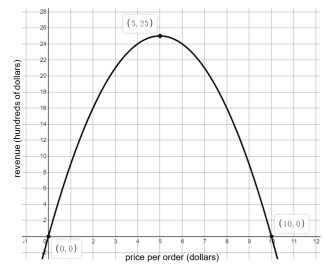
In this lesson, we explored different representations of a quadratic function.

A company is deciding how much to charge customers per order for their meal delivery service. If they charge x dollars per order, then the number of orders, in hundreds, is predicted to be $10 - x$.

Here is a table of values for a sample of prices, orders, and revenue (amount of money earned):

Price (dollars per order)	Number of orders (hundreds)	Revenue (hundreds of dollars)
0	10	0
1	9	9
2	8	16
3	7	21
4	6	24
5	5	25
6	4	24
7	3	21
8	2	16
9	1	9
10	0	0

- We multiply the price per order (x) and the number of orders $(10 - x)$ to calculate the revenue. This gives us the expression $x(10 - x)$, which is equivalent to $10x - x^2$. There is a squared term, so $10x - x^2$ is a quadratic expression.
- The expression can be used to create an equation that gives the revenue, r, as a function of the price, x. That equation is $r = 10x - x^2$.
- The graph of the equation is shown to the right.
- Using the graph, we can see that the horizontal intercepts are (0,0) and (10,0). This means that if the price per order is \$0 or \$10, the company will make \$0 in revenue. We can also see the maximum point (5,25). This means that for a price per order of \$5, the maximum revenue is 25 hundred dollars.



Quadratic expression: An expression that can be written in the form $ax^2 + bx + c$ when $a \neq 0$.

Quadratic function: A function where the output is given by a quadratic expression in the input.

Cool-down: Identifying the Quadratic (*5 minutes*)

Addressing: NC.M1.A-CED.2; NC.M1.A-REI.10

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

A company charges x dollars for tickets to a concert. The number of tickets sold (in thousands) depends on the price of the ticket and can be expressed by $(32 - x)$.

1. Write an equation to represent the revenue as a function of the ticket price, x.
2. Is the point $(7, 175)$ on the graph of the equation? Explain why or why not.

Student Reflection:

As you continue through this unit, you will learn much more about quadratics. What do you know about quadratics after today that you will want to keep in mind?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

With which math ideas from today's lesson did students grapple most? Did this surprise you, or was this what you expected?

Practice Problems

1. Identify **all** of the quadratic expressions.

x^2	$3x+2$	$4x^2-8$	$3(1.25)^x$
2^x	$-5x^2+7x-1$	$x(x+4)$	$2x$

2. (*Technology required.*) Graph the equation $-x^2+8x$ using technology.
   a. Identify the horizontal intercepts.
   b. Identify the maximum point.

3. Write an expression equivalent to $x(15-x)$ and explain why it is a quadratic expression.

4. Here are a few pairs of positive numbers whose sums are 26.

First number	Second number	Product
2	24	
6	20	
10	16	

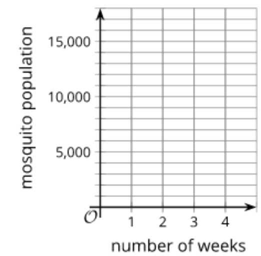
   a. Find the product of each pair of numbers.

   b. Find a pair of numbers that has a sum of 26 and will produce the largest possible product.

   c. Explain how you determined which pair of numbers has the largest product.

   (From Unit 7, Lesson 1)

5. A population of mosquitos p is modeled by the equation $p = 1,000 \cdot 2^w$, where w is the number of weeks after the population was first measured.



   a. Find and plot the mosquito population for $w = 0, 1, 2, 3, 4$.

   b. Where on the graph do you see the 1,000 from the equation for p?

   c. Where on the graph can you see the 2 from the equation?

   (From Unit 6)

6. The number of copies of a book sold the year it was released was 600,000. Each year after that, the number of copies sold decreased by $\frac{1}{2}$.

Years since published	Number of copies sold
0	
1	
2	
3	
y	

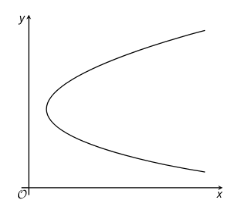
   a. Complete the table showing the number of copies of the book sold each year.

   b. Write an equation representing the number of copies, c, sold y years after the book was released.

   c. Use your equation to find c when $y = 6$. What does this mean in terms of the book?

   (From Unit 6)

7. Explain why this graph does not represent a function.

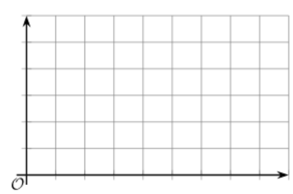


   (From Unit 5)

8. Function D gives the height of a drone t seconds after it lifts off.

   Sketch a possible graph for this function given that:

   - $D(3) = 4$
   - $D(10) = 0$
   - $D(5) > D(3)$



   (From Unit 5)

9. Solve this system of linear equations without graphing: $\begin{cases} 3y + 7 = 5x \\ 7x - 3y = 1 \end{cases}$

   (From Unit 3)

10. Select **all** the expressions equivalent to $80 - 20x$.

   a. $20(4 - 20x)$
   b. $20(4 - x)$
   c. $-4(-20 + 5x)$
   d. $-10(8 - 2x)$
   e. $16(5 - 4x)$

   (Addressing NC.7.EE.1)

Lesson 3: Comparing Quadratic and Exponential Functions

PREPARATION

Lesson Goal	Learning Target
● Use graphs, tables, and calculations to show that exponential functions eventually overtake quadratic functions.	● I can compare the rates of change for exponential functions to those for quadratic functions.

Lesson Narrative

In Unit 6, students compared linear and exponential growth and observed that exponential growth eventually overtakes linear growth (even if a quantity showing linear growth starts out with a much greater value). They examined this phenomenon and then observed that it will always happen at a large-enough input value.

In this lesson, students investigate how quantities that grow quadratically compare to those that grow exponentially. They discover and reason that increasing exponential functions also eventually surpass increasing quadratic functions. By examining successive quotients for each type of function, students see that the outputs of quadratic functions are not multiplied by the same factor each time the input increases by 1. In fact, these successive quotients get smaller as the inputs increase, while the outputs of the exponential function have the same multiplier. As they compare the two types of functions, they develop their understanding of quadratic expressions and how shape of the graph differs between the two types of functions.

What is the main purpose of this lesson? What is the one thing you want your students to take away from this lesson?

Focus and Coherence

Building On	Addressing
NC.6.EE.1: Write and evaluate numerical expressions involving whole-number exponents. **NC.6.EE.2:** Write, read, and evaluate algebraic expressions. ● Write expressions that record operations with numbers and with letters standing for numbers. ● Identify parts of an expression using mathematical terms and view one or more of those parts as a single entity. ● Evaluate expressions at specific values of their variables using expressions that arise from formulas used in real-world problems.	**NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior. **NC.M1.F-LE.3:** Compare the end behavior of linear, exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

Agenda, Materials, and Preparation

Technology is required for this lesson: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L3 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.6.EE.2

This bridge supports students in remembering the importance of the order of operations in evaluating quadratic and exponential expressions, preparing them to understand how exponential functions ultimately increase in value faster than quadratics. This task is aligned to Check Your Readiness question 3.

Student Task Statement

Different students are evaluating two expressions, $3 \cdot 6^x$ and 5^x. Analyze their work, describe any errors made in the "corrected work" column, and evaluate each expression correctly.

	Noah's work	Mai's work	Corrected work
Evaluate 5^x when x is 6.	5^x 5^6 30	5^x 5^6 $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$ $7,776$	
Evaluate $3 \cdot 6^x$ when x is 2.	$3 \cdot 6^x$ $3 \cdot 6^2$ $3 \cdot 12$ 36	$3 \cdot 6^x$ $3 \cdot 6^2$ 18^2 324	

DO THE MATH

PLANNING NOTES

Warm-up: From Least to Greatest (*5 minutes*)

Building On: NC.6.EE.1	**Building Toward:** NC.M1.F-LE.3

In this warm-up, students compare the values of exponential expressions by making use of their structure (MP7). The reasoning here prepares them to think about exponential growth later in the lesson.

Students should recognize that $9^2 < 10^2$ and $2^9 < 2^{10}$. Deciding whether 10^2 or 2^9 is greater requires some estimation or further reasoning using properties of exponents.

For example, some students may recognize that $2^4 = 16$ and $2^8 = 2^4 \cdot 2^4 = (2^4)^2$, so $2^8 = 16^2$, which is 256. Because 2^9 is greater than 2^8, it follows that 2^9 is greater than 256 and therefore greater than 10^2.

Note: Students should not use a calculator to evaluate the expressions in order to encourage them to rely on the structure of the expressions.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping. They will remain in these groups for the remainder of the lesson.
- After a few moments of quiet think time, ask students to share their thinking with a partner.

Monitoring Tip: As students discuss their thinking, listen for strategies that involve using properties of exponents or thinking about the structure of the expressions. If students are sharing how they computed each quantity, ask students, “Without evaluating each expression, how can you know how to order these?”

Student Task Statement

List these quantities in order, from least to greatest, without evaluating each expression. Be prepared to explain your reasoning.

a. 2^{10}

b. 10^2

c. 2^9

d. 9^2

Step 2

- Select students to share their responses and reasoning. Highlight explanations that:
  - show that the expressions can be compared by analyzing their structure (as noted in the Monitoring Tip)
  - show it is not necessary to know their exact values to put the expressions in order

DO THE MATH	PLANNING NOTES

Activity 1: Which One Grows Faster? (*10 minutes*)

Instructional Routines: Poll the Class; Compare and Connect (MLR7)	
Addressing: NC.M1.F-LE.3	

This activity prompts students to contrast quantities that grow exponentially and quadratically by writing equations and creating tables of values. Before students begin working, they estimate the number of squares in each pattern of figure 5 and figure 10. Making a reasonable estimate and comparing a computed value to one's estimate is often an important aspect of making sense of problems (MP1). Later, from the tables, students notice that the output of the exponential function eventually outgrows that of the quadratic function. In the next activity, they will think further about whether this is always the case.

Some students may choose to use graphing technology to study the pattern and to plot the data. Others may wish to use a calculator to compute growth factors. Provide access to Desmos. If students opt to use Desmos, they practice choosing appropriate tools strategically (MP5).

Step 1

- Display the image of the patterns for all to see and ask students to read the description of how the patterns grow. Ask students to predict which pattern will have more small squares in figure 5.
- Ask students to predict which pattern will have more small squares in figure 10.
- *Poll the Class* to collect their predictions for figure 10.
- Display the number of students who think pattern A will have more small squares and the number who think pattern B will have more small squares.

Step 2

- Suggest one partner completes the questions for pattern A and one partner completes the questions for pattern B. They can then work together to compare the patterns and make observations.

Advancing Student Thinking: Some students may write the equation for pattern B as $g(n) = 2n$. Point out that pattern B is *doubling* the number of small squares. Step 3 would have four small squares. Prompt students to test their equation when $n = 3$ to see if it gives the correct output. $g(3) = 2(3) = 6$ not 4, so a linear function does not work. Since pattern B is doubling, the function is exponential not linear. A linear pattern such as $2n$ would add two small squares at each step rather than double the number of small squares.

Student Task Statement

- In pattern A, the length and width of the rectangle grow by one small square from each figure to the next.
- In pattern B, the number of small squares doubles from each figure to the next.
- In each pattern, the number of small squares is a function of the figure number, n.

Pattern A

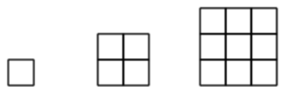


Figure 0 Figure 1 Figure 2 Figure 3

Pattern B

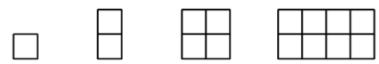


Figure 0 Figure 1 Figure 2 Figure 3

Pattern A

1. Write an equation to represent the number of small squares at figure n in pattern A.
2. Is the function linear, quadratic, or exponential?
3. Complete the table:

n, figure number	$f(n)$, number of small squares
0	
1	
2	
3	
4	
5	
6	
7	
8	

Pattern B

1. Write an equation to represent the number of small squares at figure n in pattern B.
2. Is the function linear, quadratic, or exponential?
3. Complete the table:

n, figure number	$g(n)$, number of small squares
0	
1	
2	
3	
4	
5	
6	
7	
8	

4. How would the two patterns compare if they continue to grow? Make one or two observations.

Step 3

- Facilitate a whole-class discussion. Select students to share their equations and to display their tables for all to see. Invite others to share their observations about the values in the tables.
  - Use the *Compare and Connect* routine as students share their equations and tables with the class by calling students' attention to the ways the dimensions are represented within each equation.
    - If students wrote different equations for one of the patterns, ask how the same pattern is represented by each equation.
    - Ask students to take a close look at the equations to distinguish how each pattern is represented. Amplify student words and actions that describe the connections between a specific feature of one mathematical representation and a specific feature of another representation.

RESPONSIVE STRATEGY

Demonstrate and encourage students to use color-coding and annotations to highlight connections between representations. Use one color to highlight connections between the growth factor and the graph of the quadratic function, and another color for the exponential function.

Supports accessibility for: Visual-spatial processing

– To help students understand why the value of the exponential function outgrows that of the quadratic function, consider showing tables that contrast the output values of f and g and amending each with a third column that shows their growth factors as n goes up by 1.

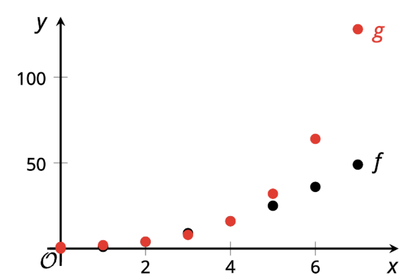
n, figure number	$f(n)$, number of squares	Growth factor (to 2 places)
0	0	
1	1	undefined
2	4	$\frac{4}{1} = 4$
3	9	$\frac{9}{4} = 2.25$
4	16	$\frac{16}{9} = 1.78$
5	25	$\frac{25}{16} = 1.56$
6	36	$\frac{35}{25} = 1.44$
7	49	$\frac{49}{36} = 1.36$
8	64	$\frac{64}{49} = 1.31$

n, figure number	$g(n)$, number of squares	Growth factor
0	1	
1	2	$\frac{2}{1} = 2$
2	4	$\frac{4}{2} = 2$
3	8	$\frac{8}{4} = 2$
4	16	$\frac{16}{8} = 2$
5	32	$\frac{32}{16} = 2$
6	64	$\frac{64}{32} = 2$
7	128	$\frac{128}{64} = 2$
8	256	$\frac{256}{128} = 2$

– Highlight the fact that a fundamental feature of an exponential function is that it changes by equal factors over equal intervals. In this exponential function, the output increases by a factor of 2 for each figure.

– In the quadratic function, the output changes by a factor of 4, then $2\frac{1}{4}$, then $1\frac{7}{9}$, and so on. Even though it started out growing faster than the exponential function, the growth factor of the quadratic function decreases for each figure and falls below 2 after a couple of figures. In the meantime, the growth factor of the exponential function stays at 2.

– Consider showing the graphs representing the two functions to help students visualize the data in the tables. This graph shows plots the outputs of f and g at whole-number inputs.



– Discuss how the graphs representing both quadratic and exponential functions curve upward. The two are very close together for small values of x. As x continues to grow, however, the values of g become much greater than those of f and continue to increase more quickly.

DO THE MATH

PLANNING NOTES

Activity 2: Comparing Two More Functions (*15 minutes*)

Instructional Routines: Collect and Display (MLR2); Discussion Supports (MLR8) - Responsive Strategy	
Addressing: NC.M1.F-LE.3; NC.M1.F-IF.7	

Students continue to compare quadratic and exponential functions in this activity. This time, they decide how to compare the functions.

If students choose graphing to perform comparisons, they practice choosing appropriate tools strategically (MP5).

Step 1

- Ask students to observe the equations representing the two functions and determine which function is exponential and which is quadratic. Invite students to share how they know. Make sure students recognize that p is quadratic and q is exponential before they begin the activity.
- Provide access to Desmos. This may be a good opportunity for students to experiment with the graphing window. If the horizontal dimension is very small (for example, $0 < x < 5$) or the vertical dimension is very large (for example, $0 < y < 3,000$), the two graphs will be hard to distinguish. As needed, remind students to think about adjusting the graphing window to make the graphs more informative.

RESPONSIVE STRATEGY

As students work with graphing technology, some may benefit from a checklist or list of steps to be able to adjust the graphing window to experiment with the horizontal and vertical dimensions.

Supports accessibility for: Organization; Conceptual processing; Attention

M **Monitoring Tip:** Monitor for different strategies students may use to compare the functions. Identify students who:

- create one or more tables of values and compare the values of $p(x)$ and $q(x)$ at increasingly large values of x
- graph the functions defined by $p(x) = 6x^2$ and $q(x) = 3^x$ and compare the graphs
- create one or more tables of values, calculate the growth factors at equal intervals of input, and then compare the growth factors

Student Task Statement

Here are two functions: $p(x) = 6x^2$ and $q(x) = 3^x$.

Investigate the output of p and q for different values of x. As the value of x increases, which function will eventually have a greater value?

Support your answer with tables, graphs, or other representations.

Are You Ready For More?

1. Jada says that some exponential functions grow more slowly than the quadratic function as x increases. Do you agree with Jada? Explain your reasoning.
2. Could you have an exponential function $g(x) = b^x$ and a quadratic function $f(x) = x^2$ so that $g(x) < f(x)$ for all values of x?

Step 2

- Facilitate a whole-class discussion by selecting students to present their strategies in the sequence listed in the Monitoring Tip. If no students chose to graph the functions, consider displaying the graphs for all to see. By presenting strategies that involve calculation of growth factors last, students will deepen their understanding of why exponential values will eventually overtake the quadratic values.
  - As students share their analysis with the class, use the *Collect and Display* routine by listening for and collecting the language students use to identify and describe how the output of the exponential function eventually outgrows that of the quadratic function.
    - Write the students' words and phrases on a visual display and update it throughout the remainder of the lesson.
    - Remind students to borrow language from the display as needed. This will help students read and use mathematical language during their partner and whole-group discussions.
  - Again, share that one way to make sense of why exponential functions eventually grow faster than quadratic functions is by thinking of the growth factors. The output of an exponential function always increases by the same factor when the input increases by 1 (for example, 3 for the exponential function q studied here). The output of a quadratic function, on the other hand, increases by smaller and smaller factors when the input increases by 1. So even though a quadratic function may take larger values than an exponential function for many inputs, the values of the exponential function will eventually overtake the quadratic.

RESPONSIVE STRATEGY

Provide the following sentence frame to use while facilitating the whole-class discussion: "As x increases in value, function ___ will eventually grow faster than function ___. I can see that in my table/graph/equation by looking at ___."

Discussion Supports (MLR8)

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to distinguish between the behaviors of exponential and quadratic functions. To gain this skill, students study and compare scenarios that can be modeled with each type of function.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

Ask students to reflect on how they analyzed the behaviors of quadratic and exponential functions. Discuss questions such as:

- "What are some ways of comparing quadratic growth and exponential growth?" (by comparing their values in a table, by graphing the equations representing them, or by comparing the growth factors as the input increases)
- "When we compared n^2 and 2^n, we saw the value of 2^n become greater than n^2 at $n = 5$. When we compared $6x^2$ and 3^x, we saw 3^x overtaking $6x^2$ by the time x reaches 5. If we compare, say, $1,000x^2$ and 2^x, will the exponential still overtake the quadratic? If so, at what x value do you think it would happen? If not, why not?" (Yes. It'd probably happen when x is between 15 and 20.)

PLANNING NOTES

Student Lesson Summary and Glossary

We have seen that the graphs of quadratic functions can curve upward. Graphs of exponential functions, with base larger than 1, also curve upward. To compare the two, let's look at the quadratic expression $3n^2$ and the exponential expression 2^n.

A table of values shows that $3n^2$ is initially greater than 2^n, but 2^n eventually becomes greater.

We also saw an explanation for why exponential growth eventually overtakes quadratic growth.

- When n increases by 1, the exponential expression 2^n always increases by a factor of 2.
- The quadratic expression $3n^2$ increases by different factors, depending on n, but these factors get smaller. For example, when n increases from 2 to 3, the factor is $\frac{27}{12}$ or 2.25. When n increases from 6 to 7, the factor is $\frac{147}{108}$ or about 1.36. As n increases to larger and larger values, $3n^2$ grows by a factor that gets closer and closer to 1.

A quantity that always doubles will eventually overtake a quantity growing by this smaller factor at each step.

n	$3n^2$	2^n
1	3	2
2	12	4
3	27	8
4	48	16
5	75	32
6	108	64
7	147	128
8	192	256

Cool-down: Comparing $5x^2$ and 2^x (*5 minutes*)

Addressing: NC.M1.F-LE.3

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

Tyler completes the table comparing values of the expressions $5x^2$ and 2^x.

x	$5x^2$	2^x
1	5	2
2	20	4
3	45	8
4	80	16

Tyler concludes that $5x^2$ will always take larger values than 2^x for the same value of x. Do you agree? Explain or show your reasoning.

Student Reflection:

Based on your participation and engagement, please reflect on one of the prompts below.

- I felt confident to engage in the work and participate today because…
- I did not feel confident to engage in the work and participate today because...

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Which students did you notice needed more guidance today? Which were more independent today? Were you surprised by any of the students?

Practice Problems

1. The table shows values of the expressions $10x^2$ and 2^x.

x	$10x^2$	2^x
1	10	2
2	40	4
3	90	8
4	160	16
8		
10		
12		

   a. Describe how the values of each expression change as x increases.

   b. Predict which expression will have a greater value when:

      i. x is 8

      ii. x is 10

      iii. x is 12

   c. Find the value of each expression when x is 8, 10, and 12.

   d. Make an observation about how the values of the two expressions change as x becomes greater and greater.

2. Function f is defined by $f(x) = 1.5^x$. Function g is defined by $g(x) = 500x^2 + 345x$.

   a. Which function is quadratic? Which one is exponential?

   b. The values of which function will eventually be greater for larger and larger values of x?

3. Create a table of values to show that the exponential expression $3(2)^x$ eventually overtakes the quadratic expression $3x^2 + 2x$.

x	$3(2)^x$	$3x^2+2x$

4. The table shows the values of 4^x and $100x^2$ for some values of x.

   Use the patterns in the table to explain why eventually the values of the exponential expression 4^x will overtake the values of the quadratic expression $100x^2$.

x	4^x	$100x^2$
1	4	100
2	16	400
3	64	900
4	256	1600
5	1024	2500

5. Here are some lengths, widths, and areas of a garden whose perimeter is 60 feet.

   a. What lengths and widths do you think will produce the largest possible area? Explain how you know.

   b. Complete the table with the missing measurements.

   (From Unit 7, Lesson 1)

Length (ft)	Width (ft)	Area (sq feet)
3	27	81
6	24	
9		
12		
15		225
18		
21		189

6. A bicycle costs $240, and it loses $\frac{3}{5}$ of its value each year.

    a. Write expressions for the value of the bicycle, in dollars, after 1, 2, and 3 years.

    b. When will the bike be worth less than $1?

    c. Will the value of the bike ever be 0? Explain your reasoning.

    (From Unit 6)

7. A farmer plants wheat and corn. It costs about $150 per acre to plant wheat and about $350 per acre to plant corn. The farmer plans to spend no more than $250,000 planting wheat and corn. The total area of corn and wheat that the farmer plans to plant is less than 1200 acres.

    This graph represents the inequality $150w + 350c \leq 250,000$, which describes the cost constraint in this situation. Let w represent the number of acres of wheat and c represent the number of acres of corn.

    a. The inequality $w + c < 1,200$ represents the total area constraint in this situation. On the same coordinate plane, graph the solution to this inequality.

    b. Use the graphs to find at least two possible combinations of the number of acres of wheat and the number of acres of corn that the farmer could plant.

    c. The combination of 400 acres of wheat and 700 acres of corn meets one constraint in the situation but not the other constraint. Which constraint does this meet? Explain your reasoning.

    (From Unit 3)

8. Jada is researching the price of vegetables at different grocery stores around her city. The mean prices per pound and standard deviation of the prices are shown in the table.

Store	Mean price per pound	Standard deviation
Veggies “R” Us	$3.56	$1.43
Veggiemart	$3.56	$0.89

    What conclusions can Jada draw from these data?

    (From Unit 4)

9. Solve the equation for x: $6x + 8y = 9$.

    (From Unit 2)

10. Evaluate each of the following expressions if $x = 3$.

    a. $4x^2$

    b. $(4x)^2$

    c. $2 \cdot 4^x$

    d. $4 \cdot 2^x$

    (Addressing NC.6.EE.1; NC.6.EE.2)

Lesson 4: Building Quadratic Functions to Describe Situations (Part One)

PREPARATION

Lesson Goals	Learning Targets
• Explain (orally and in writing) the meaning of each term in a quadratic expression that represents the height of a free-falling object. • Use tables, graphs, and equations to represent the height of a free-falling object. • Write quadratic functions to represent the height of an object falling due to gravity.	• I can explain the meaning of each term in a quadratic expression that represents the height of a free-falling object. • I can use tables, graphs and equations to represent the height of a free-falling object. • I can write quadratic functions to represent the height of an object falling due to gravity.

Lesson Narrative

This lesson begins a sequence of several lessons in which students construct quadratic functions to represent various situations. Here, they investigate the movement of free-falling objects. Students analyze the vertical distances that falling objects travel over time and see that they can be described by quadratic functions. They use tables, graphs, and equations to represent and make sense of the functions. In subsequent lessons, students build on the functions developed here to represent projectile motions, providing a context to develop understanding of the zeros, vertex, and domain of quadratic functions.

To express the relationship between distance and time, students need to see regularity in numerical values and express that regularity (MP8).

What math language will you want to support your students with in this lesson? How will you do that?

Focus and Coherence

Building On	Addressing
NC.6.EE.2: Write, read, and evaluate algebraic expressions. • Write expressions that record operations with numbers and with letters standing for numbers. • Identify parts of an expression using mathematical terms and view one or more of those parts as a single entity.	**NC.M1.A-SSE.1:** Interpret expressions that represent a quantity in terms of its context. **a.** Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents. **b.** Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression. (*continued*)

• Evaluate expressions at specific values of their variables using expressions that arise from formulas used in real-world problems.	**NC.M1.F-BF.1b:** Write a function that describes a relationship between two quantities. **b.** Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication. **NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*15 minutes*)
  - Geogebra applet "Galileo and Gravity": https://bit.ly/GalileoGravity
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L4 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional*, *5 minutes*)

Building On: NC.6.EE.2

This bridge will lead students into writing and evaluating quadratic functions by reviewing function notation in terms of a real-world linear function.

Student Task Statement

In the United States, weather temperatures are usually expressed using the Fahrenheit temperature scale. In many other countries, the weather temperatures are expressed using the Celsius temperature scale. The function $C(x) = \frac{5}{9}(x - 32)$ describes the relationship between the temperatures, where C represents the temperature in degrees Celsius and x represents the temperature in degrees Fahrenheit. If the high temperature in Charlotte averages 92° Fahrenheit in August, what is the average temperature in degrees Celsius?

DO THE MATH	PLANNING NOTES

Warm-up: An Interesting Numerical Pattern (*5 minutes*)

Instructional Routine: Notice and Wonder	
Building Towards: NC.M1.F-BF.1b	

The purpose of this warm-up is to elicit the idea that the values in the table have a predictable pattern, which will be useful when students consider the context of a falling object in a later activity. While students may *Notice and Wonder* many things about this table, the patterns are the important discussion points, rather than trying to find a rule for the function. Because the rule is not easy to uncover, studying the numbers ahead of time should prove helpful as students analyze the function later.

This prompt gives students opportunities to see and make use of structure (MP7). The specific structure they might notice is all the y values are multiples of 16 and perfect squares (with the pattern 0^2, 4^2, 8^2, $12^2 \ldots$). Some may notice the pattern is not linear and wonder whether it is quadratic.

Step 1

- Display the table and ask students what they notice and wonder. If time permits, have students share their noticings and wonderings with a partner before sharing with the full class.

Student Task Statement

Study the table. What do you notice? What do you wonder?

x	0	1	2	3	4	5
y	0	16	64	144	256	400

Step 2

- Invite students to share their observations and questions. Record and display them for all to see.
- After all responses are recorded, tell students that they will investigate these values more closely in upcoming activities.

DO THE MATH

PLANNING NOTES

Activity 1: Falling from the Sky (*10 minutes*)

Instructional Routines: Co-Craft Questions (MLR5); Discussion Supports (MLR8) - Responsive Strategy
Building Towards: NC.M1.F-BF.1b

The motion of a falling object is commonly modeled with a quadratic function. This activity prompts students to build a very simple quadratic model using given time-distance data of a free-falling rock. By reasoning repeatedly about the values in the data, students notice regularity in the relationship between time and the vertical distance the object travels, which they then generalize as an expression with a squared variable (MP8). The work here prepares students to make sense of more complex quadratic functions later (that is, to model the motion of an object that is launched up and then returns to the ground).

Step 1

- With student workbooks closed, display the image of the falling object from the task statement for all to see (without questions). Students should recognize the numbers from the warm-up. Invite students to make some other observations about the information. Ask questions such as:
  - "What do you think the numbers tell us?"
  - "Does the object fall the same distance every successive second? How do you know?"
- Using the *Co-Craft Questions* routine, display the description of the situation (without the questions):

  > A rock is dropped from the top floor of a 500-foot tall building. A camera captures the distance the rock traveled, in feet, after each second.

  Invite students to think of their own mathematical questions about the situation, then ask for volunteers to share their questions for the whole class. Listen for and amplify any questions involving the relationship between elapsed time and the distance that a falling object travels.

Step 2

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Tell students to think quietly about the first question and share their thinking with a partner.
- Consider pausing for a brief whole-class discussion before students proceed to the second question.

RESPONSIVE STRATEGY

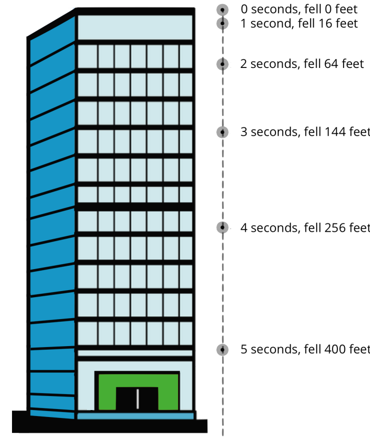
Provide the following sentence frame for students to use while sharing with their partner: "After 6 seconds, I think the rock will have fallen ___ feet because I noticed that . . . "

Discussion Supports (MLR8)

***Advancing Student Thinking*:** Some students may question why the distances are positive when the rock is falling. In earlier grades, negative numbers represented on a vertical number line may have been associated with an arrow pointing down. Emphasize that the values shown in the picture measure how far the rock fell and not the direction it is falling.

Student Task Statement

A rock is dropped from the top floor of a 500-foot tall building. A camera captures the distance the rock traveled, in feet, after each second.



1. How far will the rock have fallen after 6 seconds? Show your reasoning.

2. Jada noticed that the distances fallen are all multiples of 16. She wrote down:

$$\begin{aligned} 16 &= 16 \cdot 1 \\ 64 &= 16 \cdot 4 \\ 144 &= 16 \cdot 9 \\ 256 &= 16 \cdot 16 \\ 400 &= 16 \cdot 25 \end{aligned}$$

Then, she noticed that 1, 4, 9, 16, and 25 are 1^2, 2^2, 3^2, 4^2, and 5^2.

   a. Use Jada's observations to predict the distance fallen from an even taller building after 7 seconds. (Assume the building is tall enough that an object dropped from the top of it will continue falling for at least 7 seconds.) Show your reasoning.

   b. Write an equation for the function, with d representing the distance dropped after t seconds.

Step 3

- Discuss the equation students wrote for the last question. If not already mentioned by students, point out that the t^2 suggests a quadratic relationship between elapsed time and the distance that a falling object travels. Ask students:
  - "How do you know that the equation $d = 16t^2$ represents a function?" (For every input of time, there is a particular output.)
  - "Suppose we want to know if the rock will travel 600 feet before 6 seconds have elapsed. How can we find out?" (Find the value of d when t is 6, which is $16 \cdot 6^2$ or 576 feet. Students may also suggest finding the value of t when d is 600, which is possible but results in an equation that is more complicated to solve.)
- Explain to students that although there are only a few data points to go by in this case, and the quadratic expression $16t^2$ is a simplified model, quadratic functions are generally used to model the movement of falling objects. This expression will appear in other contexts where gravity affects the quantities being studied.

DO THE MATH	PLANNING NOTES

Activity 2: Galileo and Gravity (*15 minutes*)

Instructional Routines: Discussion Supports; Collect and Display (MLR2) - Responsive Strategy; Compare and Connect	
Building On: NC.M1.F-IF.2	**Addressing:** NC.M1.F-BF.1b; NC.M1.F-IF.7; NC.M1.A-SSE.1

In this activity, students continue to explore how quadratic functions can model the movement of a falling object. They evaluate the function seen earlier ($d = 16t^2$) at a non-integer input, and then build a new function to represent the distance from the ground of a falling object t seconds after it is dropped. To find a new expression that describes the height of the object, students reason repeatedly about the height of the object at different times and look for regularity in their reasoning (MP8).

The number 576 is chosen as the height (in feet) from which the object is dropped to make it more apparent for students that the values in the two tables (distance fallen and distance from ground) record distances measured from opposite ends. (Any value of $16t^2$ at a whole-number t could work. In this case $t = 6$ is selected.)

RESPONSIVE STRATEGY

Use this routine as students discuss their expressions with a partner to listen for and collect the language students use to identify and describe what is the same and what is different between Elena's and Diego's tables. Write the students' words and phrases on a visual display and update it with connections to the graphs introduced during the synthesis. Remind students to borrow language from the display as needed. This will help students read and use mathematical language during their paired and whole-group discussions.

Collect and Display (MLR2)

Step 1

- Suggest students remain in their pairs to work together to answer questions 1 and 2a and complete the tables. To facilitate peer discussion, consider using the *Discussion Supports* of displaying sentence stems or questions that students could use, such as:
  - "How are the two tables alike? How are they different?"
  - "What does 576 feet represent in this problem?"

Student Task Statement

Galileo Galilei, an Italian scientist, and other medieval scholars studied the motion of free-falling objects. The law they discovered can be expressed by the equation $d(t) = 16 \cdot t^2$, which gives the distance fallen in feet, d, as a function of time, t, in seconds.

An object is dropped from a height of 576 feet.

1. Evaluate $d(0.5)$ and explain what it means.
2. To find out where the object is after the first few seconds after it was dropped, Elena and Diego created different tables.
   a. How are the two tables alike? How are they different?
   b. Complete Elena's and Diego's tables. Be prepared to explain your reasoning.

Elena's table:

Time (seconds)	Distance fallen (feet)
0	0
1	16
2	64
3	
4	
t	

Diego's table:

Time (seconds)	Distance from the ground (feet)
0	576
1	560
2	512
3	
4	
t	

Step 2

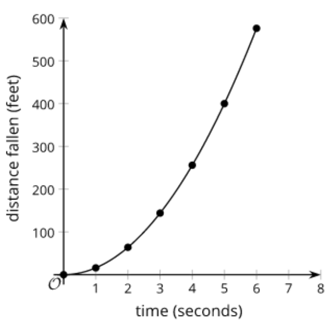
- To help students make sense of the two functions, display the tables and use the *Compare and Connect* routine to discuss their representations (tables, equations, and graphs). Ask questions such as:
  - “How did you complete the missing values in the first table?” (Substituting 3 and 4 for t in $16t^2$ gives the distances fallen after 3 and 4 seconds.)
  - “What about those in the second table?” (The distance from the ground is 576 minus the distance fallen, so we can use the values for $t = 3$ and $t = 4$ from the first table to calculate the values in the second table.)
  - “Why do the values in the first table increase and those in the other table decrease?” (The distance from the top of the building increases as the object falls farther and farther away. The distance from the ground decreases as the object falls closer and closer to it.)
  - “The expression representing the distance fallen shows $16t^2$, and the other shows $576 - 16t^2$. Why is that?” (In the first function, the distance fallen, measured from where the object is dropped, will always be positive. In the second function, what’s measured is the height from the ground, so the distance fallen needs to be subtracted from the height of the building.)
  - “If we graph the two equations that represent distance fallen and distance from the ground over time, what would the graphs look like? Try sketching the graphs.” (The first graph would increase, with a higher rate of change as time increases. The second graph would decrease at a faster rate over time until the object lands at 6 seconds.)

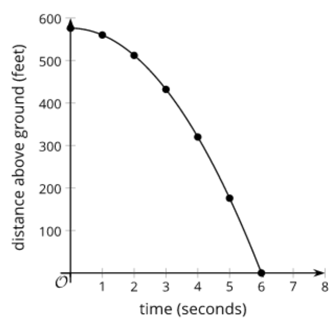
RESPONSIVE STRATEGIES

Use color-coding and annotations to highlight connections between representations in a problem. Use color-coding to illustrate how the values in each table correspond to the values in each graph. Some students may benefit from access to physical copies of the graphs that they can annotate for themselves.

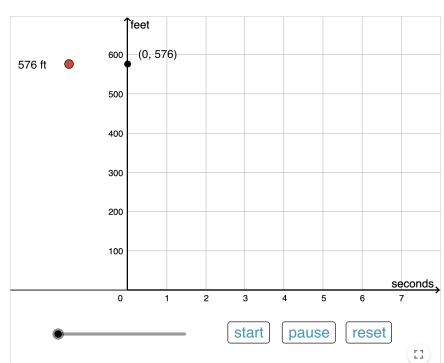
Supports accessibility for: Visual-spatial processing; Conceptual processing

- Display graphs that represent the two functions and make sure students can interpret them. For example, they should see that the y-intercept of each graph corresponds to the starting value of each function before the object is dropped.





- They should also notice that the difference in distance between successive seconds gets larger in both cases, hence the curving graphs. (If the differences were constant, the graphs would have been straight lines.)
- Display the Geogebra applet for all to see. Ask students how the graph of the height of the object is related to the path that the object takes as it falls. The Geogebra applet “Galileo and Gravity” is available here: https://bit.ly/GalileoGravity. (The height of the object decreases as the object falls.



DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to have students use tables, equations, and graphs to study the quadratic nature of free-falling objects. Students will build on this connection between free-falling objects and quadratic functions as they begin to explore projectile motion in the lessons that follow.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

To highlight the key ideas from this lesson and the connections to earlier lessons, discuss questions such as:

- "We used two different functions to describe the movement of a falling object. One function measured the distance the object traveled from its starting point, and the other measured its distance from the ground. How are the equations, tables, and graphs of these functions alike and different?" (The equations both have $16t^2$, but one is positive and the other is negative. Their graphs are both curves, but one graph curves upward and the other downward. One table shows increasing values and the other shows decreasing values, but they change by the same amounts from row to row.)
- "How can we tell that the 'distance fallen vs. time' relationship is not exponential just from looking at its table?" (When you divide successive values, the ratios get smaller.)
- "How are these functions like or unlike those representing the visual patterns in the previous lesson?" (They can all be represented by quadratic expressions. The relationships between the figure number and the number of squares were easier to see. The relationships between time and distance are not as obvious.)

PLANNING NOTES

Student Lesson Summary and Glossary

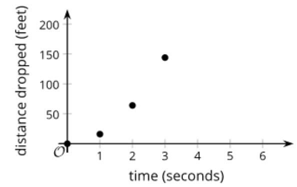
The distance traveled by a falling object in a given amount of time is an example of a quadratic function. Galileo is said to have dropped balls of different mass from the Leaning Tower of Pisa, which is about 190 feet tall, to show that they travel the same distance in the same time. In fact the equation $d(t) = 16t^2$ models the distance d, in feet, that a cannonball falls after t seconds, no matter what its mass.

Because $16 \cdot 4^2 = 256$, and the tower is only 190 feet tall, the cannonball hits the ground before 4 seconds.

Here is a table showing how far the cannonball has fallen over the first few seconds.

Time (seconds)	Distance fallen (feet)
0	0
1	16
2	64
3	144

Here are the time and distance pairs plotted on a coordinate plane:



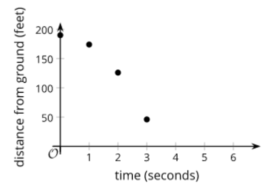
Notice that the distance fallen is increasing each second. The average rate of change is increasing each second, which means that the cannonball is speeding up over time. This comes from the influence of gravity, which is represented by the quadratic expression $16t^2$. It is the exponent 2 in that expression that makes it increase by larger and larger amounts.

Another way to study the change in the position of the cannonball is to look at its distance from the ground as a function of time.

Here is a table showing the distance from the ground in feet at 0, 1, 2, and 3 seconds.

Time (seconds)	Distance from the ground (feet)
0	190
1	174
2	126
3	46

Here are the time and distance pairs plotted on a graph:



The function rule that defines the distance from the ground as a function of time is $d(t) = 190 - 16t^2$. It tells us that the cannonball's distance from the ground is 190 feet before it is dropped and has decreased by $16t^2$ when t seconds have passed.

Cool-down: Where Will It Be? (*5 minutes*)

Addressing: NC.M1.F-BF.1b

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

The expression $16t^2$ represents the distance in feet an object falls after t seconds. The object is dropped from a height of 906 feet.

1. What is the height in feet of the object 2 seconds after it is dropped?
2. Write a function rule representing the height of the object in feet t seconds after it is dropped.

Student Reflection:

What was the most helpful method or strategy your teacher used in this lesson today?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

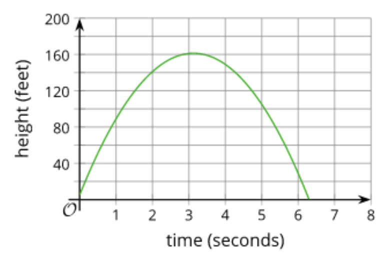
Think about a recent time from class when your students were confused. What did you do to support them in reasoning about their confusion together as a community of learners?

Practice Problems

1. A rocket is launched in the air and its height, in feet, is modeled by the function h. Here is a graph representing h.

   Select **all** true statements about the situation.

   a. The rocket is launched from a height less than 20 feet above the ground.
   b. The rocket is launched from about 20 feet above the ground.
   c. The rocket reaches its maximum height after about 3 seconds.
   d. The rocket reaches its maximum height after about 160 seconds.
   e. The maximum height of the rocket is about 160 feet.

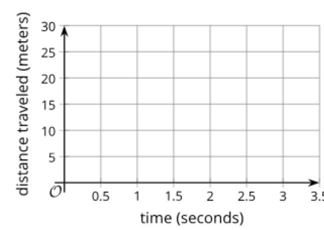


2. A baseball travels d meters t seconds after being dropped from the top of a building. The distance traveled by the baseball can be modeled by the equation $d = 5t^2$.

   a. Complete the table and plot the data on the coordinate plane.

   b. Is the baseball traveling at a constant speed? Explain how you know.

t **(seconds)**	d **(meters)**



3. A rock is dropped from a bridge over a river. Which table could represent the distance in feet fallen as a function of time in seconds?

Table A

Time (seconds)	**Distance fallen (feet)**
0	0
1	48
2	96
3	144

Table B

Time (seconds)	**Distance fallen (feet)**
0	0
1	16
2	64
3	144

Table C

Time (seconds)	**Distance fallen (feet)**
0	180
1	132
2	84
3	36

Table D

Time (seconds)	**Distance fallen (feet)**
0	180
1	164
2	116
3	36

4. A small ball is dropped from a tall building. Which equation could represent the ball's height, h, in feet, relative to the ground, as a function of time, t, in seconds?

   a. $h = 100 - 16t$
   b. $h = 100 - 16t^2$
   c. $h = 100 - 16^t$
   d. $h = 100 - \frac{16}{t}$

5. Determine whether $5n^2$ or 3^n will have the greater value when:

   a. $n = 1$

   b. $n = 3$

   c. $n = 5$

   (From Unit 7, Lesson 3)

6. Diego claimed that $10 + x^2$ is always greater than 2^x and used this table as evidence. Do you agree with Diego?

x	$10+x^2$	2^x
1	11	2
2	14	4
3	19	8
4	26	16

   (From Unit 7, Lesson 3)

7. The table shows the height, in centimeters, of the water in a swimming pool at different times since the pool started to be filled.

Minutes	Height
0	150
1	150.5
2	151
3	151.5

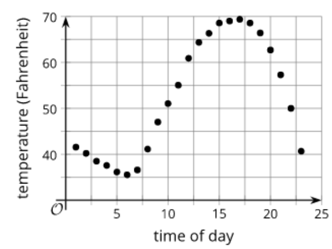
   a. Does the height of the water increase by the same amount each minute? Explain how you know.

   b. Does the height of the water increase by the same factor each minute? Explain how you know.

   (From Unit 6)

8. The temperature was recorded several times during the day. Function T gives the temperature in degree Fahrenheit, n hours since midnight.

   Here is a graph for this function.



   a. Pick two consecutive points and connect them with a line segment. Estimate the slope of that line. Explain what that estimated value means in this situation.

   b. Pick two non-consecutive points and connect them with a line segment. Estimate the slope of that line. Explain what that estimated value means in this situation.

   (From Unit 5)

9. (*Technology required.*) A study investigated the relationship between the amount of daily food waste measured in pounds and the number of people in a household. The table displays the results of the study.

Number of people in household, x	Food waste (pounds), y
2	3.4
3	2.5
4	8.9
4	4.7
4	3.5
4	4
5	5.3
5	4.6
5	7.8
6	3.2
8	12

   Use graphing technology to create the line of best fit for the data in the table.

   a. What is the equation of the line of best fit for the data? Round numbers to two decimal places.

   b. What is the slope of the line of best fit? What does it mean in this situation? Is this realistic?

   c. What is the y-intercept of the line of best fit? What does it mean in this situation? Is this realistic?

   (From Unit 4)

10. The function $F(c) = \frac{9}{5}C + 32$, where F represents degrees Fahrenheit and C represents degrees Celsius, gives temperature in degrees Fahrenheit based on the temperature in degrees Celsius. If the high temperature in Reykjavik, Iceland, on Christmas is $5°$ Celsius, what is the temperature in degrees Fahrenheit?

    (Addressing NC.6.EE.2)

Lesson 5: Building Quadratic Functions to Describe Situations (Part Two)

PREPARATION

Lesson Goals	Learning Targets
• Create graphs of quadratic functions that represent a physical phenomenon and determine an appropriate domain when graphing. • Identify and interpret (orally and in writing) the meaning of the vertex of a graph and the zeros of a function represented in tables and graphs. • Write and interpret (orally and in writing) quadratic functions that represent a physical phenomenon.	• I can create quadratic functions and graphs that represent a situation. • I can relate the vertex of a graph and the zeros of a function to a situation. • I know that the domain of a function can depend on the situation it represents.

Lesson Narrative

Previously, students used simple quadratic functions to describe how an object falls over time given the effect of gravity. In this lesson, they build on that understanding and construct quadratic functions to represent projectile motions. They learn that the graphs of these quadratic functions are called **parabolas**. In studying the key features of parabolas, they learn about the **zeros** of a function and the **vertex** of a graph. They also begin to consider appropriate domains and ranges for a function given the situation it represents.

Students use a linear model to describe the height of an object that is launched directly upward at a constant speed. Because of the influence of gravity, however, the object will not continue to travel at a constant rate (eventually it will stop going higher and will start falling), so the model will have to be adjusted (MP4). They notice that this phenomenon can be represented with a quadratic function, and that adding a squared term to the linear term seems to "bend" the graph and change its direction.

What about this topic brings you the most excitement or curiosity?

Focus and Coherence

Building On	Addressing
NC.8.F.5: Qualitatively analyze the functional relationship between two quantities. • Analyze a graph determining where the function is increasing or decreasing; linear or non-linear.	**NC.M1.A-SSE.1a:** Interpret expressions that represent a quantity in terms of its context. **a.** Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents. **NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities. (*continued*)

• Sketch a graph that exhibits the qualitative features of a real-world function. **NC.M1.F-BF.1a:** Write a function that describes a relationship between two quantities. **a.** Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table).	**NC.M1.F-BF.1b:** Write a function that describes a relationship between two quantities. **b.** Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication. **NC.M1.F-IF.5:** Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes. **NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Agenda, Materials, and Preparation

Technology is required for this lesson: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L5 Cool-down (print 1 copy per student)

LESSON

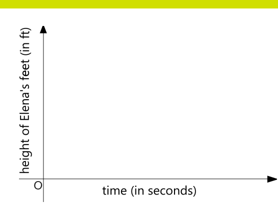
Bridge (*Optional, 5 minutes*)

Building On: NC.8.F.5

As students prepare to interpret the characteristics of quadratic functions in context, this bridge will relate to the graphical nature of qualitative functions, based on the meanings of the independent and dependent variables. It will also begin to relate time and height in problems involving gravity's pull on an object.

Student Task Statement

Elena jumps three times on a trampoline, with each jump going higher than the last! If the trampoline is 4 feet off the ground, draw a graph that could represent the height of Elena's feet after x seconds.



DO THE MATH

PLANNING NOTES

Warm-up: Sky Bound (*5 minutes*)

Building On: NC.M1.F-BF.1a

In this warm-up, students consider what happens if an object is launched up in the air unaffected by gravity. The work here serves two purposes. It reminds students that an object that travels at a constant speed can be described with a linear function. It also familiarizes students with a projectile context used in the next activity, in which students will investigate a quadratic function that more realistically models the movement of a projectile—with gravity in play.

Students who use a calculator to complete the table practice choosing tools strategically (MP5).

Step 1

- Ask a student to read the opening paragraph of the activity aloud. To help students visualize the situation described, consider displaying a picture of a cannon pointing straight up, 10 feet above ground. Ask students to consider what a speed of 406 feet per second means in more concrete terms. How fast is that?
- Students may be more familiar with miles per hour. Tell students that the speed of 406 feet per second is about 277 miles per hour.

Student Task Statement

A cannon is 10 feet off the ground. It launches a cannonball straight up with a velocity of 406 feet per second.

Imagine that there is no gravity and that the cannonball continues to travel upward with the same velocity.

1. Complete the table with the heights of the cannonball at different times.

Seconds	0	1	2	3	4	5	t
Distance above ground (feet)	10						

2. Write an equation to model the distance in feet, d, of the ball t seconds after it was fired from the cannon if there was no gravity.

Step 2

- Ask students how the pattern of change in the table relates to the equation that describes the height of the cannonball if there were no gravity. Even without graphing, students should notice that the height of the cannonball over time is a linear function given the repeated addition of 406 feet every time t increases by 1.
- Tell students that, in the next activity, they will look at some actual heights of the cannonball.

DO THE MATH

PLANNING NOTES

Activity 1: Tracking a Cannonball (*15 minutes*)

Instructional Routine: Discussion Supports (MLR8)
Addressing: NC.A-SSE.1a; NC.M1.F-BF.1b

Prior to this course, students learned that an object traveling at a constant speed can be described with a linear function whose graph is a straight line. Here, they see a model that accounts for the fact that an object that is launched straight up at a constant speed does not keep going at the same rate when the influence of gravity is taken into account. Adding a quadratic term to a linear function has an effect of "bending" the graph, as the output values are no longer changing at a constant rate.

To generalize the relationship between time and distance, students reason repeatedly with numerical values and look for regularity (MP8). If students opt to use spreadsheet or graphing technology, they practice choosing appropriate tools strategically (MP5).

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping. Students will remain in these groups for Activity 2.
- Give students a minute or two of quiet time to think about the first question, and then time to share their observations with their partner. Tell students that they will need to reference their work in the warm-up.
- Some students may choose to use a calculator to extend the pattern, and subsequently to use graphing technology to plot the data. Make Desmos accessible, if requested.

***Advancing Student Thinking*:** When comparing the tables, some students may make observations that lack the detail needed to write an equation for the actual height. Prompt them to rewrite the outputs for the actual height in terms of the hypothetical height ($400 = 416 - 16$, $758 = 822 - 64$, $1084 = 1228 - 144$, and so on). Show them values of $16t^2$ from a previous lesson to help them see and extend the pattern to write the equation.

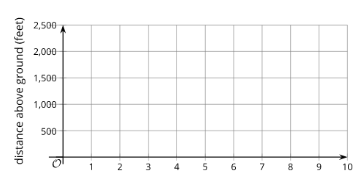
Student Task Statement

Earlier, you completed a table that represents the height of a cannonball, in feet, as a function of time, in seconds, if there was no gravity.

1. This table shows the actual heights of the ball at different times.

Seconds	0	1	2	3	4	5
Distance above ground (feet)	10	400	758	1,084	1,378	1,640

Compare the values in this table with those in the table you completed earlier. Make at least two observations.



a. Plot the two sets of data you have on the same coordinate plane.

b. How are the two graphs alike? How are they different?

2. Write an equation to model the actual distance d, in feet, of the ball t seconds after it was fired from the cannon. If you get stuck, consider the differences in distances and the effects of gravity from a previous lesson.

Step 2

- Facilitate a whole-class discussion by inviting students to share their observations about the two tables (the one from the warm-up and the one here) and how the two graphs compare. Highlight responses that suggest that the values in the second table account for the effect of gravity.

Use the following *Discussion Supports* to amplify mathematical uses of language to describe comparisons between the tables and graphs.

  – After students share an observation, invite them to repeat their reasoning using mathematical language relevant to the lesson (for example, "increasing at a decreasing rate" or "without gravity, the graph is linear because the rate of change is constant, and with gravity, the graph is quadratic because the vertical position is decreased by $16t^2$ due to gravity").

  – Consider inviting the remaining students to build on the reasoning of their classmates to provide additional opportunities for all students to produce language as they interpret the reasoning of others.

RESPONSIVE STRATEGY

Use color-coding and annotations to highlight connections between representations in a problem. For example, teachers may model for students how to color-code the values in the table along with the corresponding points on the graph or how to highlight the actual height values and the hypothetical data values in the graph using different colors to show the contrast in values.

Supports accessibility for: Visual-spatial processing

- Help students see how the output for each t value varies across the two tables. When t is 1, the output in feet in the second table is 16 less than in the first table. When t is 2, there is a difference of 64 feet. When t is 3, that difference is 144 feet, and so on. The values 16, 64, 144, . . . correspond to the expression $16t^2$ that we saw in the previous lesson (the distance fallen in feet as a function of time in seconds), so we can represent the values in the second table with the equation $d = 10 + 406t - 16t^2$. Ask students:
  – "What do the 10, $406t$, and $-16t^2$ mean in this situation?" (The 10 is the vertical position of the cannonball before it was launched: 10 feet above ground. In $406t$, the 406 tells us the vertical velocity at which it was shot up. The $-16t^2$ accounts for the effect of gravity on the height of the cannonball after it was shot up.)
  – "Why do you think the graph that represents $d = 10 + 406t$ changes from a straight line to a curve when $-16t^2$ is added to the equation?" (Before that term was added, the height increased by 406 feet every second. Adding $-16t^2$ decreases how much the cannonball travels up by some amount, but that amount gets larger each successive second. Eventually, the cannonball stops increasing in height and starts to fall.)

DO THE MATH	PLANNING NOTES

Activity 2: Graphing Another Cannonball (*10 minutes*)

Instructional Routines: Graph It; Discussion Supports (MLR8) - Responsive Strategy	
Addressing: NC.A-SSE.1a; NC.M1.A-CED.2; NC.M1.F-IF.5; NC.M1.F-IF.7	

In this *Graph It* activity, students explore another model of a projectile motion. They graph and interpret a quadratic function in context and begin considering a reasonable domain and range for the function. Along the way, they practice reasoning concretely and abstractly (MP2). By the end of the lesson, they relate the vertex of the graph to the maximum height of the cannonball and the positive zero of the function to the time when the cannonball hits the ground.

Step 1

- Provide access to devices that can run Desmos.
- Give students a few minutes of quiet work time and then a few minutes to share their thinking with their partner.

***Advancing Student Thinking*:**

- Students may approach the estimations in the third question in different ways (including by eyeballing). If desired, demonstrate how to use the Desmos to trace the graph and identify the coordinates of any point on it (which may include values that are precise or values rounded to a specified decimal place). Or, first observe how students go about estimating and give additional guidance as needed.
- To support students with question 4, ask students: "Is the equation a good model for predicting the height of the cannonball 10 seconds after it is fired? What about 1 minute after it is fired?"
- Some students may have difficulty getting the graph to show up on their screen. If needed, demonstrate how to adjust the graphing boundaries within Desmos.

Student Task Statement

The function defined by $d = 50 + 312t - 16t^2$ gives the height in feet of a cannonball t seconds after the ball leaves the cannon.

1. What do the terms 50, $312t$, and $-16t^2$ tell us about the cannonball?
2. Use graphing technology to graph the function. Adjust the graphing window to the following boundaries: $0 \leq x \leq 25$ and $0 \leq y \leq 2,000$.
3. Observe the graph and:
   a. Describe the shape of the graph. What does it tell us about the movement of the cannonball?
   b. Estimate the maximum height the ball reaches. When does this happen?
   c. Estimate when the ball hits the ground.
4. What domain is appropriate for this function? Explain your reasoning.
5. What range is appropriate for this function? Explain your reasoning.

Are You Ready For More?

If the cannonball were fired at 800 feet per second, would it reach a mile in height? Explain your reasoning.

Step 2

- Facilitate a whole-class discussion by inviting students to share their observations and interpretations of the graph. Highlight the following points:
  - The graph representing a quadratic function is a parabola, which is a special kind of "U" shape. We will learn more about the geometry of this shape in a later course, but for now, notice that there is a point when the graph changes direction, from going up as x increases to going down (or changes from going down to going up). We call this point the vertex of the graph.
  - In the previous activity, we plotted a limited set of points, so we could not tell where the vertex of the graph was. In this graph, we are able to identify the vertex. In this situation, the vertex tells us the maximum height that the cannonball reaches and the number of seconds after launch that it took before it starts to fall.
  - In this graph, we can also see that the height of the cannonball is 0 when t is a little less than 20. That point, the horizontal intercept, relates to what is called the zero of the function. A zero of a function is an input value that produces 0 for the output. In this situation, the zero tells us when the cannonball hits the ground.
  - Even though we can continue the graph beyond $t = 20$, in this situation, any output values beyond that point would not have any meaning. After the cannonball hits the ground, the function is no longer appropriate for modeling the movement of the cannonball. Likewise, the function is not appropriate before $t = 0$ or before the cannon is fired. In this situation, a domain between 0 and just below 20 seconds is appropriate.
  - The graph shows a second zero, which occurs at a negative x-value. However, negative x-values are outside the domain of the function (because the ball was not moving before time zero), so this zero does not make sense.
  - The maximum height reached by the cannonball is 1,571 feet above ground, and the ball never goes below ground. Therefore, for this situation, a range between (and including) 9 and 1,571 feet is appropriate.

RESPONSIVE STRATEGY

Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: vertex of the graph, zero of the function, and parabola.

Discussion Supports (MLR8)

DO THE MATH	**PLANNING NOTES**

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to understand where formulas for projectile motion come from, and in particular why quadratic functions are needed. Students are also introduced to the terms "zero" and "vertex" as they relate to parabolas, and articulate that sometimes it is necessary to restrict the domain and range of a function to more appropriately align with the context.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

To reinforce the connections between the parameters of a quadratic expression and the situation it describes, ask students:

- “So far, we’ve seen different expressions that represent vertical distances. Here are three expressions that all represent distance, in feet, as a function of time, in seconds, for an object that is dropped or launched. What does each of them tell us? Draw a diagram to illustrate the distances, if helpful.”
    - $16t^2$ (the distance an object travels t seconds after being dropped)
    - $400-16t^2$ (the height of an object that is dropped from a height of 400 feet)
    - $50+100t-16t^2$ (the height of an object that is shot up from 50 feet above the ground at a vertical speed of 100 feet per second, t seconds after being launched)
- “If each expression defines a function, what does the zero of that function tell us?”
    - $16t^2$ (The zero is the time when the object has traveled a distance of 0 feet. This happens at $t=0$, before the object is dropped.)
    - $400-16t^2$ (The zero is the time when the height of the object is 0 feet, which is when it hits the ground.)
    - $50+100t-16t^2$ (The zero is the time when the height of the object is 0 feet, which is also when it hits the ground.)

Explain to students that the models seen here are simplified models, and they ignore other factors such as air resistance, so the models that scientists use to study physical phenomena are likely to be more complex than what they’ve seen here.

If time permits, consider addressing a common misconception: that a graph of a quadratic function that represents distance-time relationship shows the physical trajectory of the object. Ask students to draw a sketch of what a bystander would see if they are facing the cannon as the ball is being launched. If needed, revisit the Geogebra applet “Galileo and Gravity” https://www.geogebra.org/m/druqqfyq (from the previous lesson) to further emphasize this difference.

Clarify that the graph represents the height of the object as a function of time, not the path that the object travels. In the examples given here, the object just goes straight up and straight down.

PLANNING NOTES

Student Lesson Summary and Glossary

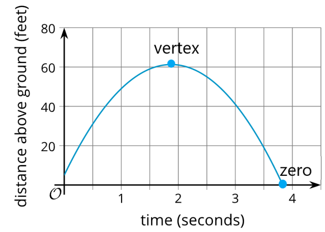
In this lesson, we looked at the height of objects that are launched upward and then come back down because of gravity.

An object is thrown upward from a height of 5 feet with a velocity of 60 feet per second. Its height, $h(t)$, in feet, after t seconds is modeled by the function $h(t)=5+60t-16t^2$.

- The linear expression $5+60t$ represents the height the object would have at time t if there were no gravity. The object would keep going up at the same speed at which it was thrown. The graph would be a line with a slope of 60, which comes from the constant speed of 60 feet per second.

- The expression $-16t^2$ represents the effect of gravity, which eventually causes the object to slow down, stop, and start falling back again.

Here is the graph of h:



The graph representing any quadratic function is a special kind of "U" shape called a **parabola**. You will learn more about the geometry of parabolas in a future course.

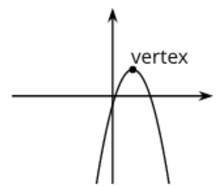
Notice the parabola intersects the vertical axis at 5, which means the object was thrown into the air from 5 feet off the ground. The graph indicates that the object reaches its peak height of about 60 feet after a little less than 2 seconds. That peak is the point on the graph where the function reaches a maximum value. At that point, the curve changes direction, and the output of the function changes from increasing to decreasing. We call that point the **vertex** of the graph. Every parabola has a vertex, because there is a point where it changes direction—from increasing to decreasing, or the other way around.

The object hits the ground a little before 4 seconds. That time corresponds to the horizontal intercept of the graph. An input value that produces an output of 0 is called a **zero** of the function. A zero of the function h is approximately 3.8 because $h(3.8) \approx 0$.

In this situation, input values less than 0 seconds or more than about 3.8 seconds would not be meaningful, so an appropriate domain for this function would include all values of t between 0 and about 3.8.

Parabola: The "U"-shaped graph representing any quadratic function.

Vertex : The vertex of the graph of a quadratic function is the point where the graph changes from increasing to decreasing or vice versa. It is the highest or lowest point on the graph.



Zero (of a function): An input that yields an output of zero. If other words, if $f(a) = 0$, then a is a zero of f.

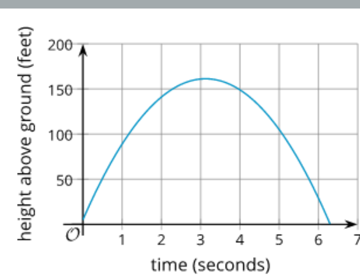
Cool-down: Rocket in the Air (*5 minutes*)

Addressing: NC.A-SSE.1a; NC.M1.F-IF.7

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

The height, h, of a stomp rocket (propelled by a short blast of air) above the ground after t seconds is given by the equation $h(t) = 5 + 100t - 16t^2$. Here is a graph that represents h.



1. How does the 5 in the equation relate to the graph?
2. What does $100t$ in the equation mean in terms of the rocket?
3. What does the $-16t^2$ mean in terms of the rocket?
4. About when does the rocket hit the ground?

Student Reflection:

What is something you still do not fully understand and need more explanation or practice?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What did you say, do, or ask during the lesson debrief that helped students be clear on the learning of the day? How did understanding the cool-down of the lesson before you started teaching today help you synthesize that learning?

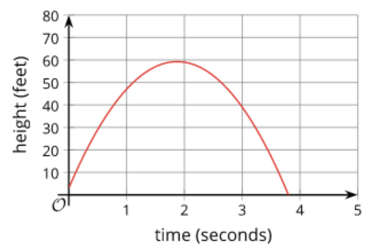
Practice Problems

1. The height of a diver above the water is given by $h(t) = -5t^2 + 10t + 3$, where t is time measured in seconds and $h(t)$ is measured in meters.

   Select **all** statements that are true about the situation.

   a. The diver begins 5 meters above the water.
   b. The diver begins 3 meters above the water.
   c. The function has 1 zero that makes sense in this situation.
   d. The function has 2 zeros that make sense in this situation.
   e. The graph that represents h starts at the origin and curves upward.
   f. The diver begins at the same height as the water level.

2. The height of a baseball, in feet, is modeled by the function h given by the equation $h(t) = 3 + 60t - 16t^2$. The graph of the function is shown.



   a. About when does the baseball reach its maximum height?

   b. About how high is the maximum height of the baseball?

   c. About when does the ball hit the ground?

3. (*Technology required.*) Two rocks are launched straight up in the air. The height of rock A is given by the function f, where $f(t) = 4 + 30t - 16t^2$. The height of rock B is given by g, where $g(t) = 5 + 20t - 16t^2$. In both functions, t is time measured in seconds, and height is measured in feet.

   Use graphing technology to graph both equations. Determine which rock hits the ground first and explain how you know.

4. Each expression represents an object's distance from the ground in meters as a function of time, t, in seconds.

   Object A: $-5t^2 + 25t + 50$

   Object B: $-5t^2 + 50t + 25$

   a. Which object was launched with the greatest vertical speed?

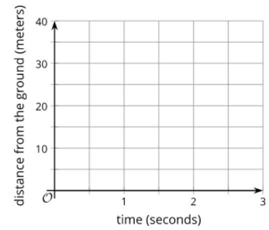
   b. Which object was launched from the greatest height?

5. Han accidentally drops his water bottle from the balcony of his apartment building. The equation $d = 32 - 5t^2$ gives the distance from the ground, d, in meters, after t seconds.

   a. Complete the table and plot the data on the coordinate plane.

   b. Is the water bottle falling at a constant speed? Explain how you know.

t (seconds)	d (meters)
0	
0.5	
1	
1.5	
2	



(From Unit 7, Lesson 4)

6. The function f is defined by $f(x) = 2^x$, and the function g is defined by $g(x) = x^2 + 16$.

   a. Find the values of f and g when x is 4, 5, and 6.

   b. Will the values of f always be greater than the values of g? Explain how you know.

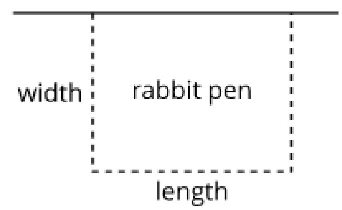
(From Unit 7, Lesson 3)

7. Tyler is building a pen for their rabbit on the side of the garage. They need to fence in three sides of the pen and want to use 24 ft of fencing.

   a. The table shows some possible lengths and widths. Complete each area.

   b. Which length-and-width combination should Tyler choose to give their rabbit the most room?

Length (ft)	Width (ft)	Area (sq ft)
8	8	
10	7	
12	6	
14	5	
16	4	

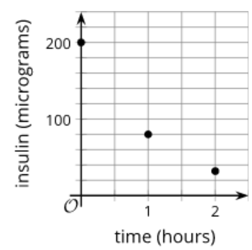


(From Unit 7, Lesson 1)

8. The graph shows how much insulin, in micrograms (mcg), is in a patient's body after receiving an injection.

   a. Write an equation giving the number of mcg of insulin, m, in the patient's body h hours after receiving the injection.

   b. After 3 hours, will the patient still have at least 10 mcg of insulin in their body? Explain how you know.



(From Unit 6)

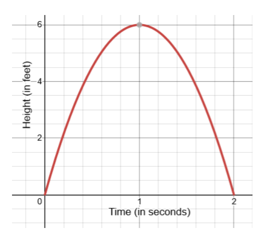
9. Lin says that a solution to the equation $2x - 6 = 7x$ must also be a solution to the equation $5x - 6 = 10x$.

   Write a convincing explanation about why this is true.

   (From Unit 3)

10. The graph plots the path of a high jumper at Parabola High School competing in the state championships. What do you know about the jump from the graph?

    (Addressing NC.8.F.5)



Lesson 6: Building Quadratic Functions to Describe Situations (Part Three)

PREPARATION

Lesson Goals	Learning Targets
• Choose an appropriate domain for a quadratic function representing a context and explain (orally) the reason why it was selected. • Interpret (orally and in writing) the vertex of a graph and the zeros of a quadratic function in context. • Write and graph equations to model economic situations characterized by quadratic functions.	• I can choose a domain that makes sense in a revenue situation. • I can relate the vertex of a graph and the zeros of a function to revenue and profit situations. • I can model revenue with quadratic functions and graphs.

Lesson Narrative

In the previous two lessons, students analyzed quadratic functions that modeled projectile motion. In this lesson, they encounter a quadratic relationship in an economic context, profit as a function of quantity sold.

Students also deepen their understanding of the zeros and the domain and range of a quadratic function, and of the vertex of its graph. They interpret these features in various contexts, reasoning concretely and abstractly (MP2).

What is the main purpose of this lesson? What is the one thing you want your students to take away from this lesson?

Focus and Coherence

Building On	Addressing
NC.8.F.4: Analyze functions that model linear relationships. • Understand that a linear relationship can be generalized by $y = mx + b$. • Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. • Construct a graph of a linear relationship given an equation in slope-intercept form.	**NC.M1.F-IF.5:** Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes. *(continued)*

• Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values. **NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.	**NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Agenda, Materials, and Preparation

Technology isn't required for this lesson, but there are opportunities for students to choose to use appropriate technology to solve problems. It is ideal if each student has accessibility to their own device.

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
  - 2 different colored highlighters per pair of students made available, as needed
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L6 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.F.4

Before students relate x-intercepts to zeroes of quadratic functions in this lesson, they will determine the x-intercept and y-intercept of a linear function and relate them to a context in the bridge task. This bridge is aligned to Check Your Readiness question 7.

Student Task Statement

Mrs. Gillis gives her class a set of 36 math challenges to complete during the first quarter. Andre decides to complete four each week. The function $y = 36 - 4x$ represents the number of challenges Andre has remaining to complete, y, after x weeks.

1. What is the y-intercept of this function? What does it represent?
2. What is the x-intercept of this function? What does it represent?

DO THE MATH	PLANNING NOTES

Warm-up: Graphs of Four Functions (*5 minutes*)

Instructional Routine: Which One Doesn't Belong?	
Building On: NC.M1.F-IF.4	

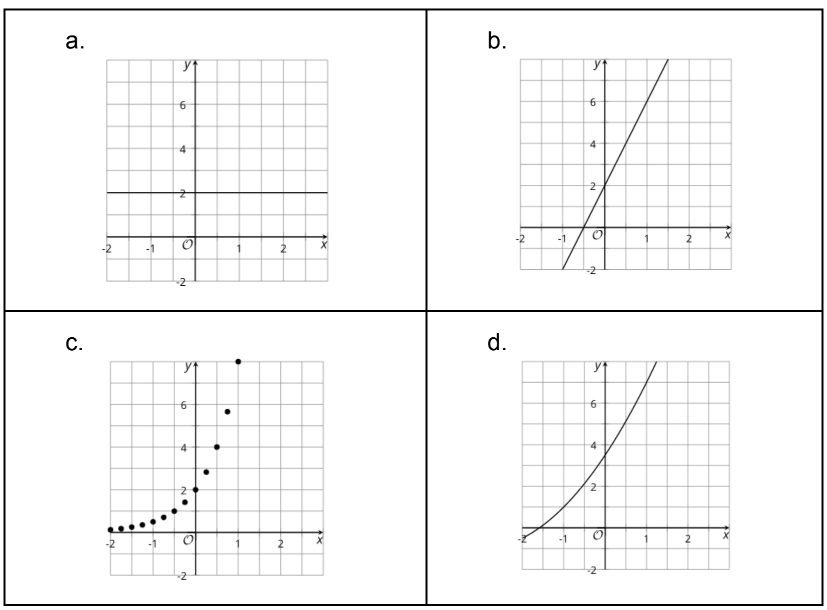
This *Which One Doesn't Belong?* warm-up prompts students to analyze and compare the features of the graphs of four functions. In making comparisons, students have a reason to use language precisely (MP6). The activity also enables the teacher to hear the terminology students know and how they talk about characteristics of linear, exponential, and quadratic functions and their graphs.

Step 1

- Ask students to arrange themselves in small groups or use visibly random grouping. Display the graphs for all to see.
- Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, ask each student to share their reasoning why a particular item does not belong, and together find at least one reason each item doesn't belong.

Student Task Statement

Which one doesn't belong? Explain your reasoning.



Step 2

- Ask each group to share one reason why a particular item does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given make sense to the class.
- During the discussion, ask students to explain the meaning of any terminology they use, such as "discrete," "increasing," x-intercept," and y-intercept." Also, press students on any incorrect claims.

DO THE MATH	PLANNING NOTES

Activity 1: Pharmaceutical Profiting (*10 minutes*)

Instructional Routines: Co-Craft Questions (MLR5) - Responsive Strategy; Discussion Supports (MLR8)	
Addressing: NC.M1.F-IF.7	

In this activity, students encounter a quadratic function in a business context. They study the relationship between the profit a pharmaceutical company earns and the number of vials of insulin it sells, and see that the relationship can be described with a quadratic function. In order to answer the questions, students must use appropriate tools strategically (MP5), make sense of problems (MP1), and reason abstractly and quantitatively (MP2).

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Read the context that is above the questions while displaying or pointing out the photograph. Ask for a volunteer to describe in their own words what "vials of insulin" are, making sure all students understand the context for the equation they will analyze.
- Ask the class, "What is profit?" (Profit is the difference between the amount of money earned from selling a product and the amount of money spent in producing that same product, assuming the difference is positive. The opposite of profit, or when the difference is negative, is called a "loss." "Breaking even" occurs when the difference is zero.)

RESPONSIVE STRATEGY

Use the this routine by giving students 1 minute to come up with a mathematical question that could be asked about this situation. Then invite 2-3 students to quickly share their questions aloud with the class. Move on to Step 2, but consider scribing students' questions for all to see as students work on the activity.

Co-Craft Questions (MLR5)

Step 2

- Give students 5 minutes to answer questions 1–3 independently before sharing their responses with their partner. Using the *Discussion Supports* of providing sentence frames, facilitate a discussion between partners as they share their thinking with one another for questions 2 and 3:
  - "The profit function, $p(v)$, is positive ($p(v) > 0$) when ____ is between ____ and ____. This means that the pharmaceutical company only makes a profit when _____."
  - "The profit function, $p(v)$, is negative ($p(v) < 0$) when ____ is between ____ and ____. This means that the pharmaceutical company loses money when _____."
- Ask partners to complete question 4 collaboratively and record their reasoning in their own workbook.

Monitoring Tip: As students work, listen for discussion between partners around whether or not to include negative v-values or whether or not to use p or v values to define the intervals. Select specific students and let them know they may be asked to share later with the whole class their strategies or reasoning in determining positive/negative intervals. Include at least one student who doesn't typically volunteer.

Advancing Student Thinking: Some students may identify where the number of vials sold is positive and negative, instead of positive and negative profit. Remind these students that the notation $p(v)$ means that profit earned is a function of the number of vials sold. Therefore, when asked to find where the function is positive, we are interestested in where the profit is positive. Similarly, students may confuse positive/negative intervals with intervals of increase/decrease. Remind these students that a function that is increasing on an interval has a positive rate of change throughout the interval but could still contain negative output values. It may be helpful to encourage students to articulate or shade with two different colored highlighters the region of the coordinate plane that contains positive y-values, and the region of the coordinate plane that contains negative p-values.

Students may also describe the positive/negative intervals using p values, instead of v values. Use the context of the situation to explain why describing these intervals using p values is not as helpful to companies using the function to make decisions. In other words, identifying that profit is positive when they are gaining above $0 does not tell companies how to earn a positive profit, whereas identifying that profit is positive when we sell between 0 and 500 vials of insulin allows companies to make an informed decision.

Student Task Statement

The profit of a pharmaceutical company's insulin is modeled by the equation $p(v) = -0.3v^2 + 150v$, where p represents profit (in millions of dollars), and v represents the number of vials (in millions) of insulin sold.

(Image source[1])

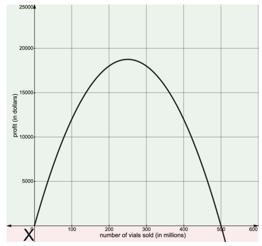
1. Graph the function $p(v)$ in Desmos.
2. Identify an interval where the function is positive ($p(v) > 0$). Interpret the meaning of this interval in the context of the situation the function describes.
3. Identify an interval where the function is negative ($p(v) < 0$). Interpret the meaning of this interval in the context of the situation the function describes.
4. If the company doesn't make much insulin, people are willing to pay more for that insulin because it is so hard to find. If the company makes a lot of insulin, there is more insulin than people really need, so the company cannot charge as much. With your partner, discuss why the profit would be small when the company makes too much or too little insulin.

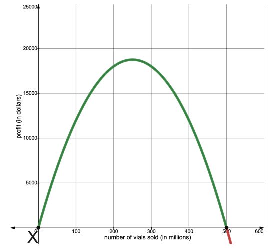
Step 3

- Facilitate a whole-class discussion by inviting students previously selected to share their strategies and reasoning used to determine the positive/negative intervals with the whole class. Use the following questions to guide discussion:
  - "When we are looking for intervals where the function is positive or negative, are we looking for positive and negative vials or profit?" (We are looking for profit to be positive/negative because profit is a function of the number of vials.)
  - "How are intervals where a function is positive or negative different than intervals of increase/decrease?" (A function that is increasing on an interval has a positive rate of change throughout the interval, whereas a function that is positive on an interval has positive output values throughout the interval. Similarly, a function that is decreasing on an interval has a negative rate of change throughout the interval, whereas a function that is negative on an interval has negative output values throughout the interval.)

[1] Barbhuiya, T. (2021). Free injection. *Unsplash.* https://unsplash.com/photos/msQB97gUxY0

- “What strategies did you use to help you identify where the function is positive/negative?” (Using colored markers or highlighters helps us to better distinguish between positive or negative intervals.)
    - Consider displaying a visual of this strategy:





- “When describing the positive/negative interval of the profit function, was it more meaningful to use quantities related to the number of vials sold or the company’s profit?” (Number of vials sold; identifying that profit is positive when they are gaining above $0 only tells us something we already know, whereas identifying that profit is positive when we sell between 0 and 500 million vials of insulin allows companies to make an informed decision.)
- “Why might identifying where the profit function is positive be beneficial for pharmaceutical companies?” (A positive profit means the company is making money, but a negative profit means they are losing money.)
- “If pharmaceutical companies are motivated to make a profit, what are the implications for the availability of the medication?” (Everyone who needs the medication may not be able to have it.)
- If time permits, invite partner pairs to share with the class their reasoning for why profit would be small when the company makes very little insulin, and when it makes a lot of insulin. (question 4). Connect this to the shape of the graph.
- If time permits, explain that insulin is a lifesaving drug for people who have diabetes. Ask students, “If the company can charge more when there is a shortage of insulin, should it?” (Yes, the company can’t sell as much insulin, so it needs to charge more to stay in business in order to make more insulin when possible. No, raising the prices means that only richer people can afford insulin, and the less wealthy people would have to do without.)

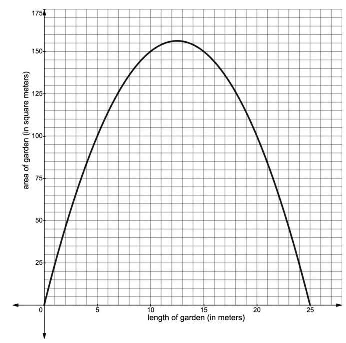
DO THE MATH	PLANNING NOTES

Activity 2: Domain, Range, Vertex, and Zeros (15 minutes)

Instructional Routine: Compare and Connect (MLR7)
Addressing: NC.M1.F-IF.5; NC.M1.F-IF.7

This activity serves two main goals. It prompts students to look closely at appropriate domains and ranges for quadratic functions given the situations they represent. It also gives them an opportunity to identify or estimate the vertex of the graph and the zeros of the functions, and then to interpret them in context.

Step 1



- Share with students that in a previous lesson, they saw a function representing the area of a garden, in square meters, as a function of its length, in meters, where the garden had a set perimeter of 50 meters. Display the graph of the equation $A = l(25 - l)$ using graphing technology and ask the following questions:
  - "What domain is appropriate for the length of the garden?" (The length of the garden must be a positive value because it represents a distance. Length can not be 0 (or 25) because that would force the width to be 25 (or 0), creating only a one dimensional space. A length beyond 25 would force width to be negative for a total perimeter of 50 meters, which is also impossible. So an appropriate domain would be $0 < l < 25$.)
  - "What range is appropriate for the length of the garden?" (The area of the garden cannot be negative. Additionally, no matter what combination of lengths and widths we tried in this activity, the area never exceeded 156.25 square meters. Therefore, an appropriate range would be $0 < A \leq 156.25$.)
  - "What are the zeros of the function? What do they tell us in this situation?" (0 and 25. They tell us the lengths that would create an area of zero (no garden).)
  - "What is the vertex of the graph representing the function? What does it tell us in this situation?" (It appears to be at $(12.5, 156.25)$. It tells us the length that would generate the largest area possible for the garden.)
- Tell students they will now think about the domain, vertex, and zeros of a few quadratic functions we have seen so far.

Step 2

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Consider asking each group to work on only one or two functions and then to share their findings with the class, or choose only a couple of functions for the class to investigate.
- Use the *Compare and Connect* routine to prepare students for the whole-class discussion. Ask students to select one of the functions (guiding students so that all three functions will be represented in the class display) and to prepare a visual display of their work. Students should consider how to display their reasoning so that another student can interpret what they see. Suggest that students should add notes and details to the graphs or functions to help communicate their thinking. Provide 2–3 minutes of quiet think time for students to read and interpret each other's work before a whole-class discussion.

RESPONSIVE STRATEGY

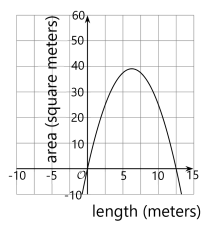
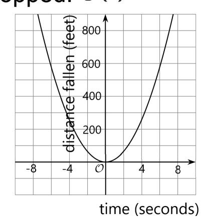
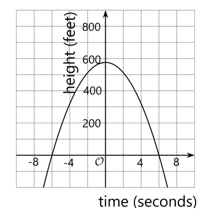
Represent the same information through different modalities by using diagrams. To support students in thinking of an appropriate domain for a given situation, suggest that they draw a diagram (or series of diagrams) to help visualize some input/output pairs that make sense.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Advancing Student Thinking: Some students may confuse zeros and horizontal intercepts (x-intercepts). Watch for students that write the zeros as ordered pairs such as $(25, 0)$ rather than 25. Emphasize that while these two terms are related, there is difference. A zero is an input value that makes the function's output 0, and the horizontal intercept is the point where the graph of the function meets the horizontal axis. A zero of a function is the x-coordinate of an x-intercept of its graph.

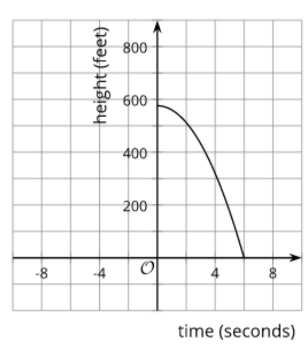
Student Task Statement

Here are three sets of descriptions and equations that represent some familiar quadratic functions. Graphs related to each function are also shown, though they may not reflect a reasonable domain for each function. For each function, complete the table.

	1. The area of rectangle with a perimeter of 25 meters and a side length x: $A(x) = x \cdot \frac{(25-2x)}{2}$ 	2. The distance in feet that an object has fallen t seconds after being dropped: $g(t) = 16t^2$ 	3. The height in feet of an object t seconds after being dropped: $h(t) = 576 - 16t^2$ 
Describe a domain that is appropriate for the situation. Think about any upper or lower limits for the input, as well as whether all numbers make sense as the input.			
Describe a range that is appropriate for the situation.			
Describe how the graph should be modified to show the domain (and range) that makes sense.			
Identify or estimate the vertex on the graph. Describe what it means in the situation.			
Identify or estimate the zeros of the function. Describe what it means in the situation.			

Step 3

- Invite groups to compare and connect their responses and explanations. If not already discussed or displayed by students, show examples of graphs that are each adjusted for a domain or range appropriate for the function represented.
- Explain that the graphs of quadratic functions may or may not show the vertex, depending on the situation it represents.
- Here is the graph representing the function defined by $h(t) = 576 - 16t^2$ (the third function), adjusted for the domain appropriate in the situation. In the graph, the y-intercept is the vertex, but because negative values for t are not applicable, we don't see a "turn" in the graph (where the output changes from increasing to decreasing).



- The function h models the height of an object t seconds after being dropped. Because a negative number of seconds is not meaningful here, assuming that object stops once it hits the ground (at 6 seconds), an appropriate domain for the function would be $0 \leq t \leq 6$.

DO THE MATH	PLANNING NOTES

Lesson Debrief *(5 minutes)*

The purpose of this lesson is for students to practice finding and interpreting the zero(s) and vertex of functions in a variety of contexts. Additionally, students determine a reasonable domain and range for various functions based on context.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

To help students consolidate the key ideas in this lesson, consider asking them to complete one or more of the following sentence stems:

- In general, the zeros of a function tell us… For example: _____.
- In general, the vertex of a graph that represents a function tells us… For example: ______.
- To determine an appropriate domain for a function, we need to consider…
- To determine an appropriate range for a function, we need to consider…

PLANNING NOTES

Student Lesson Summary and Glossary

As you have seen, quadratic functions often come up when studying revenue. (Recall that “revenue” means the money collected when someone sells something.) Interpreting the key features of these functions can help businesses make decisions about how much to charge.

Suppose we are selling raffle tickets and deciding how much to charge for each ticket. When the price of the tickets is higher, typically fewer tickets will be sold.

Let’s say that with a price of d dollars, it is possible to sell $600 - 75d$ tickets. A function that models the revenue r collected is $r(d) = d(600 - 75d)$, or $r(d) = 600d - 75d^2$. Here is a graph that represents the function.

If the greatest revenue is $1,200, and the revenue collected cannot be negative, then the range of the function r is between 0 and 1200.

We can also see that the domain of the function r is between 0 and 8. Clearly the cost of the tickets cannot be negative. If the cost of the tickets were more than $8, the expression for the number of tickets sold, $600 - 75d$, becomes negative. Since the number of tickets sold cannot be negative, this tells us our model does not work for $d > 0$.

The graph shows that the zeros of the function r are 0 and 8. These are the prices for which we would make no money from the raffle.

As the raffle organizers, we are most interested in setting the price of tickets to make the most money. We can see this in the vertex of the graph, (4, 1200). If we charge $4 for the tickets, we will produce the maximum revenue of $1200.

Cool-down: Making the Greatest Revenue (*5 minutes*)

Addressing: NC.M1.F-IF.5; NC.M1.F-IF.7

Cool-down Guidance: Points to Emphasize
If students are still struggling to interpret the graphs, spend time before the Mid-Unit Assessment ensuring that students can interpret the x- and y- intercepts, and the vertex. Practice problems, in addition to feedback on cool-downs and highlighted examples, can support student thinking.

Cool-down

This graph represents the revenue in dollars that a company expects if they sell their product for p dollars.

1. Based on this model, which price would generate more revenue for the company, \$5 or \$17? Explain how you know.
2. At what price should the company sell their product if they wish to make as much revenue as possible? How much revenue will they make?
3. What is an appropriate domain for the function? Explain how you know.
4. What is an appropriate range for the function? Explain how you know.

Student Reflection:

This lesson called for a great deal of sharing your thinking and strategies for working through the activities. On a scale of 1 to 5, how confident were you to speak aloud and share with your peers?

How can your teacher support you in becoming more confident in mathematics?

1 - Not confident at all
2 - Barely confident
3 - No completely confident but not anxious
4 - Fairly confident
5 - Very confident

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What was the best question you asked students today? Why would you consider it the best one based on what students said or did?

Practice Problems

1. Based on past musical productions, a theater predicts selling $400 - 8p$ tickets when each ticket is sold at p dollars.

   a. Complete the table to find out how many tickets the theater expects to sell and what revenues it expects to receive at the given ticket prices.

   b. For which ticket prices will the theater earn no revenue? Explain how you know.

   c. At what price should the theater sell the tickets if it must earn at least $3,200 in revenue to break even (to not lose money) on the musical production? Explain how you know.

Ticket price (dollars)	Number of tickets sold	Revenue (dollars)
5		
10		
15		
20		
30		
45		
50		
p		

2. A company sells running shoes. If the price of a pair of shoes in dollars is p, the company estimates that it will sell $50,000 - 400p$ pairs of shoes.

   Write an expression that represents the revenue in dollars from selling running shoes if a pair of shoes is priced at p dollars.

3. (*Technology required.*) The profit that a company makes selling an item (in thousands of dollars) depends on the price of the item (in dollars). If s is the sale price of the item, then profit, $P(s)$, can be represented by the function rule: $P(s) = -2s^2 + 24s - 54$.

   a. Graph the function $P(s)$ in Desmos.

   b. Identify an interval where the function is positive ($P(s) > 0$). Interpret the meaning of this interval in the context of the situation the function describes.

   c. Identify an interval where the function is negative ($P(s) < 0$). Interpret the meaning of this interval in the context of the situation the function describes.[2]

4. The function f represents the revenue in dollars the school can expect to receive if it sells $220 - 12x$ coffee mugs for x dollars each.

   Here is the graph of f.

   Select **all** the statements that describe this situation.

   a. At $2 per coffee mug, the revenue will be $196.
   b. The school expects to sell 160 mugs if the price is $5 each.
   c. The school will lose money if it sells the mugs for more than $10 each.
   d. The school will earn about $1,000 if it sells the mugs for $10 each.
   e. The revenue will be more than $700 if the price is between $4 and $14.
   f. The expected revenue will increase if the price per mug is greater than $10.

5. (*Technology required.*) A small marshmallow is launched straight up in the air with a slingshot. The function h, given by the equation $h(t) = 5 + 20t - 5t^2$, describes the height of the marshmallow in meters as a function of time, t, in seconds, since it was launched.

   a. Use graphing technology to graph the function h.

   b. About when does the marshmallow reach its maximum height?

   c. About how long does it take before the marshmallow hits the ground?

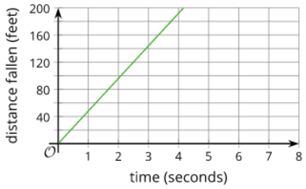
   d. What domain makes sense for the function h in this situation?

   (From Unit 7, Lesson 4)

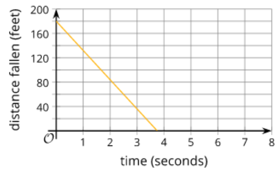
[2] Adapted from https://tasks.illustrativemathematics.org/

6. A rock is dropped from a bridge over a river. Which graph could represent the distance fallen, in feet, as a function of time in seconds? Explain your reasoning.

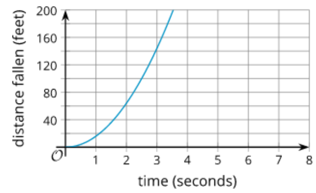
Graph A



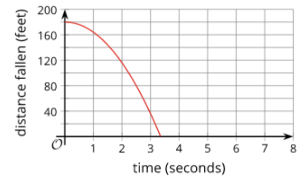
Graph B



Graph C



Graph D



(From Unit 7, Lesson 4)

7. A bacteria population, p, is modeled by the equation $p = 100,000 \cdot 2^d$, where d is the number of days since the population was first measured.

Assuming the population growth has been the same prior to the day it was first measured, select **all** the predictions that are true statements in this situation.

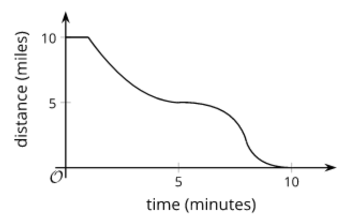
a. $100,000 \cdot 2^{-2}$ represents the bacteria population 2 days before it was first measured.
b. The bacteria population 3 days before it was first measured was 800,000.
c. The population was more than 1,000 one week before it was first measured.
d. The population was more than 1,000,000 one week after it was first measured.
e. The bacteria population 4 days before it was first measured was 6,250.

(From Unit 6)

8. Simplify the expression, writing with all positive exponents: $\frac{18x^5y^{-2}z^3}{12x^6y^4z^{-2}}$

(From Unit 6)

9. Here is a graph of Han's distance from home as he drives.



Identify the intercepts of the graph and explain what they mean in terms of Han's distance from home.

(From Unit 5)

10. Throughout the month of January, the weather in Charlotte gets colder. According to timeanddate.com, on January 1, 2021, the high temperature was $54°$ Fahrenheit, and it dropped about $0.5°$F per day for 30 days, until January 31, 2021. Mai models the temperature in Charlotte by function $t = -0.5d + 54$, where t represents the temperature and d represents the number of days passed.

a. What is the vertical intercept of this function? What does it represent?

b. What is the horizontal intercept of this function? What does it represent?

c. Is the horizontal intercept of this function realistic? Why or why not?

(Addressing NC.8.F.4)

Lesson 7: Equivalent Quadratic Expressions

PREPARATION

Lesson Goals	Learning Target
• Use area diagrams to reason about the product of two sums and to write equivalent expressions. • Use the distributive property to write equivalent quadratic expressions.	• I can rewrite quadratic expressions in different forms by using an area diagram or the distributive property.

Lesson Narrative

By now students have encountered a variety of situations that can be modeled with quadratic functions. They are also familiar with some features of the expressions, tables, and graphs that represent such functions. This lesson transitions students from reasoning concretely and contextually about quadratic functions to reasoning about their representations in ways that are more abstract and formal (MP2).

In earlier grades, students reasoned about multiplication by thinking of the product as the area of a rectangle where the two factors being multiplied are the side lengths of the rectangle. In this lesson, students use this familiar reasoning to expand expressions such as $(x+4)(x+7)$, where $x+4$ and $x+7$ are side lengths of a rectangle with each side length decomposed into x and a number. They use the structure in the diagrams to help them write equivalent expressions in expanded form: for example, $x^2+11x+28$ (MP7). Students recognize that finding the sum of the partial areas in the rectangle is the same as applying the distributive property to multiply out the terms in each factor.

After this lesson, students move to thinking more abstractly about such diagrams. Rather than reasoning in terms of area, they use the diagrams to organize and account for all terms when applying the distributive property.

The terms "standard form" and "factored form" are not yet used and will be introduced in an upcoming lesson, after students have had some experience working with the expressions.

What connections from what students shared and discussed in previous lessons do you want to be explicit about in this lesson?

Focus and Coherence

Building On	Addressing	Building Towards
NC.6.EE.3: Apply the properties of operations to generate equivalent expressions without exponents **NC.7.EE.1:** Apply properties of operations as strategies to: • Add, subtract, and expand linear expressions with rational coefficients. • Factor linear expression with an integer GCF.	**NC.M1.A-APR.1:** Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions.	**NC.M1.F-IF.8:** Use equivalent expressions to reveal and explain different properties of a function.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L7 Cool-down (print 1 copy per student)

LESSON

Warm-up: Diagrams of Products (*5 minutes*)

Building On: NC.6.EE.3

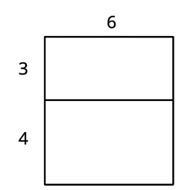
In this warm-up, students recall that an area diagram can be used to illustrate multiplication of a number and a sum. This work prepares them to use diagrams to reason about the product of two sums that are variable expressions.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Give students quiet work time and then time to share their work with a partner.

Student Task Statement

1. Explain why the diagram shows that $6(3+4)=6\cdot 3+6\cdot 4$.
2. Draw a diagram to show that $5(x+2)=5x+10$.



Step 2

- Invite students to share their partner's explanation of the diagram.
- Make sure students understand that the expressions $6\cdot 3+6\cdot 4$ and $6(3+4)$ are two ways of representing the area of the same rectangle.
  - $6\cdot 3+6\cdot 4$ treats the area of the largest rectangle as a *sum* of the areas of the two smaller rectangles that are 6 by 3 and 6 by 4.
  - The other expression, $6(3+4)$ describes the area of the largest rectangle as a *product* of its two side lengths, which are 6 and the sum of 3 and 4.
- Ask students to share their diagram for $5(x+2)=5x+10$. Make sure to highlight:
  - the way the diagrams represent the area of the largest rectangle as the sum of the areas of the two smaller rectangles with areas of $5x$ and 10
  - the way the diagrams represent the area of a large rectangle that is 5 by $x+2$
  - that when we express $5(x+2)$ as $5x+10$, we are applying the distributive property

DO THE MATH	PLANNING NOTES

Activity 1: Drawing Diagrams to Represent More Products (*15 minutes*)

Instructional Routines: Take Turns; Critique, Correct, Clarify (MLR3)	
Building On: NC.6.EE.3; NC.7.EE.1	**Addressing:** NC.M1.A-APR.1

This activity builds on students' prior knowledge of writing equivalent expressions. They expand the product of a number (or a variable) and a sum by drawing diagrams and applying the distributive property. Unlike the work in earlier grades, however, some of the resulting expressions are quadratic expressions. The visual and algebraic reasoning here builds towards the next activity, where they will expand expressions containing two sums.

Step 1

- Remind students that multiplying out the factors in an expression like $5(x+2)$ and writing it as $5x+10$ is often called "expanding an expression."
- Keep students in partners from the warm-up and have them work towards completing the task. Have students *Take Turns* with their partner for question 2, with one explaining what to do and the other recording the work.

RESPONSIVE STRATEGY

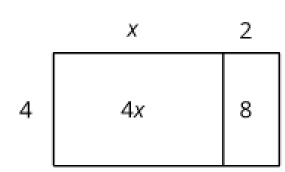
Activate or supply background knowledge. Some students may benefit from additional support to learn how to draw appropriate diagrams. Consider providing access to some blank, or partially completed diagrams to start with.

Supports accessibility for: Visual-spatial processing; Organization

Monitoring Tip: Monitor for students who
- represent the expression with a diagram and connect the diagram to the equivalent expression
- take a more general algebraic approach and use the distributive property (without a diagram)

Student Task Statement

Applying the distributive property or expanding $4(x+2)$ gives us $4x+8$, so we know the two expressions are equivalent. We can use a rectangle with side lengths $(x+2)$ and 4 to illustrate the multiplication.



1. Draw a diagram to show that $n(2n+5)$ and $2n^2+5n$ are equivalent expressions.
2. For each expression, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.

   a. $6(\frac{1}{3}n+2)$

   b. $p(4p+9)$

   c. $5r(r+\frac{3}{5})$

   d. $(0.5w+7)w$

Step 2

- Facilitate the *Critique, Correct, Clarify* routine by saying and displaying the following first draft statement: "I know that $3a(a+2)$ equals $3a+6a$ because I have to distribute to add the $3a$."
  - Give students 1 minute of quiet think time to begin to identify parts of the statement that are incomplete, incorrect, or unclear. Then invite two or three students to share their ideas aloud. As students share, annotate the statement to indicate the parts of the statement that students identify, without adding actual edits or corrections. If needed, point students' attention to the end of the statement so that 'to add the $3a$' is underlined or circled to indicate that part needs clarifying.
  - Provide students 2–3 minutes to work individually or in pairs to develop a second draft of the statement that is more complete, more correct, and more clear.
  - Invite one or two students to share their second drafts aloud with the class. Scribe as they share for all to see. Ask students to speak slowly and to repeat or clarify their words as they share, and invite the rest of the class to help as soon as it becomes clear that additional clarification or correction is needed. This process of collective public editing produces a "class third draft."
  - Listen for and amplify the language students use to describe what should happen when $(a+2)$ is multiplied by $3a$ and explain why the product should include a square term. This will help students understand how to write equivalent quadratic expressions using the distributive property.
- Next, have a student who drew a diagram for question 2a share their diagram. Ensure this is displayed for all to see. Then, invite students to connect the reasoning communicated in the "class third draft" just produced to the displayed diagram.
- Highlight for students:
  - The distributive property allows us to write equivalent expressions.
  - The distributive property is helpful in writing equivalent expressions for a product when a factor has more than 2 terms. For example, we can show $3x(2+x+4y)$ by drawing a rectangle with side lengths $3x$ and $2+x+4y$, and find the areas of the sub-rectangles: $6x$, $3x^2$, and $12xy$. Or we can distribute the multiplication of $3x$ to each term in the sum, which gives $6x+3x^2+12xy$.
  - In the next activity, we will look at how to multiply out (or expand) the factors in an expression when each factor is a sum.

DO THE MATH	PLANNING NOTES

Activity 2: Using Diagrams to Find Equivalent Quadratic Expressions (*15 minutes*)

Instructional Routine: Take Turns	
Addressing: NC.M1.A-APR.1	**Building Towards:** NC.M1.F-IF.8

In earlier lessons, students saw quadratic functions expressed in different ways—some written as products of two factors (for example, $d(25-d)$ or $16x^2$), and some written as sums (for example, $10+406t-16t^2$ or n^2+1). Here, they see that quadratic expressions that are products of two factors can be written as sums, and that rectangular area diagrams can be used to help us write equivalent expressions. As they use diagrams to transform expressions, they notice and make use of structure (MP7).

Step 1

- Give students a moment of quiet time to think about how to find an equivalent expression for $(10+2)(10+a)$. Invite students to share their strategies, which may include:
  - drawing a rectangle with side lengths $10+2$ (or 12) and $10+a$, partitioning it into sub-rectangles, finding the partial areas, and adding them
  - rewriting $10+2$ as 12, and then writing $12(10+a)$ or $120+12a$
  - multiplying each term in one factor with each term in the other factor, and then combining the partial products: $100+10a+20+2a=120+12a$
- Explain that some of the same strategies they used to expand $(10+2)(10+a)$ can be used to multiply two sums that each contains a variable, such as $(x+1)(x+3)$.

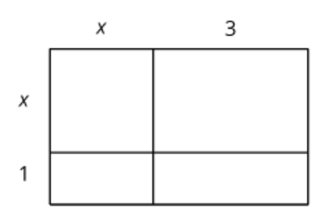
Allow students to continue working in the same pairs to discuss their reasoning as they work through the activity. Have students *Take Turns* with their partner for question 2, with one explaining what to do and the other recording the work. Also consider pausing the class after the second question. Make sure students notice how the partial areas in the diagram correspond to the expressions they produce. Noticing this structure will enable them to write equivalent expressions without drawing a diagram in the last question.

Advancing Student Thinking: Some students may be unfamiliar with decomposing a rectangular diagram into sub-rectangles, especially when the side lengths represent variable expressions like $2x+1$ and $x+3$. They may benefit from seeing an example involving only numbers. Show that to reason about $21 \cdot 13$, we can draw a rectangular diagram with side lengths $20+1$ and $10+3$, decompose the rectangle to separate the tens and ones on each side, compute the four partial areas separately, and then find the sum of those partial areas.

Some students may write $(x+5)^2$ as (x^2+5^2). Remind them that $(x+5)^2=(x+5)(x+5)$.

Student Task Statement

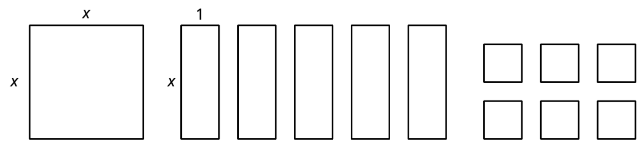
1. Here is a diagram of a rectangle with side lengths $x+1$ and $x+3$. Use this diagram to show that $(x+1)(x+3)$ and x^2+4x+3 are equivalent expressions.



2. Draw diagrams to help you write an equivalent expression for each of the following:
   a. $(x+5)^2$
   b. $2x(x+4)$
   c. $(2x+1)(x+3)$
   d. $(x+m)(x+n)$
3. Write an equivalent expression for each expression without drawing a diagram:
   a. $(x+2)(x+6)$
   b. $(x+5)(2x+10)$

Are You Ready For More?

1. Is it possible to arrange an x-by-x square, five x-by-1 rectangles, and six 1-by-1 squares into a single large rectangle? Explain or show your reasoning.
2. What does this tell you about an equivalent expression for $x^2 + 5x + 6$?



3. Is there a different non-zero number of 1-by-1 squares that we could have used instead that would allow us to arrange the combined figures into a single large rectangle?

Step 2

- Invite students to share some of their diagrams and discuss how they generalized their work with diagrams to write equivalent expressions without diagrams. Ask questions such as:
  - "To expand $(x+2)(x+6)$, which variables or numbers do we multiply first? Which do we multiply next? Is there a particular order we should follow?" (It doesn't matter, as long as each term in one factor is distributed to each term in the other factor. Drawing arrows can help us keep track of the distribution.)

$$
\begin{aligned}
&(x+2)(x+6) \\
&= x(x+6) + 2(x+6) \\
&= x^2 + 6x + 2x + (2)(6) \\
&= x^2 + (6+2)x + (2)(6)
\end{aligned}
$$

  - "Is $x^2 + 6x + 2x + 12$ equivalent to $(x+2)(x+6)$?" (Yes, but the $6x$ and $2x$ can also be combined to get a shorter expression: $x^2 + 8x + 12$.)
  - "Look at all of the expanded expressions. How are they alike?" (They all have a term with a squared variable, a linear term (or two) with a variable, and a constant term with no variable. Each constant term is a product of the two constant terms in the factors.)
  - "How are they different?" (Some of the squared terms have a coefficient of 2 or another number. One of the expressions shows only variables.)
- Before asking this question, consider writing $(x+88)(x+22) = x^2 + \underline{\quad\quad} x + (88 \cdot 22)$ for all to see. Then ask:
  - "If we expand $(x+88)(x+22)$, we know that the term with a squared variable will be x^2, and the constant term will be $88 \cdot 22$. How do we know what the remaining term(s) will be?" (It will be $88x$ and $22x$ or $(88+22)x$.
- Before asking this question, display two area diagrams for $(x+2)(x+6)$ for all to see before asking this question. Draw one with $(x+2)$ along the top edge and one with $x+6$ along the top edge. Then ask:
  - "Does it matter which side of the diagram we label $x+2$?" (No. Both diagrams represent rectangles with the same area, and both diagrams give the same expanded expression: $x^2 + 8x + 12$.)

RESPONSIVE STRATEGY

Use color-coding and annotations to highlight connections between representations in a problem. For example, use three different colors to make visible connections between the rectangle side lengths and the areas of each section. This will also help students keep track of each term of the product.

Supports accessibility for: Visual-spatial processing

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to have students become comfortable using the distributive property to rewrite the product of two sums in expanded form. The lesson makes use of structure first with area models. Through discussions and reasoning, students may arrive at generalizations connecting the use of distributive property and the structure of quadratic expressions.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

Invite students to reflect on the strategies for writing equivalent quadratic expressions by asking questions such as:

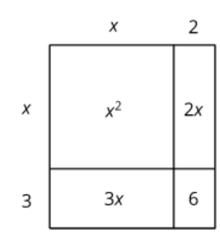
- "In what ways are area diagrams useful for expanding expressions like $(x+4)(x+9)$? Are there any drawbacks to drawing diagrams?" (The diagrams can help us see and keep track of the different parts of the factors that need to be multiplied and make sure they are all accounted for. Drawing a diagram every time we want to expand an expression would be time consuming.)
- "In what ways is using the distributive property helpful? Are there any drawbacks?" (Expanding expressions with the distributive property is probably quicker than drawing diagrams, but if we don't keep track of the terms closely, we may miss some terms that need to be multiplied.)
- "Which strategy would you choose to expand $(x+11)(2x+3)$? Why?"

PLANNING NOTES

Student Lesson Summary and Glossary

A quadratic function can often be defined by many different but equivalent expressions. For example, we saw earlier that the predicted revenue, in thousands of dollars, from selling a downloadable movie at x dollars can be expressed as $x(18-x)$, which can also be written as $18x-x^2$. The former is a product of x and $18-x$, and the latter is a difference of $18x$ and x^2, but both expressions represent the same function.

Sometimes a quadratic expression is a product of two factors that are each a linear expression, for example $(x+2)(x+3)$. We can write an equivalent expression by thinking about each factor, the $(x+2)$ and $(x+3)$, as the side lengths of a rectangle, and each side length decomposed into a variable expression and a number.



Multiplying $(x+2)$ and $(x+3)$ gives the area of the rectangle. Adding the areas of the four sub-rectangles also gives the area of the rectangle. This means that $(x+2)(x+3)$ is equivalent to $x^2+2x+3x+6$, or to x^2+5x+6.

Notice that the diagram illustrates the distributive property being applied. Each term of one factor (say, the x and the 2 in $x+2$) is multiplied by every term in the other factor (the x and the 3 in $x+3$).

$$\begin{aligned}&(x+2)(x+3)\\ &= x(x+3)+2(x+3)\\ &= x^2+3x+2x+(2)(3)\\ &= x^2+(3+2)x+(2)(3)\end{aligned}$$

In general, when a quadratic expression is written in the form of $(x+p)(x+q)$, we can apply the distributive property to rewrite it as $x^2+px+qx+pq$ or $x^2+(p+q)x+pq$.

Cool-down: Writing Equivalent Expressions (*5 minutes*)

Addressing: NC.M1.A-APR.1

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

1. Use a diagram to show that $(3x+1)(x+2)$ is equivalent to $3x^2+7x+2$.
2. Is $(x+4)^2$ equivalent to $2x^2+8x+8$? Explain or show your reasoning.

Student Reflection:

What are some of the similarities between creating equivalent expressions in prior lessons and grade levels and the equivalent expressions you created in this lesson?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Why is it important for students to be able to write and use different forms to represent and analyze quadratic functions?

Practice Problems

1. Draw a diagram to show that $(2x+5)(x+3)$ is equivalent to $2x^2+11x+15$.

2. Match each quadratic expression that is written as a product with an equivalent expression that is expanded.

a. $(x+2)(x+6)$ b. $(2x+8)(x+2)$ c. $(x+8)(x+4)$ d. $(x+2)(2x+6)$	1. $x^2+12x+32$ 2. $2x^2+10x+12$ 3. $2x^2+12x+16$ 4. $x^2+8x+12$

3. Select **all** expressions that are equivalent to x^2+4x.

   a. $x(x+4)$
   b. $(x+2)^2$
   c. $(x+x)(x+4)$
   d. $(x+2)^2-4$
   e. $(x+4)x$

4. Tyler drew a diagram to expand $(x+5)(2x+3)$.

   a. Explain Tyler's mistake.

   b. What is the correct expanded form of $(x+5)(2x+3)$?

	$2x$	3
x	$2x^2$	$3x$
5	$7x$	8

5. Based on past concerts, a band predicts selling $600-10p$ concert tickets when each ticket is sold at p dollars.

   a. Complete the table to find out how many concert tickets the band expects to sell and what revenues it expects to receive at the given ticket prices.

   b. In this model, at what ticket prices will the band earn no revenue at all?

   c. At what price should the band sell the tickets if it must earn at least 8,000 dollars in revenue to break even (to not lose money) on a given concert. Explain how you know.

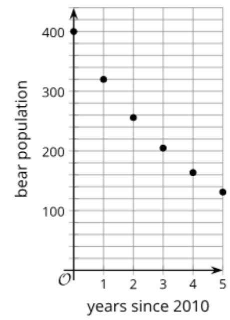
   (From Unit 7, Lesson 6)

Ticket price (dollars)	Number of tickets	Revenue (dollars)
10		
15		
20		
30		
35		
45		
50		
60		
p		

6. Explain why the values of the exponential expression 3^x will eventually overtake the values of the quadratic expression $10x^2$.

   (From Unit 7, Lesson 3)

7. A population of bears decreases exponentially.



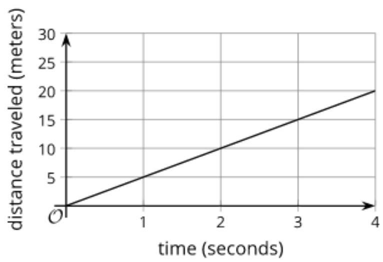
   a. What is the annual decay factor for the bear population? Explain how you know.
   b. Using function notation, represent the relationship between the bear population, b, and the number of years since the population was first measured, t. That is, find a function f so that $b = f(t)$.

   (From Unit 6)

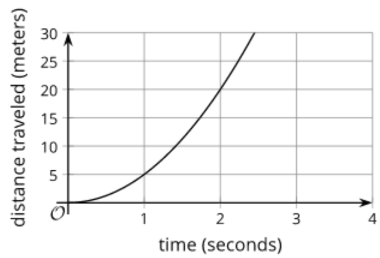
8. A baseball travels d meters in t seconds after being dropped from the top of a building. The distance traveled by the baseball can be modeled by the equation $d = 5t^2$.

   Which graph could represent this situation? Explain how you know.

Graph A



Graph B



   (From Unit 6)

9. Equations defining functions a, b, c, d, and f are shown here.

   Select **all** the equations that represent exponential functions.

   a. $a(x) = 2^3 \cdot x$
   b. $b(t) = (\frac{2}{3})^t$
   c. $c(m) = \frac{1}{5} \cdot 2^m$
   d. $d(x) = 3x^2$
   e. $f(t) = 3 \cdot 2^t$

   (From Unit 6)

10. Consider a function q defined by $q(x) = x^2$. Explain why negative values are not included in the range of q.

   (From Unit 5)

Lesson 8: Standard Form and Factored Form

PREPARATION

Lesson Goals	Learning Targets
• Comprehend the terms “standard form” and “factored form” (in written and spoken language). • Use rectangular diagrams to reason about the product of two differences or of a sum and difference and to write equivalent expressions. • Use the distributive property to write quadratic expressions given in factored form in standard form.	• I know the difference between “standard form” and “factored form.” • I can rewrite quadratic expressions given in factored form in standard form using either the distributive property or a diagram.

Lesson Narrative

Previously, students used area diagrams to expand expressions of the form $(x+p)(x+q)$ and generalized that the expanded expressions take the form of $x^2+(p+q)x+pq$. In this lesson, they see that the same generalization can be applied when the factored expression that contains a sum and a difference (when p or q is negative) or two differences (when both p and q are negative).

Although they have encountered an algebraic approach, students still benefit from drawing diagrams to expand unfamiliar factored expressions. Area diagrams are intuitive for visualizing the product of two sums, but they are less intuitive for visualizing the product of two differences (for example, $(x-5)^2$) or the product of a sum and a difference (for example, $(x+3)(x-4)$). Subtraction can be represented by removing parts of a rectangle and finding the area of the remaining region, but this strategy can get complicated when both factors are differences.

At this point, students transition from thinking about rectangular diagrams concretely, in terms of area, to thinking about them more abstractly, as a way to organize the terms in each factor. (Students made similar transitions from area diagrams to abstract diagrams in middle school, for example, when they learned to distribute the multiplication of a number or a variable—positive and negative—over addition and subtraction.)

Students also learn to use the terms **standard form** and **factored form**, and the terms **coefficient, linear term**, and **constant term** as they relate to standard form. When classifying quadratic expressions by their form, students refine their language and thinking about quadratic expressions (MP6). In an upcoming lesson, students will graph quadratic expressions of these forms and study how features of the graphs relate to the parts of the expressions.

What strategies or representations do you anticipate students might use in this lesson?

Focus and Coherence

Building On	Addressing	Building Towards
NC.4.NBT.5: Multiply a whole number of up to three digits by a one-digit whole number, and multiply up to two two-digit numbers with place value understanding using area models, partial products, and the properties of operations. Use models to make connections and develop the algorithm. **NC.6.NS.9:** Apply and extend previous understandings of addition and subtraction. • Describe situations in which opposite quantities combine to make 0. • Understand $p+q$ as the number located a distance qq from p, in the positive or negative direction depending on the sign of q. Show that a number and its additive inverse create a zero pair. • Understand subtraction of integers as adding the additive inverse, $p-q=p+(-q)$. Show that the distance between two integers on the number line is the absolute value of their difference. • Use models to add and subtract integers from -20 to 20 and describe real-world contexts using sums and differences.	**NC.M1.A-APR.1:** Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions. **NC.M1.A-SSE.1a:** Interpret expressions that represent a quantity in terms of its context. **a.** Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.	**NC.M1.F-IF.8:** Use equivalent expressions to reveal and explain different properties of a function.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
    - M1.U7.L8 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.4.NBT.5

In this lesson, students extend the diagrams from the previous lesson to work with negative values. This bridge provides students an opportunity to look at different types of diagrams, revealing different partial products yet still resulting in the correct final answer. Multiplication of negative values with a diagram are also considered as a preview to the work of the lesson.

Student Task Statement

1. Here are two diagrams that could be used to multiply $47 \cdot 62$. Which diagram do you think is most helpful and why?[1]

Diagram A

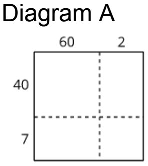
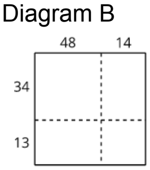
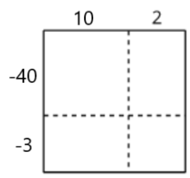


Diagram B



[1] Adapted from IM K–5, Grade 4, Unit 6, Practice Section B https://curriculum.illustrativemathematics.org/K5/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

2. To multiply $-43 \cdot 12$, a student drew this diagram, found the product for each of the four boxes, and then found their sum. Does this diagram work with a negative factor? Explain.



3. Would a diagram work to multiply $-36 \cdot -19$? If so, draw the diagram.

DO THE MATH

PLANNING NOTES

Warm-up: Opposites Attract (*5 minutes*)

Instructional Routines: Math Talk; Discussion Supports (MLR8) - Responsive Strategy	
Building On: NC.6.NS.9	

This *Math Talk* encourages students to think about the fact that subtracting a number is equivalent to adding the opposite of that number (that is, $100 - 5 = 100 + -5$) and to rely on the structure of the expressions on each side of the equal sign to mentally solve problems. The understandings elicited here will be helpful later in the lesson when students reason that $(x-1)(x-4)$ is equivalent to $(x+-1)(x+-4)$, which in turn helps them expand the expression using diagrams and the distributive property.

Some students may find the value of n by reasoning. For example, to find n in $40 - 8 = 40 + n$, they see that the left side of the equation is 32 and reason that the number being added to 40 must be negative, and that it must be -8. Others may carry out the steps for solving equations, for example, by subtracting 40 from both sides, or by adding 8 to both sides and then subtracting 48 from both sides.

Any of these strategies are fine. The key point is to recognize that the number being added on one side is the opposite of the value being subtracted from the other side. If after a couple of questions students show an understanding of this, and if time is limited, it is not essential for students to complete all questions.

As students solve these equations mentally, they practice looking for and making use of structure (MP7).

Step 1

- Display one problem at a time.
- Give students quiet think time for each problem and ask them to give a signal when they have an answer and a strategy.
- Keep all problems displayed throughout the talk.

RESPONSIVE STRATEGY

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Solve each equation mentally.

1. $40 - 8 = 40 + n$
2. $25 + -100 = 25 - n$
3. $3 - \frac{1}{2} = 3 + n$
4. $72 - n = 72 + 6$

Step 2

- Facilitate the Math Talk by asking students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:
  - "Who can restate ____'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone solve the problem in a different way?"
  - "Does anyone want to add on to ____'s strategy?"
  - "Do you agree or disagree? Why?"
- If not already made clear in students' explanations, highlight that subtracting a number gives the same outcome as adding the opposite of that number. This may be helpful to rewrite quadratic expressions where one or both factors are differences. For example $(x-5)(x+2)$ can be rewritten as $(x+-5)(x+2)$.

RESPONSIVE STRATEGY

Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because...." or "I noticed ____ so I...." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Discussion Supports (MLR8)

DO THE MATH	PLANNING NOTES

Activity 1: Finding Products of Differences (*15 minutes*)

Addressing: NC.M1.A-APR.1	**Building Towards:** NC.M1.F-IF.8

In this activity, students encounter quadratic expressions in factored form where at least one of the factors is a difference.

Some students may find reasoning with a diagram helpful while others may apply the distributive property without the use of a diagram. Both approaches are welcome. Previously, students thought about the parts of the diagram as representing length, width, and area. With the introduction of negative numbers, these diagrams no longer have a physical interpretation. Nonetheless, students see that using a diagram will still produce the correct product of two factors, and can be a useful way to organize their work. If time is limited, note that it is not necessary to wait until all students are finished with all problems to begin the discussion.

Monitoring Tip: Take note of students who:

- find it useful to reason with a diagram
- transfer what they learned when multiplying two sums in the preceding lesson—that $(x+p)(x+q)$ is equivalent to $x^2+(p+q)x+pq$—and use negative numbers for p and q

Step 1

- Remind students of the diagrams they've used to expand expressions such as $x(x+5)$ to write equivalent expressions. Tell students that we are now going to try making an "area" diagram where one of the side lengths involves subtraction. Start with the expression $x(x-5)$. Show students that this can be rewritten as $x(x+-5)$. Then label the diagram as follows, perhaps asking students what should go in the inner boxes after labeling the outer boxes:

	x	-5
x	x^2	$-5x$

- Tell students that the diagram shows $x(x-5)=x(x+-5)=x^2+-5x=x^2-5x$.

RESPONSIVE STRATEGIES

To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure that students have been able to transition from using area diagrams to the rectangle diagrams that allow for negative values. Some students may benefit from additional support to learn how to draw appropriate diagrams. Consider providing access to some blank or partially completed diagrams to help students get started.

Supports accessibility for: Memory; Organization

Step 2

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Provide students with 2–3 minutes of quiet think time to work independently on the first question and then share what they have come up with with their partner.
- Partners work together to complete the task.

Advancing Student Thinking: Some students will write the expression in the last question as x^2-2^2. Remind them that just as x^2 means $x \cdot x$, the expression $(x-2)^2$ means $(x-2)(x-2)$.

Student Task Statement

1. Show that $(x-1)(x-1)$ and x^2-2x+1 are equivalent expressions by drawing a diagram or applying the distributive property. Show your reasoning.
2. For each expression, write an equivalent expression. Show your reasoning.

   a. $(x+1)(x-1)$ b. $(x-2)(x+3)$ c. $(x-2)^2$

Step 3

- Facilitate a whole-class discussion, beginning with having a student who used a diagram display their work. Have a student who did not use a diagram share their work on the same problem. Ask students:
  - "What connections do you see between the two approaches?" (The partial products of the diagram are revealed by those who apply the distributive property to distribute the multiplication of each term in one factor to each term in the other factor. The order may be different but each reasoning should reveal the same partial products.)

$$\begin{aligned}
&(x-1)(x-1)\\
&=(x+-1)(x+-1)\\
&=x(x+-1)+-1(x+-1)\\
&=x^2+-1x+-1x+(-1)(-1)\\
&=x^2+-2x+1\\
&=x^2-2x+1
\end{aligned}$$

  - "How are the expressions $(x-1)(x-1)$ and $(x+-1)(x+-1)$ alike and different?" (The expressions are equivalent; the first expression shows the product of two differences, and the second expression shows the product of two sums.)

- Display the following reasoning to students. Give students time to read the diagram and ask questions, or pick out steps to ask about based on your observations of students during work time.

DO THE MATH	PLANNING NOTES

Activity 2: What Is the Standard Form? What Is the Factored Form? (*10 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1)	
Addressing: NC.M1.A-SSE.1a	**Building Towards:** NC.M1.F-IF.8

Students have seen quadratic expressions in both standard form and factored form since the beginning of the unit. In this activity, they learn to distinguish the expressions by their forms and to refer to each form by its formal name. Refining their language about the different forms prepares students to be more precise in their thinking about the graphs of quadratic functions in later lessons (MP6).

Sometimes, when modeling physical phenomena, it is more natural to write the constant term first or to write factored form using a variable with a negative coefficient. (See Activity 1 in Lesson 9 for examples of both). For these reasons, we still consider quadratic expressions containing the terms ax^2+bx+c written in a different order to be in standard form, and we consider expressions like $(-x+1)(x-2)$ to be in factored form.

Step 1

- Keep students in groups from previous activities.

- Provide students with 2–3 minutes of quiet work time and then time to share their work with a partner. Pairs should share their thinking for question 1 before engaging in the Stronger and Clearer routine for question 2.

Student Task Statement

The quadratic expression x^2+4x+3 is written in standard form.

Here are some other quadratic expressions. The expressions on the left are written in standard form, and the expressions on the right are not.

Written in standard form:	Not written in standard form:
x^2-1	$(2x+3)x$
x^2+9x	$(x+1)(x-1)$
$\frac{1}{2}x^2$	$3(x-2)^2+1$
$4x^2-2x+5$	$-4(x^2+x)+7$
$-3x^2-x+6$	$(x+8)(-x+5)$
$1-x^2$	

1. What are some characteristics of expressions in standard form?
2. $(x+1)(x-1)$, $(2x+3)x$, and $(x+8)(-x+5)$ in the right column are quadratic expressions written in **factored form**. Why do you think that form is called factored form?

Are You Ready For More?

Which quadratic expression can be described as being both standard form and factored form? Explain how you know.

Step 2

- Use the *Stronger and Clearer Each Time* routine to focus students' attention on the second question, "Why do you think that form is called factored form?"
  - Provide 2–3 minutes for students to meet in pairs to share their initial responses or initial thinking about this question. As pairs are sharing, provide them with the following prompts for feedback that will help add detail to strengthen and clarify each other's ideas and writing:
    - "Can you say more about what each expression means?"
    - "I understand ____, but can you clarify ____?"
  - Provide 1–2 minutes for students to meet with a second partner, share responses and thinking, and give and receive feedback on their ideas and writing.
  - Finally, provide students 2–3 minutes to revise their initial response to problem 2. Encourage students to make their responses stronger and clearer by incorporating the feedback and any good ideas from their partners.

Step 3

- Display the two columns of expressions for all to see. Solicit students' ideas on the features of each form. Record their responses for all to see. Invite other students to express agreement or disagreement or to clarify their fellow students' responses.
- Define a quadratic expression in standard form explicitly as $ax^2 + bx + c$. Explain that we refer to a as the coefficient of the squared term x^2, b as the coefficient of the linear term x, and c as the constant term. Mention that we usually write the terms in that order, but an expression like $6 + 3x - 5x^2$ is still considered to be in standard form, with $a = -5$, $b = 3$, and $c = 6$.
- As time permits, use the following questions. Ask students:
  - "How would you write $(2x + 3)x$ in standard form?" $(2x^2 + 3x)$
  - "The expression $2x^2 + 3x$ only has two terms. Is it still in standard form?" (Yes, there is no constant term, which means c is 0.)
  - "How would you write $-4(x^2 + \frac{1}{4}x) + 7$ in standard form?" ($-4x^2 - x + 7$)
  - "What are the values of the coefficients a and b?" (-4 and -1)
  - "What is the value of the constant term?" (7)
- Clarify that a quadratic expression in factored form is a product of two factors that are each a linear expression. For example, $(x + 1)(x - 1)$, $(2x + 3)x$, and $x(4x)$ all have two linear expressions for their factors. An expression with two factors that are linear expressions and a third factor that is a constant, for example: $2(x + 2)(x - 1)$, is also in factored form. We will continue to look at this form of a quadratic expression in future lessons.

DO THE MATH	PLANNING NOTES

Lesson Debrief (5 minutes)

The purpose of this lesson is for students to extend the concept of multiplying two sums to multiplying two differences or a sum and a difference. Students are formally introduced to the terminology "factored form" and "standard form" to more precisely define the expressions they have been working with.

<table>
<tr>
<td>
Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

To help students consolidate the ideas in this lesson, consider asking the following questions:

● "What does it mean to expand a factored expression?" (to multiply out all the terms in each factor)
</td>
<td>PLANNING NOTES</td>
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- "Show how a diagram can help us expand $(x+4)(x-10)$." (The partial products are x^2, $4x$, $-10x$, and -40. The sum of those products is $x^2-6x-40$.)
- "How can we expand $(x+4)(x-10)$?" ($(x^2-6x-40)$)
- "How would you explain to a friend who is absent today how to write $(x-10)(x-5)$ in standard form? What strategy (or strategies) would you suggest?"
- "Give a few different examples of quadratic expressions in standard form and a few in factored form. Ask a partner if they agree that your examples are indeed in those forms."
- "Is the expression x^2+1 in standard form? (Yes, it is not the product of two linear factors. In ax^2+bx+c, $a=1$, $b=0$, and $c=1$.)
- "How can we write $x(x-6)+2x$ in standard form?" (x^2-4x)

Student Lesson Summary and Glossary

A quadratic function can often be represented by many equivalent expressions. For example, a quadratic function f might be defined by $f(x)=x^2+3x+2$. The quadratic expression x^2+3x+2 is said to be in **standard form**.

> **Standard form** (of a quadratic expression)**:** The standard form of a quadratic expression is ax^2+bx+c, where a, b, and c are constants, and a is not 0.

In standard form, we refer to a as the **coefficient** of the squared term x^2, b as the **coefficient** of the **linear term** x, and c as the **constant term**. In $f(x)=x^2+3x+2$, $a=1$, $b=3$, and $c=2$.

> **Coefficient:** In an algebraic expression, the coefficient of a variable is the constant the variable is multiplied by. If the variable appears by itself then it is regarded as being multiplied by 1 and the coefficient is 1. The coefficient of x in the expression $3x+2$ is 3. The coefficient of p in the expression $5+p$ is 1.
>
> **Constant term:** In an expression like $5x+2$, the number 2 is called the constant term because it doesn't change when x changes. In the expression $5x-8$, the constant term is -8, because we think of the expression as $5x+(-8)$. In the expression $12x-4$, the constant term is -4.
>
> **Linear term:** The linear term in a quadratic expression (in standard form) ax^2+bx+c, where a, b, and c are constants, is the term bx. (If the expression is not in standard form, it may need to be rewritten in standard form first.)

The function f can also be defined by the equivalent expression $(x+2)(x+1)$. When the quadratic expression is a product of two factors where each one is a linear expression, this is called the **factored form**. The expression $3(x-7)(x+1)$ is also in factored form: the product of a number and two linear expressions.

> **Factored form** (of a quadratic expression)**:** A quadratic expression that is written as the product of a constant times two linear factors is said to be in factored form. For example, $2(x-1)(x+3)$ and $(5x+2)(3x-1)$ are both in factored form.

An expression in factored form can be rewritten in standard form by expanding it, which means multiplying out the factors. In a previous lesson, we saw how to use a diagram and to apply the distributive property to multiply two linear expressions, such as $(x+3)(x+2)$. We can do the same to expand an expression with a sum and a difference, such as $(x+5)(x-2)$, or to expand an expression with two differences, such as $(x-4)(x-1)$.

To represent $(x-4)(x-1)$ with a diagram, we can think of subtraction as adding the opposite:

$$\begin{aligned}
&(x-4)(x-1)\\
&= (x + -4)(x + -1)\\
&= x(x + -1) + -4(x + -1)\\
&= x^2 + -1x + -4x + (-4)(-1)\\
&= x^2 + -5x + 4\\
&= x^2 - 5x + 4
\end{aligned}$$

	x	-4
x	x^2	$-4x$
-1	$-x$	4

Cool-down: From One Form to Another (*5 minutes*)

Addressing: NC.M1.A-APR.1

Cool-down Guidance: Points to Emphasize
Use student work to identify and address common errors and strategies to use to avoid errors (for example, making a diagram, re-writing expressions with subtraction as equivalent addition problems). Consider making an anchor chart that highlights these strategies to support student work with factoring later in the unit.

Cool-down

For each expression, write an equivalent expression in standard form. Show your reasoning.

1. $(2x+5)(x+1)$
2. $(x-2)(x+2)$

Student Reflection:

What was the best support or encouragement you received in math class today? Who gave you that support or encouragement?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Which teaching or support strategies are you most proud of using thus far in this unit? Which are you looking forward to changing going forward?

Practice Problems

1. Write each quadratic expression in standard form. Draw a diagram if needed.
   a. $(x+4)(x-1)$
   b. $(2x-1)(3x-1)$
2. Consider the expression $8-6x+x^2$.
   a. Is the expression in standard form? Explain how you know.
   b. Is the expression equivalent to $(x-4)(x-2)$? Explain how you know.
3. Which quadratic expression is written in standard form?
   a. $(x+3)x$
   b. $(x+4)^2$
   c. $-x^2-5x+7$
   d. $x^2+2(x+3)$
4. Explain why $3x^2$ can be said to be in both standard form and factored form.
5. (*Technology required*.) Two rocks are launched straight up in the air. In both functions, t is time measured in seconds, and height is measured in feet.
   - The height of rock A is given by the function f, where $f(t)=4+30t-16t^2$.
   - The height of rock B is given by function g, where $g(t)=5+20t-16t^2$.

   Use graphing technology to graph both equations.
   a. What is the maximum height of each rock?
   b. Which rock reaches its maximum height first? Explain how you know.

   (From Unit 7, Lesson 5)

6. A football player throws a football. The function h given by $h(t) = 6 + 75t - 16t^2$ describes the football's height in feet t seconds after it is thrown.

   Select **all** the statements that are true about this situation.

   a. The football is thrown from ground level.
   b. The football is thrown from 6 feet off the ground.
   c. In the function h, $-16t^2$ represents the effect of gravity.
   d. The outputs of h decrease and then increase in value.
   e. The function h has 2 zeros that make sense in this situation.
   f. The vertex of the graph of h gives the maximum height of the football.

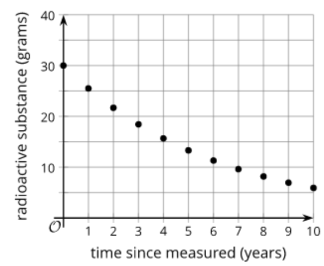
   (From Unit 7, Lesson 5)

7. Jada dropped her sunglasses from a bridge over a river. Which equation could represent the distance, y, fallen in feet as a function of time, t, in seconds?

   a. $y = 16t^2$
   b. $y = 48t$
   c. $y = 180 - 16t^2$
   d. $y = 180 - 48t$

   (From Unit 7, Lesson 4)

8. The graph shows the number of grams of a radioactive substance in a sample at different times after the sample was first analyzed.



   a. What is the average rate of change for the substance during the 10-year period?

   b. Is the average rate of change a good measure for the change in the radioactive substance during these 10 years? Explain how you know.

   (From Unit 6)

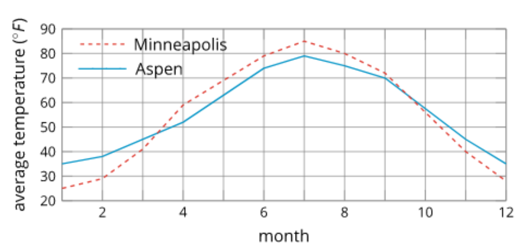
9. Each day after an outbreak of a new strain of the flu virus, a public health scientist receives a report of the number of new cases of the flu reported by area hospitals.

Days since outbreak	2	3	4	5	6	7
Number of new cases of the flu	20	28	38	54	75	105

   Would a linear or exponential model be more appropriate for these data? Explain how you know.

   (From Unit 6)

10. $A(t)$ is the average high temperature in Aspen, Colorado, t months after the start of the year. $M(t)$ is the temperature in Minneapolis, Minnesota, t months after the start of the year. Temperature is measured in degrees Fahrenheit.



   a. What does $A(8)$ mean in this situation? Estimate $A(8)$.

   b. Which city had a higher average temperature in February?

   c. Were the two cities' average high temperatures ever the same? If so, when?

   (From Unit 5)

11. Show two different strategies to multiply 23×54.

   (Addressing NC.4.NBT.5)

Lesson 9: Graphs of Functions in Standard and Factored Forms

PREPARATION

Lesson Goals	Learning Targets
• Connect (orally and in writing) a quadratic expression given in factored form and the intercepts of its graph. • Interpret (orally and in writing) the meaning of x-intercepts and y-intercepts on a graph of a quadratic function that represents a context.	• I know how the numbers in the factored form of a quadratic expression relate to the intercepts of its graph. • I can explain the meaning of the intercepts on a graph of a quadratic function in terms of the situation it represents.

Lesson Narrative

This lesson serves two goals. The first is to relate the work in the past couple of lessons on quadratic expressions back to the quadratic functions that represent situations. Now students have additional insights that enable them to show (algebraically) that two different expressions can define the same function.

The second goal is to prompt students to notice connections between different forms of quadratic expressions and features of the graphs that represent the expressions. Students are asked to identify the x- and y-intercepts of graphs representing expressions in standard and factored form. They observe that some numbers in the expressions are related to the intercepts and hypothesize about the patterns they observe (MP7). This work sets the foundation for upcoming lessons, in which students look more closely at how the parameters of quadratic expressions are related to their graphs.

What teaching strategies will you be focusing on during this lesson?

Focus and Coherence

Building On	Addressing	Building Towards
NC.7.EE.3: Solve multi-step real-world and mathematical problems posed with rational numbers in algebraic expressions. • Apply properties of operations to calculate with positive and negative numbers in any form. • Convert between different forms of a number and equivalent forms of the expression as appropriate.	**NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior. **NC.M1.A-APR.3:** Understand the relationships among the factors of a quadratic expression, the solutions of a quadratic equation, and the zeros of a quadratic function.	**NC.M1.F-IF.8a:** Use equivalent expressions to reveal and explain different properties of a function. **a.** Rewrite a quadratic function to reveal and explain different key features of the function.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
    - M1.U7.L9 Cool-down (print 1 copy per student)

LESSON

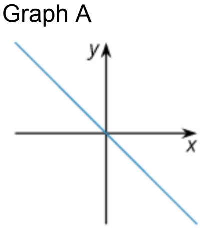
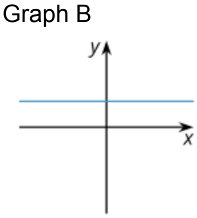
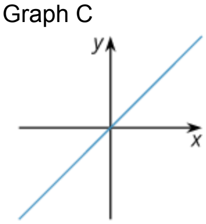
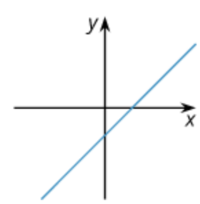
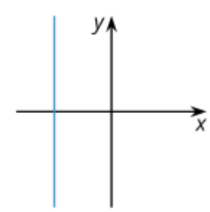
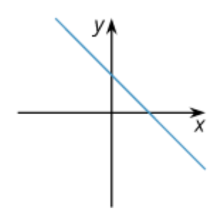
Bridge (*Optional, 5 minutes*)

Building On: NC.M1.A-REI.10

The purpose of this bridge is to reinforce for students that the form and parts of an equation can provide information about the graph of a linear equation, including the slope, the intercepts, or the position relative to the axes. This may be a useful reminder prior to students exploring intercepts for a graph of a quadratic function in this lesson.

Student Task Statement

Here are some equations and graphs. Without technology, match each graph to one or more equations that it *could* represent. Be prepared to explain how you know.

Graph A	Graph B	Graph C	Graph D	Graph E	Graph F
					

- $y = 8$
- $y = 3x - 2$
- $x + y = 6$
- $0.5x = -4$
- $y = x$
- $-\frac{2}{3}x = y$
- $12 - 4x = y$
- $x - y = 12$
- $2x + 4y = 16$
- $3x = 5y$

DO THE MATH

PLANNING NOTES

Warm-up: Quadratic Quandary (*5 minutes*)

Instructional Routine: Which One Doesn't Belong?	
Building On: NC.M1.F-IF.7	

This warm-up prompts students to carefully analyze and compare graphs of quadratic functions using the *Which One Doesn't Belong?* routine. In making comparisons, students have a reason to use language precisely (MP6).

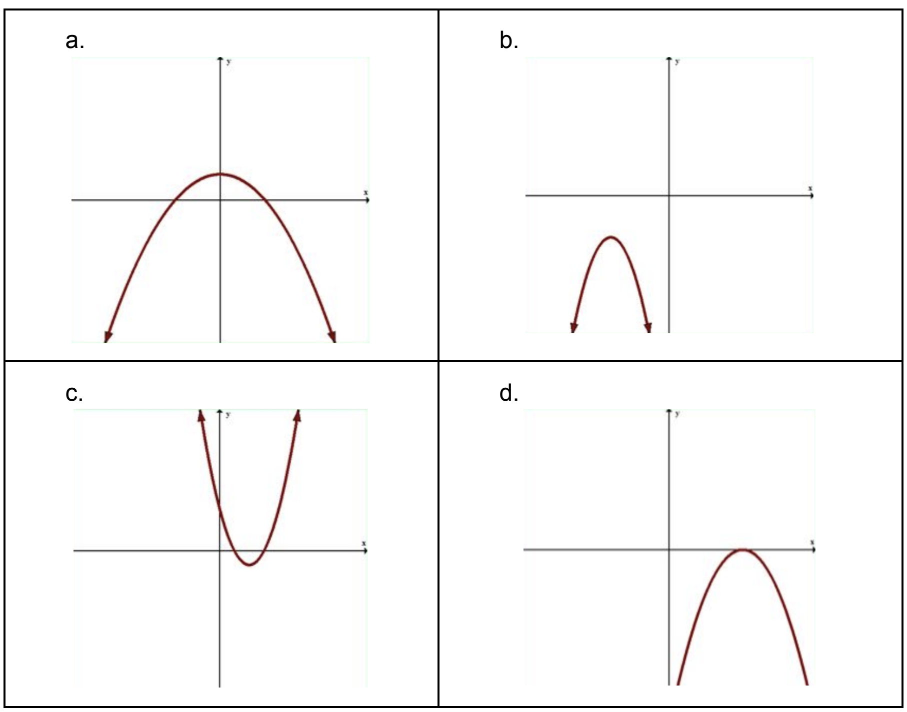
The work here prepares students to think about key features of graphs of quadratic functions.

Step 1

- Ask students to arrange themselves in small groups or use visibly random grouping.
- Display the graphs for all to see.
- Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, ask each student to share their reasoning as to why a particular graph does not belong, and together find at least one reason each item doesn't belong.

Student Task Statement

Which graph doesn't belong? Explain your reasoning.[1]



Step 2

- In a whole-class discussion, ask each group to share one reason why one of the graphs doesn't belong. Provide space for students to agree, disagree, or ask questions of other groups.
- Tell students that graphs of quadratic functions can provide information about the quadratic expressions and vice versa. These connections will be explored in this lesson and upcoming ones.

[1] Adapted with permission from www.wodb.ca.

DO THE MATH	PLANNING NOTES

Activity 1: Revisiting Projectile Motion (*10 minutes*)

Addressing: NC.M1.F-IF.7	**Building Towards:** NC.M1.F-IF.8a

In the past few lessons, students have reasoned symbolically and formally about quadratic expressions. This activity ties that work back to the quadratic functions that represent situations from earlier in the unit.

Students are given two expressions that represent a familiar quadratic function. They identify these expressions by their form and justify why the two are equivalent, using what they learned about expanding factored expressions. Students also study the graph that represents the function and interpret the features of the graph in context. Students are asked to interpret the meaning of the horizontal and vertical intercepts rather than the x- and y-intercepts since the variables x and y are not used to define the function.

Step 1

- Ask students to arrange themselves in groups of two or use visibly random grouping. Students will remain in these groups for the remainder of the lesson.
- Provide 3–4 minutes of quiet work time and then time for partners to share their thinking with each other.

RESPONSIVE STRATEGY

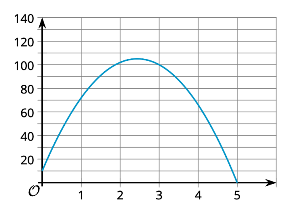
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down.

Supports accessibility for: Language; Social-emotional skills

Advancing Student Thinking: Some students may not recognize that the quadratic terms are the same if they identify the squared term as $16t^2$ rather than $-16t^2$. Some may ignore the subtraction sign in $10+78t-16t^2$ or associate it with the $78t$ rather than the $16t^2$. Show them a simpler expression such as $5-10$ and rewrite it as $5+-10$. Then, ask them to rewrite the expression defining h in a similar way.

Student Task Statement

In an earlier lesson, we saw that an equation such as $h(t)=10+78t-16t^2$ can model the height of an object thrown upward from a height of 10 feet with a vertical velocity of 78 feet per second.



1. Is the expression $10+78t-16t^2$ written in standard form? Explain how you know.
2. Jada said that the equation $g(t)=(-16t-2)(t-5)$ also defines the same function, written in factored form. Show that Jada is correct.
3. Here is a graph representing both $g(t)=(-16t-2)(t-5)$ and $h(t)=10+78t-16t^2$.
    a. Identify or approximate the vertical and horizontal intercepts.
    b. What do each of these points mean in this situation?

Step 2

- Facilitate a whole-class discussion. Make sure that:
  - Students see that the expression $10+78t-16t^2$ is in standard form, even though it is written as $c+bx+ax^2$. Clarify that -16 is a (the coefficient of the squared term), 78 is b (the coefficient of the linear term), and 10 is c (the constant term).
  - Students can explain the equivalence of $(-16t-2)(t-5)$ and $10+78t-16t^2$ using a strategy from earlier lessons (for example, drawing a diagram or applying the distributive property).
- Ask students if they notice any connections between the two equations and the features of the graph. Some may notice the 10 in the standard form of the equation tells us the vertical intercept is $(0,10)$. Others may predict that the subtraction of 5 has something to do with the horizontal intercepts or may not see any connections. Any observation is fine at this point, as students will look closely at equations and graphs starting in the next activity.

DO THE MATH	PLANNING NOTES

Activity 2: Relating Expressions and Their Graphs (*15 minutes*)

Instructional Routines: Take Turns; Collect and Display (MLR2)	
Addressing: NC.M1.F-IF.7; NC.M1.A-APR.3	**Building Towards:** NC.M1.F-IF.8a

The goal of this activity is to uncover the connections between the x- and y-intercepts on the graph and the parameters of quadratic expressions in standard and factored form. Earlier in the unit, students learned that the zeros of a quadratic function are x-values that produce y-value of 0, and that the zeros of a function are the x-coordinates of the x-intercepts of the graph. This idea comes into focus in this activity.

Note that in the examples given here, the y-coordinate of the y-intercept is always equal to the product of the zeros, but this is not the case with all graphs representing quadratic functions. (For example, the graph representing $y=2x^2-8$ intercepts the x-axis at 2 and -2, but the y-intercept is $(0,-8)$.) If students make this observation, consider acknowledging that this seems to be true for these graphs and prompting students to revisit this observation in upcoming lessons (in which students will also study graphs) to see if it is always true.

Step 1

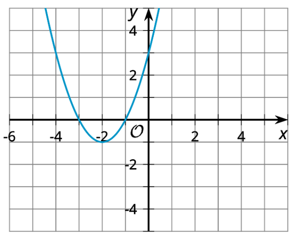
- Give students a few minutes of quiet time to complete the first two questions.
- Students should *Take Turns* sharing their responses for the parts of question 1, one partner per function unless there is disagreement, then both share for question 2, and collaboratively respond to question 3. Use the *Collect and Display* routine to scribe the words and phrases students use as they discuss their observations in question 2 with a partner and as they complete the last question.

Student Task Statement

Here are pairs of expressions in standard and factored forms. Each pair of expressions define the same quadratic function, which can be represented with the given graph.

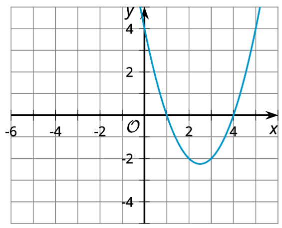
1. Identify the x-intercepts and the y-intercept of each graph.

Function f
x^2+4x+3
$(x+3)(x+1)$



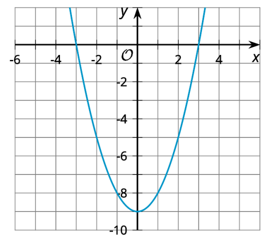
x-intercepts: ______
y-intercept: ______

Function g
x^2-5x+4
$(x-4)(x-1)$



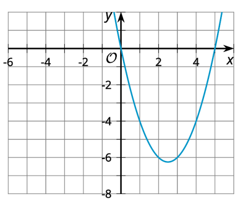
x-intercepts: ______
y-intercept: ______

Function h
x^2-9
$(x-3)(x+3)$



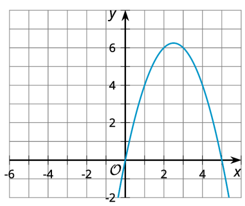
x-intercepts: ______
y-intercept: ______

Function i
x^2-5x
$x(x-5)$



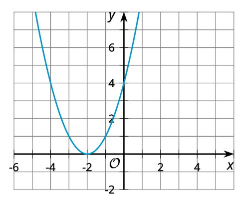
x-intercepts: ______
y-intercept: ______

Function j
$5x-x^2$
$x(5-x)$



x-intercepts: ______
y-intercept: ______

Function k
x^2+4x+4
$(x+2)(x+2)$



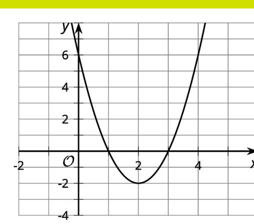
x-intercept: ______

2. What do you notice about the x-intercepts, the y-intercept, and the numbers in the expressions defining each function? Make a couple of observations.

3. Here is an expression that models function p, another quadratic function: $(x-9)(x-1)$. Predict the x-intercepts and the y-intercept of the graph that represent this function.

Are You Ready For More?

Find the values of a, p, and q that will make $y=a(x-p)(x-q)$ be the equation represented by the graph.



Step 2

- Invite students to share their observations about how the numbers in the quadratic expressions relate to the intercepts of the graphs. Refer to any relevant collected and displayed student language, and update with additional language students use during this discussion.
- Ask students to share their predictions for the x- and y-intercepts of the graph of function p, defined by $(x-9)(x-1)$. Discuss with students:
  - "How did you find the x-intercept of the graph of function p without graphing? (By looking at the intercepts in other graphs with similar factors; in the examples, expressions of the form $(x-a)(x-b)$ have $(a,0)$ and $(b,0)$ for the x-intercepts.)
  - "How did you find the y-intercept?" (By writing the expression in standard form and seeing what the constant term is; by evaluating the expression at $x=0$.)

RESPONSIVE STRATEGY

Use color-coding and annotations to highlight connections between representations in a problem. For example, use color to illustrate students' observations of how the numbers in the quadratic expressions relate to the intercepts of the graphs.

Supports accessibility for: Visual-spatial processing

- Demonstrate graphing $p(x)=(x-9)(x-1)$ using the technology available in your classroom. Point to the intercepts. If using Desmos, show students that you can click on the intercepts to reveal the coordinates of the points.
- Remind students that earlier in the unit, we learned that the x-intercepts of a graph tell us the zeros of the function or the input values that produce an output of 0. Highlight that because an expression in factored form can tell us about the x-intercept of the graph, this form is also handy for telling us about the zeros of the function that the expression represents.
- Using technology, graph $l(x)=x(4x-14)$ and $m(x)=(2x+5)(x-2)$. Ask students to share with their partner what is similar and different about these functions compared to the ones they investigated in this activity. (One factor in each of these function rules has a coefficient of x that is not 1 (like 4 in $4x$), whereas the function rules in the activity were of the form $(x\pm a)(x\pm b)$; for the functions in the activity task, we could "see" the x-intercepts/ zeros in the function rule, but in these functions, we cannot see this so obviously.)
- At this point, students are just noticing that numbers in the expression have something to do with intercepts on the graph. Tell students that, in the next lesson, we will explore why they are related.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to see that quadratic expressions can give us clues about the intercepts of their graphs, and graphs can give us insights about the expressions they represent.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

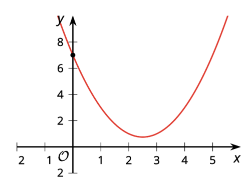
Facilitate a class discussion asking the following questions:

- "If we graph $y=(x-7)(x-3)$ and $y=x^2-10x+21$, will we end up with the same graph? How do you know?" (Yes. The two expressions that define y are equivalent. Expanding $(x-7)(x-3)$ gives $x^2-7x-3x+21$ or $x^2-10x+21$.)
- "Where do you predict the intercepts of the graph will be? How do you make your predictions?" (The y-intercept will be $(0,21)$, and the x-intercepts will be $(7,0)$ and $(3,0)$. In the examples in the lesson, we saw that the numbers in the factored form give a clue about the x-intercepts, and the constant term in the standard form gives a clue about the y-intercept. Also, evaluating y when $x=0$ gives us the y-intercept.)
- "What do the x-intercepts of a graph tell us about the quadratic function it represents?" (They tell us the zeros of the function.)
- "A ball is thrown in the air. The quadratic function $f(t)=-16t^2+12t+15$ gives the height of a ball, in feet, at time t seconds. One of the x-intercepts of the graph is $(1.41,0)$. What does this point tell us about the ball?" (It took 1.41 seconds for the ball to hit the ground.)

PLANNING NOTES

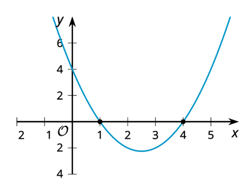
Student Lesson Summary and Glossary

Different forms of quadratic functions can tell us interesting information about the function's graph. When a quadratic function is expressed in standard form, it can tell us the y-intercept of the graph representing the function. For example, the graph representing $y=x^2-5x+7$ has y-intercept $(0,7)$. This makes sense because the y-coordinate is the y-value when x is 0. Evaluating the expression at $x=0$ gives $y=0^2-5(0)+7$, which equals 7.

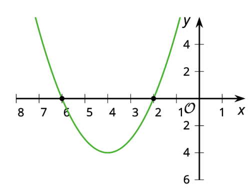


When a function is expressed in factored form, it can help us see the x-intercepts of its graph. Let's look at the functions f given by $f(x)=(x-4)(x-1)$ and g given by $g(x)=(x+2)(x+6)$.

If we graph $y=f(x)$, we see that the x-intercepts of the graph are $(1,0)$ and $(4,0)$. Notice that "1" and "4" also appear in $f(x)=(x-4)(x-1)$, and they are subtracted from x.



If we graph $y=g(x)$, we see that the x-intercepts are at $(-2,0)$ and $(-6,0)$. Notice that "2" and "6" are also in the equation $g(x)=(x+2)(x+6)$, but they are added to x.



The connection between the factored form and the x-intercepts of the graph tells us about the zeros of the function (the input values that produce an output of 0). In the next lesson, we will further explore these connections between different forms of quadratic expressions and the graphs representing them.

Cool-down: Making Connections (*5 minutes*)

Addressing: NC.M1.F-IF.7

Cool-down Guidance: Points to Emphasize
If students struggle on this cool-down, use the warm-up in Lesson 10 to emphasize the x-coordinates by using substitution to verify that x-intercepts are where $f(x) = 0$.

Cool-down

The equations $y = x^2 + 6x + 8$ and $y = (x+2)(x+4)$ both define the same quadratic function. Without graphing, identify the x- and y-intercepts of the graph. Explain how you know.

Student Reflection:

Math confidence is built by making mistakes and learning from them. Consider your own work in math class today. What was your own favorite mistake and how did you correct it?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Reflect on times you observed students listening to one another's ideas today in class. What norms would help each student better attend to their classmates' ideas in future lessons?

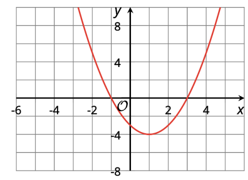
Practice Problems

1. A quadratic function f is defined by $f(x) = (x-7)(x+3)$.

   a. Without graphing, identify the x-intercepts of the graph of f. Explain how you know.

   b. Expand $(x-7)(x+3)$ and use the expanded form to identify the y-intercept of the graph of f.

2. Here is a graph that represents a quadratic function. Which expression could define this function?



   a. $(x+3)(x+1)$
   b. $(x+3)(x-1)$
   c. $(x-3)(x+1)$
   d. $(x-3)(x-1)$

3.

   a. What is the y-intercept of the graph of the equation $y = x^2 - 5x + 4$?

   b. An equivalent way to write this equation is $y = (x-4)(x-1)$. What are the x-intercepts of this equation's graph?

4. Noah said that if we graph $y = (x-1)(x+6)$, the x-intercepts will be at $(1, 0)$ and $(-6, 0)$. Explain how you can determine, without graphing, whether Noah is correct.

5. Write each quadratic expression in standard form. Draw a diagram if needed.

   a. $(x-3)(x-6)$

   b. $(x-4)^2$

   c. $(2x+3)(x-4)$

   d. $(4x-1)(3x-7)$

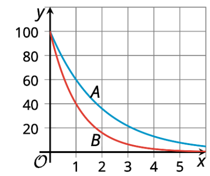
   (From Unit 7, Lesson 8)

6. A company sells a video game. If the price of the game in dollars is p, the company estimates that it will sell $20,000 - 500p$ games.

   Which expression represents the revenue in dollars from selling games if the game is priced at p dollars?

   a. $(20,000 - 500p) + p$
   b. $(20,000 - 500p) - p$
   c. $\frac{20,000 - 500p}{p}$
   d. $(20,000 - 500p) \cdot p$

   (From Unit 7, Lesson 6)

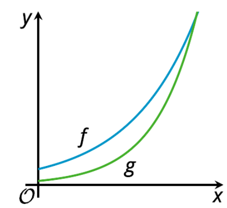


7. Here are graphs of the functions f and g given by $f(x) = 100 \cdot (\frac{3}{5})^x$ and $g(x) = 100 \cdot (\frac{2}{5})^x$.

   Which graph corresponds to f and which graph corresponds to g? Explain how you know.

   (From Unit 6)

8. Here are graphs of two functions f and g.



   An equation defining f is $f(x) = 100 \cdot 2^x$.

   Which of these could be an equation defining the function g?

   a. $g(x) = 25 \cdot 3^x$
   b. $g(x) = 50 \cdot (1.5)^x$
   c. $g(x) = 100 \cdot 3^x$
   d. $g(x) = 200 \cdot (1.5)^x$

   (From Unit 6)

9. Elena plays the piano for 30 minutes each practice day. The total number of minutes, p, that Elena practiced last week is a function of n, the number of practice days.

   Find the domain and range for this function.

   (From Unit 5)

10. I have 24 pencils and 3 cups. The second cup holds one more pencil than the first. The third holds one more than the second. How many pencils does each cup contain?[2]

    (Addressing NC.8.EE.7)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 10: Graphing from the Factored Form

PREPARATION

Lesson Goals	Learning Targets
• Create graphs of quadratic functions that are in factored form. • Given a quadratic function in factored form, explain how to determine the vertex and y-intercept of its graph.	• I can graph a quadratic function given in factored form. • I know how to find the vertex and y-intercept of the graph of a quadratic function in factored form without graphing it first.

Lesson Narrative

In an earlier lesson, students noticed a connection between the numbers in a quadratic expression in factored form (for example, the "2" and "8" in $(x+2)(x-8)$) and the x-intercepts of the graph. In this lesson, they explore that connection further.

Prior to this point, students have not looked closely at how the addition and subtraction symbols in the factors affect the x-intercepts. They also have not considered how or why the connection works, or whether it is a reliable way to determine the x-intercepts. In this lesson, they verify their observations by evaluating the expressions at certain x values and seeing if they produce an output of 0.

Students also explore what the factored form can tell us about the vertex and the y-intercept of a graph representing a quadratic function.

Share some ways you see this lesson connecting to previous lessons in this unit. What connections will you want to make explicit?

Focus and Coherence

Addressing	Building Towards
NC.M1.F-IF.7: Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior. **NC.M1.F-IF.9:** Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).	**NC.M1.F-IF.8a:** Use equivalent expressions to reveal and explain different properties of a function. **a.** Rewrite a quadratic function to reveal and explain different key features of the function.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
  - Colored pencils
- **Activity 2** (*15 minutes*)
  - Technology is required for this activity: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L10 Cool-down (print 1 copy per student)

LESSON

Warm-up: Finding Coordinates (*5 minutes*)

Addressing: NC.M1.F-IF.7

This warm-up refreshes the work in an earlier lesson. It prepares students to deepen their understanding of the factored form and the intercepts of a graph that represents a quadratic function.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping. Students will remain in these groups for Activity 1.
- Give students quiet work time and then time to share their work with a partner.

M **Monitoring Tip:** As students work, notice how they find the y-coordinate of the y-intercept. Identify students who do so by evaluating $w(0)$ and let them know they may be asked to share during the whole-class discussion.

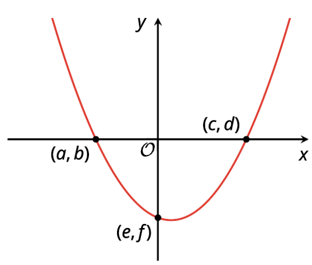
Advancing Student Thinking: Some students may think that the numerical values in the equation correspond directly to the x-intercepts in the graph and incorrectly state that $a = -2$ and $c = 1.6$. Remind them that a graph shows all pairs of x- and y-values that make the equation true. Consider asking these students to try substituting -2 for x and evaluating the expression to verify that $w(-2) = 0$.

Students will have opportunities to attend to the signs or the operations in quadratic expressions in factored form, so it is not essential that this misconception is corrected at this moment.

Student Task Statement

Here is a graph of a function w defined by $w(x) = (x + 1.6)(x - 2)$. Three points on the graph are labeled.

Find the values of a, b, c, d, e, and f. Be prepared to explain your reasoning.



Step 2

- Facilitate a whole-class discussion. Invite students to share their responses and reasoning. If not mentioned in students' explanations, emphasize that:
  - b and d must be 0 because the y-coordinate of the x-intercepts of the graph of any function is 0.
  - e must be 0 because the x-coordinate of the y-intercept of the graph of any function is 0.
  - a and c correspond to the "1.6" and "2" in the factored expression. Students may reason that the graph shows that the positive x-intercept is farther away from 0 than the negative x-intercept, suggesting that c is 2 and a is -1.6. They may also use their observations from an earlier lesson: that when a factor in the expression shows a negative sign or subtraction, the corresponding intercept takes a positive value, and when a factor shows a positive sign or addition, the corresponding intercept takes a negative value. Either explanation is reasonable at this point.
  - $(0, f)$ is the y-intercept, so the value of f can be found by evaluating $w(0)$. This value is -3.2 because $(0+1.6)(0-2) = (1.6)(-2) = -3.2$.

DO THE MATH	PLANNING NOTES

Activity 1: Comparing Two Graphs (*15 minutes*)

Instructional Routine: Poll the Class
Addressing: NC.M1.F-IF.7; NC.M1.F-IF.9

In this activity, students continue to make sense of their earlier observations about the connection between the factored form of a quadratic expression and the x-intercepts of its graph. They study two very similar expressions: $x(x+4)$ and $x(x-4)$. They evaluate each at different x-values. The expressions are chosen so that students can shift their focus from the numbers to the operations, attending to precision (MP6) along the way.

Students notice that for $x(x+4)$, the x values that produce a 0 output are 0 and -4, and for $x(x-4)$, those values are 0 and 4. This helps to explain why the signs of the x-intercepts are the opposite of the signs in the factors. In an upcoming unit, they will further develop this understanding algebraically, in terms of the zero product property. For now, it suffices that they verify the x-intercepts by evaluating the quadratic expression at the predicted values and checking that the outputs are 0.

Students also observe, by looking for regularity in repeated reasoning, that the horizontal location of the vertex of the graph can be identified once the x-intercepts are known: it is exactly halfway between the intercepts (MP8). This suggests that a quadratic expression in factored form can help us "see" both the x-coordinates of the x-intercepts and the x-coordinate of the vertex of the graph. Some students may notice the horizontal location of the vertex could be determined using the halfway point between any two points with the same y-coordinate because the graphs have reflection symmetry across a vertical line through the vertex.

In this activity, students are generalizing the relationship between the zeros on the graph of a function and the factored form of the function, so technology is not an appropriate tool.

Step 1

- Display for all to see the equations defining f and g: $f(x) = x(x+4)$ and $g(x) = x(x-4)$.
- *Poll the Class* to gather some predictions about the graphs that represent the two functions and display the results for all to see:
  - In what ways would the graphs be alike?
  - In what ways would they be different?
- While students are in the same groups as in the warm-up, ask partners to split the work on the first question (with one person analyzing function f and the other person analyzing function g) and then work together to plot the points and make observations in the last question.
- Distribute colored pencils so that each student has access to two different colors for creating their graphs in the last question.
- Since this activity was designed to be completed without technology, ask students to put away any devices.

RESPONSIVE STRATEGY

To support development of organizational skills, check in with students after they have completed the tables for f(x) and g(x). Invite students to share the values and any patterns they may have noticed with a partner. This will give students an opportunity to fix any errors that may impact the outcome of each graph.

Supports accessibility for: Memory; Organization

Advancing Student Thinking: If needed, remind students that the vertex of a graph is the point at which the graph changes direction from increasing to decreasing, or from decreasing to increasing.

Student Task Statement

Consider two functions defined by $f(x) = x(x+4)$ and $g(x) = x(x-4)$.

1. Complete the table of values for each function. Then, determine the x-intercepts and vertex of each graph. Be prepared to explain how you know.

x	$f(x)$
-5	5
-4	
-3	
-2	-4
-1	-3
0	
1	
2	
3	
4	32
5	

x-intercepts: ______

Vertex: ______

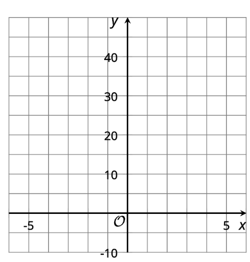
x	$g(x)$
-5	45
-4	
-3	
-2	12
-1	5
0	
1	
2	
3	-3
4	
5	

x-intercepts: ______

Vertex: ______

2. Plot the points from the tables on the same coordinate plane. (Consider using different colors or markings for each set of points so you can tell them apart.)

   Then, make a couple of observations about how the two graphs compare.



Step 2

- Facilitate a whole-class discussion focused on how students used the completed tables to help them find the x-intercepts and the vertex of each graph. If not already mentioned in students' explanations, highlight that:
  - The x-values that produce 0 for the output are the zeros of the function. They tell us where the graph intersects the x-axis. For $f(x) = x(x+4)$, those values are 0 and -4, so the x-intercepts are $(0,0)$ and $(-4,0)$. For $g(x) = x(x-4)$, those values are 0 and 4, so the x-intercepts are $(0,0)$ and $(4,0)$.

  - The vertex tells us the point where the function reaches its maximum or minimum value. We can see the minimum value in each table. In the first table, $f(-2)$ is the lowest value. Because the values on one side of $f(-2)$ mirror those on the other, we can reason that -4 is the lowest value of the function f. In the second table, $g(2)$ is the lowest value.
  - The vertex is halfway between each pair of x-intercepts.

- Ask students how they think the graphs of the functions given by $p(x) = (x - 1)(x - 3)$ and $q(x) = (x + 1)(x + 3)$ will compare.
  - "Where will their x-intercepts be?" ($(1, 0)$ and $(3, 0)$ for the graph of function p, and $(-1, 0)$ and $(-3, 0)$ for the graph of function q.)
  - "Where will the x-coordinate of the vertex of each graph be?" (2 for the graph of p and -2 for that of q.)
    - If helpful, remind students of the connection between finding the average of two numbers and finding midpoints they learned earlier in the semester, and how averaging the x-coordinates can be used to identify halfway.
  - "How do we find the y-coordinate of the vertex?" (Evaluate $p(2)$ and $q(-2)$.)
  - "The y-intercept of both graphs is $(0, 3)$. What is another point on each graph with the same y-coordinate?" (The point 2 units to the right of the vertex of p will have same y-coordinate, so $(4, 3)$. The point $(-4, 3)$ is on the graph of q.)
- Clarify that a table of values won't always show the maximum or minimum values of a function. It also won't always help us identify the x-intercepts of a graph or the zeros of a quadratic function, especially if the zeros are not integers or the given expressions are more complex. Students will learn other ways to find the zeros of any quadratic function later in the course.

DO THE MATH	PLANNING NOTES

Activity 2: What Do We Need to Sketch a Graph? (*15 minutes*)

Instructional Routines: Graph It; Collect and Display (MLR2)	
Addressing: NC.M1.F-IF.7	**Building Towards:** NC.M1.F-IF.8a

In this *Graph It* activity, students apply what they learned in earlier lessons to identify the x-intercepts and the vertex of the graphs representing several quadratic functions. For example, they saw that the x-coordinate of the vertex is always halfway between those of the x-intercepts, and the coordinate pairs on one side of the vertex mirror those on the other side. These observations help students locate the vertex of a graph once the x-intercepts are known, and ultimately to sketch a graph of a quadratic function using at least three identifiable points.

The given expressions here are in factored form, but some are unlike what students have previously seen, so students will need to transfer and generalize the reasoning strategies from earlier work. They also use graphing technology to graph the functions and check their predictions.

In a later lesson, students will use the symmetry of the graph of a quadratic function to sketch graphs when they know the vertex and one additional point rather than the vertex and x-intercepts.

Step 1

- Ask students to arrange themselves in groups of three or use visibly random grouping.
- Provide access to devices that can run Desmos or other graphing technology to use for the first two questions.
- Give students a few minutes of quiet time to think about the first question. Then, ask them to discuss their response and to complete the second question with their group. (Emphasize that students are expected to make the predictions in the first question *before* using their graphing tool.) To save time, consider asking groups to split the graphing work (each group member graphs only one function, and then the group analyzes the graphs together).
- Ask students to sketch the graph of a function for question 3 without using technology. Instruct that students may use technology to check their sketch afterwards, if desired.
- Use the *Collect and Display* routine while students are working. Listen for and collect the language and gestures students use to justify their predictions during small-group discussions about functions f, g, and h. Capture and display language that reflects a variety of ways to determine the coordinates of the points that help them to draw the graph. Remind students to borrow language from the display as needed. This will help students read and use mathematical language during their partner and whole-group discussions.

RESPONSIVE STRATEGY

Some students may benefit from a checklist or list of steps to be able to use while using the graphing technology.

Supports accessibility for: Organization; Conceptual processing; Attention

Monitoring Tip: As students work on finding the y-value of the vertex in the third question, look for those who use the x-coordinate as an input and evaluate to get the output y. Ask these students to be prepared to share during the debrief.

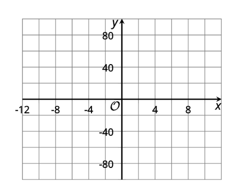
Student Task Statement

1. The functions f, g, and h are given. Predict the x-intercepts and the x-coordinate of the vertex of each function.

Equation	x-intercepts	x-coordinate of the vertex
$f(x) = (x + 3)(x - 5)$		
$g(x) = 2x(x - 3)$		
$h(x) = (x + 4)(4 - x)$		

2. Use graphing technology to graph the functions f, g, and h. Use the graphs to check your predictions.

3. Without technology, sketch a graph that represents the equation $y = (x - 7)(x + 11)$ and that shows the x-intercepts and the vertex. Think about how to find the y-coordinate of the vertex. Be prepared to explain your reasoning.



Are You Ready For More?

The quadratic function f is given by $f(x) = x^2 + 2x + 6$.

1. Find $f(-2)$ and $f(0)$.
2. What is the x-coordinate of the vertex of the graph of this quadratic function?
3. Does the graph have any x-intercepts? Explain or show how you know.

Step 2

- Facilitate a whole-class discussion focused on how students determined the x-intercepts and the x-coordinate of the vertex of a graph, and how the coordinates of these points could help them sketch the graph. During this discussion, refer to any relevant student language collected and displayed, and update the display with new words and phrases as needed. Ask questions such as:
  - "For $g(x) = 2x(x-3)$, how did you find the x-intercepts without graphing?" (The $(x-3)$ suggests that one x-intercept is $(3,0)$, and evaluating $g(3)$ does give an output of 0. The factor $2x$ suggests that the second x-intercept is $(0,0)$, because $2(0)$ is 0, and multiplying 0 by any number gives 0.)
  - "How did you find the x-value of the vertex?" (By finding the halfway point between 0 and 3, which is 1.5.)
  - "How would you find the y-coordinate of the vertex?" (By evaluating $g(1.5)$, which gives -4.5.)
  - "The expression that defines function h has the factor $(4-x)$, where the constant term appears first and x is subtracted from it. Did this affect how you determined the x-intercepts? How so?" (It wasn't as easy to see as when the x comes first. Setting $(4-x)$ equal to 0 helped determine the x-intercept.)
  - "How did you sketch the graph representing $y = (x-7)(x+11)$?" (By finding the x-intercepts and the vertex. The intercepts are $(7,0)$ and $(-11,0)$, so the x-coordinate of the vertex is -2, and the y-coordinate is $(-2-7)(-2+11)$ or -81. Those points are enough to sketch a graph.)

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to understand the connection between factors of the form $x \pm a$ in a quadratic expression and the x-intercepts of the graph, as well as to understand the processes for finding the vertex and y-intercept of a quadratic function given in factored form.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

Present students with the following questions:

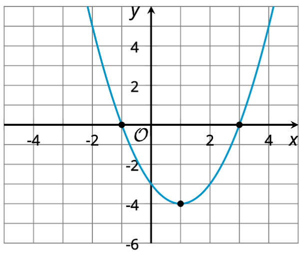
A quadratic function k is defined by $k(x) = (x - 100)(x + 20)$. How would you explain to a friend who is absent today how to use the equation (without graphing) to:

- Find the zeros of the function?
- Find the x-intercepts of the graph?
- Find the x-coordinate of the vertex?
- Find the y-coordinate of the vertex?
- Find the y-intercept of the graph?

PLANNING NOTES

Student Lesson Summary and Glossary

The function f given by $f(x) = (x + 1)(x - 3)$ is written in factored form. Recall that this form is helpful for finding the zeros of the function (where the function has the value 0) and telling us the x-intercepts on the graph representing the function.



Here is a graph representing f. It shows 2 x-intercepts at $x = -1$ and $x = 3$.

If we use -1 and 3 as inputs to f, what are the outputs?

- $f(-1) = (-1 + 1)(-1 - 3) = (0)(-4) = 0$
- $f(3) = (3 + 1)(3 - 3) = (4)(0) = 0$

Because the inputs -1 and 3 produce an output of 0, they are the zeros of the function f. And because both x values have 0 for their y value, they also give us the x-intercepts of the graph (the points where the graph crosses the x-axis, which always have a y-coordinate of 0). So, the zeros of a function have the same values as the x-coordinates of the x-intercepts of the graph of the function.

The factored form can also help us identify the vertex of the graph, which is the point where the function reaches its minimum value. Notice that the x-coordinate of the vertex is 1, and that 1 is halfway between -1 and 3. Once we know the x-coordinate of the vertex, we can find the y-coordinate by evaluating the function at that x-coordinate: $f(1) = (1 + 1)(1 - 3) = 2(-2) = -4$. So the vertex is at $(1, -4)$.

When a quadratic function is in standard form, the y-intercept is clear: its y-coordinate is the constant term c in $ax^2 + bx + c$. To find the y-intercept from factored form, we can evaluate the function at $x = 0$, because the y-intercept is the point where the graph has an input value of 0: $f(0) = (0 + 1)(0 - 3) = (1)(-3) = -3$.

Cool-down: Sketching a Graph (*5 minutes*)

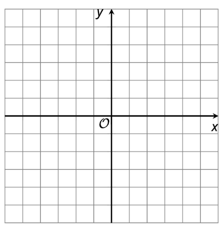
Addressing: NC.M1.F-IF.7

Cool-down Guidance: Points to Emphasize
If students struggle to identify the x- and y-intercepts, use the Card Sort in Lesson 11 to highlight key ideas based on student work in the cool down.

Since this activity was designed to be completed without technology, ask students to put away any devices.

Cool-down

The function f is given by $f(x) = (x-2)(x+4)$. Without using graphing technology, answer the following questions.



1. What are the x-intercepts of the graph representing f?
2. What are the x- and y-coordinates of the vertex of the graph?
3. What is the y-intercept?
4. Sketch a graph that represents f.

Student Reflection:

Graphing quadratic functions is easiest for me when it is in (*standard form or factored form*) _______ because...

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Which students had opportunities to share their thinking during whole-class discussion? How did you select these students?

Practice Problems

1. Select **all** true statements about the graph that represents $y = 2x(x - 11)$.

   a. Its x-intercepts are at $(-2, 0)$ and $(11, 0)$.
   b. Its x-intercepts are at $(0, 0)$ and $(11, 0)$.
   c. Its x-intercepts are at $(2, 0)$ and $(-11, 0)$.
   d. It has only one x-intercept.
   e. The x-coordinate of its vertex is -4.5.
   f. The x-coordinate of its vertex is 11.
   g. The x-coordinate of its vertex is 4.5.
   h. The x-coordinate of its vertex is 5.5.

2. Select **all** equations whose graphs have a vertex with x-coordinate 2.

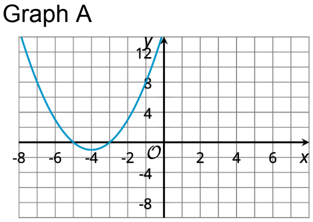
   a. $y = (x - 2)(x - 4)$
   b. $y = (x - 2)(x + 2)$
   c. $y = (x - 1)(x - 3)$
   d. $y = x(x + 4)$
   e. $y = x(x - 4)$

3. Determine the x-intercepts and the x-coordinate of the vertex of the graph that represents each equation.

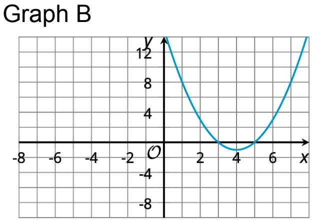
Equation	x**-intercepts**	x**-coordinates of the vertex**
$y = x(x - 2)$		
$y = (x - 4)(x + 5)$		
$y = -5x(3 - x)$		

4. Which graph below is the graph of the equation $y = (x - 3)(x + 5)$?

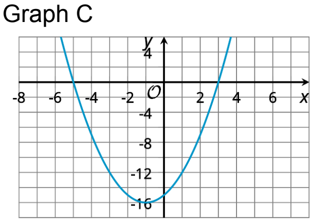
Graph A



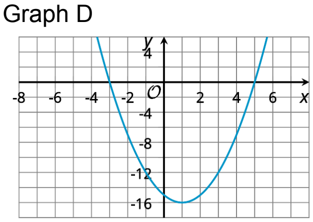
Graph B



Graph C



Graph D



5.

   a. What are the x-intercepts of the graph of $y = (x - 2)(x - 4)$?

   b. Find the coordinates of the vertex of the graph. Show your reasoning.

   c. Sketch a graph of the equation $y = (x - 2)(x - 4)$.

6. What are the x-intercepts of the graph of the function defined by $(x - 2)(2x + 1)$?

   a. $(2, 0)$ and $(-1, 0)$
   b. $(2, 0)$ and $(-\frac{1}{2}, 0)$
   c. $(-2, 0)$ and $(1, 0)$
   d. $(-2, 0)$ and $(\frac{1}{2}, 0)$

7. Is $(s+t)^2$ equivalent to $s^2+2st+t^2$? Explain or show your reasoning.

   (From Unit 7, Lesson 7)

8. A company sells calculators. If the price of the calculator in dollars is p, the company estimates that it will sell $10,000-120p$ calculators.

   Write an expression that represents the revenue in dollars from selling calculators if a calculator is priced at p dollars.

   (From Unit 7, Lesson 6)

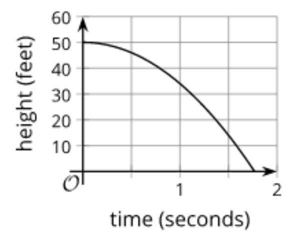
9. Which function could represent the height in meters of an object thrown upwards from a height of 25 meters above the ground t seconds after being launched?

   a. $f(t)=-5t^2$
   b. $f(t)=-5t^2+25$
   c. $f(t)=-5t^2+25t+50$
   d. $f(t)=-5t^2+50t+25$

   (From Unit 7, Lesson 5)

10. A basketball is dropped from the roof of a building, and its height in feet is modeled by the function h.

    Here is a graph representing h.

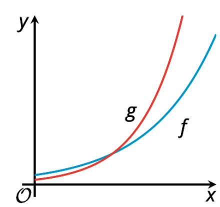


    Select **all** the true statements about this situation.

    a. When $t=0$, the height is 0 feet.
    b. The basketball falls at a constant speed.
    c. The expression that defines h is linear.
    d. The expression that defines h is quadratic.
    e. When $t=0$, the ball is about 50 feet above the ground.
    f. The basketball lands on the ground about 1.75 seconds after it is dropped.

    (From Unit 7, Lesson 4)

11. Here are graphs of two exponential functions f and g.



    The function f is given by $f(x)=100\cdot 2^x$ while g is given by $g(x)=a\cdot b^x$.

    Based on the graphs of the functions, what can you conclude about a and b?

    (From Unit 6)

12. Suppose G takes a student's grade and gives a student's name as the output. Explain why G is not a function.

    (From Unit 5)

Lesson 11: Graphing the Standard Form (Part One)

PREPARATION

Lesson Goals	Learning Targets
• Comprehend (orally and in writing) how the a and c in $y = ax^2 + bx + c$ are visible on the graph. • Connect (orally and in writing) different representations of quadratic functions (equations, tables, and graphs). • Use technology to explore how the parameters of quadratic expressions in standard form are visible on the graph.	• I can explain how the a and c in $y = ax^2 + bx + c$ affect the graph of the equation. • I understand how graphs, tables, and equations that represent the same quadratic function are related. • I can use technology to visualize the parameters of a quadratic expression in standard form.

Lesson Narrative

Students just explored the connections between quadratic functions expressed in factored form and their graphs. In this lesson, they experiment with the graphs of quadratic functions expressed in standard form and reason about how the parameters of the expressions—specifically the coefficient of the squared term and the constant term—relate to features of the graphs. Students use technology to change these values and produce the graphs. They study the effects and generalize their observations (MP8).

Then, students practice identifying equivalent quadratic expressions in standard and factored form and their corresponding graph. To do so, they look for and make use of structure (MP7). The work here strengthens students' understanding of the ties across various representations of quadratic functions.

Note that when students graphed equations in factored form earlier, they dealt mostly with monic quadratic expressions, in which the coefficient a in $ax^2 + bx + c$ is 1, and the factored expression is in the form of $(x + m)(x + n)$. Limiting the examples to such expressions enables students to see more easily the connections between the numbers in the factors and the x-intercepts of the graphs. When graphing from the standard form, that limitation is not necessary, as the connections between the coefficient a and the features of the graph can be studied and revealed more straightforwardly.

(Later in this unit, students will learn to use the zero product property to solve quadratic equations. At that time, they will revisit the factored form and how it reveals the x-intercepts of the graph, including for non-monic quadratic expressions that can be written as $(px + m)(qx + n)$.)

What are you excited for your students to be able to do after this lesson?

Focus and Coherence

Building On	Addressing
NC.8.EE.1: Develop and apply the properties of integer exponents to generate equivalent numerical expressions. **NC.M1.F-IF.9:** Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).	**NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior. **NC.M1.A-SSE.1a:** Interpret expressions that represent a quantity in terms of its context. **a**. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

Agenda, Materials, and Preparation

Technology is required for this lesson in Activity 1 and 2: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*Optional, 10 minutes*)
- **Activity 3** (*15 minutes*)
  - Representations of Quadratic Functions card sort (print 1 copy for every 2 students and cut up in advance)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L11 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.EE.1

This bridge will prepare students to accurately evaluate expressions with an exponent of 2 and either negative bases or negative coefficients, preparing them to evaluate quadratic functions to determine the effects of the a value, or leading coefficient, in this lesson. This bridge is aligned to Check Your Readiness question 3.

Student Task Statement

For each of the following functions with the given inputs, predict whether the output will be positive or negative. Then, evaluate the functions. Were you correct? Why or why not?

Function	Prediction	Evaluation	Correct prediction? Explain why or why not.
a. $y = x^2$, where $x = 5$			
b. $y = x^2$, where $x = -5$			
c. $y = 2x^2$, where $x = 5$			

(*continued*)

d. $y = 2x^2$, where $x = -5$			
e. $y = -2x^2$, where $x = 5$			
f. $y = -2x^2$, where $x = -5$			

DO THE MATH

PLANNING NOTES

Warm-up: Matching Graphs to Linear Equations (*5 minutes*)

Building On: NC.M1.F-IF.9

This warm-up activates students' prior knowledge about how the parameters of a linear expression are visible on its graph, preparing students to make similar observations about quadratic expressions and their graphs.

Step 1

- Give students 3 minutes of quiet think time to determine their matching graphs and equations.

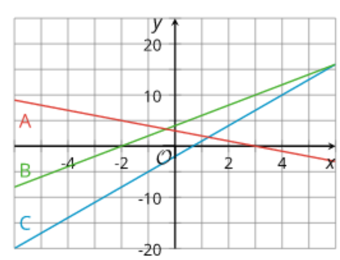
Monitoring Tip: Students may approach the matching task in different ways:

- By starting with the graphs and thinking about corresponding equations. For example, they may notice that graph A has a negative slope and must therefore correspond to $y = 3 - x$, the only equation whose linear term has a negative coefficient. They may notice that the slope of C is greater than that of B, so C must correspond to $y = 3x - 2$.
- By starting with the equations and then visualizing the graphs. For example, they may see that $y = 3x - 2$ has a constant term of -2, so it must correspond to C, and that graph B intercepts the y-axis at a higher point than graph A, so B must correspond to $y = 2x + 4$.

Invite students with contrasting approaches to share during discussion.

Student Task Statement

Graphs A, B, and C represent three linear equations: $y = 2x + 4$, $y = 3 - x$, and $y = 3x - 2$. Which graph corresponds to which equation? Explain your reasoning.



Equation	Graph	Explain your reasoning
$y = 2x + 4$		
$y = 3 - x$		
$y = 3x - 2$		

Step 2

- Select students to share how they matched the equations and the graphs. As students refer to the numbers that represent the slope and y-intercept in the equations, encourage students to use the words "coefficient" and "constant term" in their explanations.
- To support students' vocabulary development, and to prepare them for the lesson, consider writing the equations from the warm up for all to see and identifying the coefficient and constant terms in each equation.
- Highlight that the equations and the graphs are connected in more than one way, so there are different ways to know what a graph would look like given its equation, or what an equation would entail given its graph.
    - For example, you can graph $y = 3x - 2$ either by looking at the coefficients to determine the slope and y-intercept, or by substituting $x = 0$ and $y = 0$ to find both intercepts.
- Tell students that we will look at such connections between the expressions and graphs that represent quadratic functions.

DO THE MATH	PLANNING NOTES

Activity 1: Quadratic Graphs Galore (*10 minutes*)

Instructional Routines: Graph It; Collect and Display (MLR2)
Addressing: NC.M1.F-IF.7; NC.M1.A-SSE.1a

This *Graph It* activity enables students to see how the coefficient of the squared term and the constant term in a quadratic expression in standard form can be seen on the graph. Students start by graphing $y = x^2$ using technology. They then experiment with adding positive and negative constant values to x^2 and multiplying x^2 by positive and negative coefficients. They generalize their observations afterwards. Along the way, students practice looking for and expressing regularity through repeated reasoning (MP8).

In their small groups, students have opportunities to explain their reasoning and critique the reasoning of others (MP3).

Step 1

- Provide access to devices that can run Desmos.
- Ask students to arrange themselves in groups of four or use visibly random grouping.
- Ask the group to decide which member will experiment with each of the four changes listed in the activity and then report the results to their group.
- Remind students to adjust their graphing window as needed. Instruct students that creating a slider, in Desmos, might be a helpful tool for this activity. For students who need reminding, they can create sliders by typing a letter to represent a parameter, such as $f(x) = x^2 + a$ to change the constant term.

RESPONSIVE STRATEGIES

Represent the same information through different modalities by using individual sketches of each function. Provide students with a graphic organizer that provides space to include sketches and observations for each equation. Some students may benefit from additional support to learn what types of details are helpful in a sketch of a graph.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Use the *Collect and Display* routine to support small-group discussion.

- As students share what they notice about how the graphs change or stay the same depending on how they change the function, listen for and amplify language students use to describe the features of the graph such as “opens upward,” “opens downward,” “steeper,” “wider,” etc.
- Post the collected language in the front of the room so that students can refer to it throughout the rest of the activity and lesson.

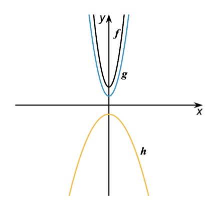
Student Task Statement

Using graphing technology, graph $y = x^2$ and then experiment with each of the following changes to the function. Record your observations (include sketches, if helpful).

1. Adding different constant terms to x^2 (for example: $x^2 + 5$, $x^2 + 10$, $x^2 - 3$, etc.).
2. Multiplying x^2 by different positive coefficients greater than 1 (for example: $3x^2$, $7.5x^2$, etc.).
3. Multiplying x^2 by different negative coefficients less than or equal to -1 (for example: $-x^2$, $-4x^2$, etc.).
4. Multiplying x^2 by different coefficients between -1 and 1 (for example: $\frac{1}{2}x^2$, $-0.25x^2$, etc.).

Are You Ready For More?

Here are the graphs of three quadratic functions. What can you say about the coefficients of x^2 in the expressions that define f (in black at the top center), g (in blue on the top outside), and h (in yellow at the bottom)? Can you identify them? How do they compare?



Step 2

- Invite students to share their observations, and if possible, demonstrate their graphs with sliders for all to see. Refer to and update any relevant student language collected and displayed. Tell students that people often describe the shape when a is positive as a parabola that “opens upward” and the shape when a is negative as a parabola that “opens downward.”
- For each change to the expression (for example, adding a constant, or multiplying x^2 by a positive number) and the observed change on the graph, solicit students’ ideas about why the graph transformed that way. For example, ask: “Why do you think subtracting a number from x^2 moves the graph down?”
- Discuss questions such as:
  - “The points $(1, 1)$, $(2, 4)$, and $(3, 9)$ are three points on the graph representing x^2. When we add 3 to x^2 how do the y-values for $x = 1$, $x = 2$, and $x = 3$ change?” (Their y-values increase by 3: $(1, 4)$, $(2, 7)$, $(3, 12)$.) “What about when we subtract 3 from x^2?” (They decrease by 3: $(1, -2)$, $(2, 1)$, $(3, 6)$.)
  - “How do the y-values change when you multiply x^2 by a positive number, say, 3?” (The y-values for $3x^2$ triple those of x^2. For $x = 1$, $x = 2$, and $x = 3$, the points will be $(1, 3)$, $(2, 12)$, and $(3, 27)$.)
  - “How do the tripled y-values affect the graph? (They stretch the graph up vertically, making the graph appear narrower.)
- To help students make stronger connections between the parameters of a quadratic expression and the features of its graph, consider the optional activity included in this lesson.

DO THE MATH

PLANNING NOTES

Activity 2: What Do These Tables Reveal? (*Optional, 10 minutes*)

Addressing: NC.M1.F-IF.7

Earlier, students made a series of observations about how changing the parameters of quadratic expressions transformed the graphs. They may not have fully recognized why the graphs changed the way they did. This optional activity prompts students to explain their earlier observations and further understand the behaviors of the graphs in relation to the quadratic expressions.

Students evaluate quadratic expressions with different coefficients and constant terms and compare the values to those expressions to the values of x^2. Upon studying the table of values, they see, for example, that adding a constant term increases the value of x^2 by that number, moving the corresponding points on the graph up by that amount. They see that multiplying the squared term by 2 or $\frac{1}{2}$ changes the output values by a factor of 2 or $\frac{1}{2}$, which moves the points for x^2 to a position twice or half as high on the graph.

Step 1

- Provide students with a calculator and about 7 minutes of quiet work time.

Student Task Statement

1.
   a. Complete the table with values of $x^2 + 10$ and $x^2 - 3$ at different values of x.

   b. Earlier, you observed the effects on the graph of adding or subtracting a constant term from x^2. Study the values in the table. Use them to explain why the graphs changed the way they did when a constant term is added or subtracted.

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
x^2+10							
x^2-3							

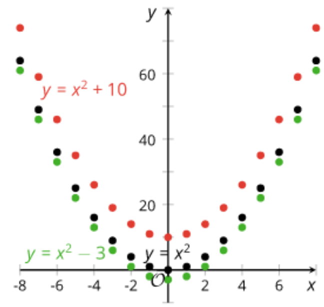
2.
   a. Complete the table with values of $2x^2$, $\frac{1}{2}x^2$, and $-2x^2$ at different values of x. (You may also use a spreadsheet tool, if available.)

   b. You also observed the effects on the graph of multiplying x^2 by different coefficients. Study the values in the table. Use them to explain why the graphs changed the way they did when x^2 is multiplied by a number greater than 1, by a negative number less than or equal to -1, and by numbers between -1 and 1.

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$2x^2$							
$\frac{1}{2}x^2$							
$-2x^2$							

Step 2

- Invite students to share their analyses on how the values in the tables relate the behaviors of the graphs they saw earlier.
- Consider plotting the points on Desmos to make explicit the connections between the values in the tables and the graphs.



DO THE MATH	PLANNING NOTES

Activity 3: Card Sort: Representations of Quadratic Functions (*15 minutes*)

Instructional Routines: Card Sort; Discussion Supports (MLR8) - Responsive Strategy
Addressing: NC.M1.F-IF.7; NC.M1.A-SSE.1a

In this *Card Sort* activity, students apply what they learned about the connections between quadratic expressions and the graphs representing them. They also practice identifying equivalent quadratic expressions in standard and factored forms. Students are given a set of cards containing equations and graphs. They sort them into sets of three cards wherein each set contains two equivalent equations and a graph that all represent the same quadratic function. They also explain to a partner how they know the cards belong together. A sorting task gives students opportunities to analyze the different equations, graphs, and structures closely and make connections (MP2, MP7) and to justify their decisions as they practice constructing logical arguments (MP3).

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping. Give each group a set of pre-printed cards.
- Tell students to take turns sorting the cards into sets that represent the same quadratic function.
  - The person whose turn it is to compile a set should explain how they know the cards belong together. (The person who has the last turn should also explain why the cards belong together, aside from the fact that they are the last remaining cards.)
  - The partner should listen and ask for clarification or discuss any disagreement.

RESPONSIVE STRATEGY

Provide the following sentence frames for students to use while engaging in the Card Sort: "I know these cards go together because I figured out that . . . ." and "I think these cards go together because I can see that . . . ."

Discussion Supports (MLR8)

- If time permits, before partners record anything, ask them to compare their sorted sets with another group of students and discuss any disagreements.
- Once all the cards are sorted, ask students to record their findings in the given graphic organizer.

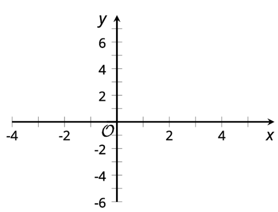
M **Monitoring Tip:** As students work, monitor how they go about making the matches. Some students may begin by studying features of the graph and then relate them to the equations. Others may work the other way around. Identify students with contrasting approaches so they can share in the discussion.

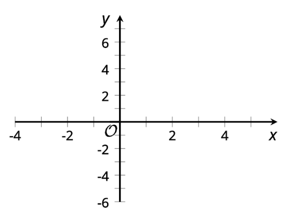
Advancing Student Thinking**:** Some students may think a factor such as $(x-1)$ relates to an x-intercept of $(-1,0)$. In earlier lessons, students learned that the zeros of the function are the x-coordinates of the x-intercepts. Show students the equations $x-1=0$ and $x+1=0$ and ask them to solve each equation and relate the solutions back to making the expression $(x+1)(x-1)$ equal 0. Some students may benefit from seeing the expression $(x-1)$ written as $x+-1$ to further emphasize that the expression takes the value 0 when x is the opposite of -1. Students will continue this work when solving quadratic equations later in the unit.

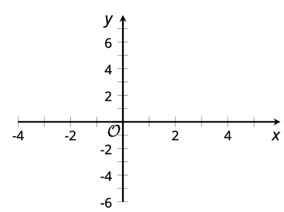
Student Task Statement

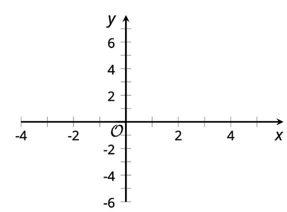
Your teacher will give your group a set of cards. Each card contains a graph or an equation.

- Take turns with your partner to sort the cards into sets so that each set contains two equations and a graph that all represent the same quadratic function.
- For each set of cards that you put together, explain to your partner how you know they belong together.
- For each set that your partner puts together, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
- Once all the cards are sorted and discussed, record the equivalent equations, sketch the corresponding graph, and write a brief note or explanation about why the representations were grouped together.

<table>
<tr>
<td>Standard form: _______
Factored form: _______


Explanation: _______</td>
<td>Standard form: _______
Factored form: _______


Explanation: _______</td>
<td>Standard form: _______
Factored form: _______


Explanation: _______</td>
<td>Standard form: _______
Factored form: _______


Explanation: _______</td>
</tr>
</table>

Step 2

- Invite students to share how they found pairs of equivalent equations and how they matched the equations to the graphs.
- Highlight these explanations:
  - The y-intercept of the graph helps to find the equation in standard form (or vice versa).
  - The x-intercepts of the graph help find the equation in factored form (or vice versa).
  - The quadratic expression in factored form can be expanded to find the equivalent expression in standard form.

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to make connections between quadratic equations and their corresponding graphs. Specifically, students discovered how adding (or subtracting) a constant from the x^2 term shifts the graph up or down, how multiplying the x^2 term by a coefficient greater than 1 (or between 0 and 1) stretches (or shrinks) the graph vertically by that factor, and how multiplying the x^2 term by a negative coefficient reflects the graph so that it opens downward. Students connect changes observed on the graph to changes in the outputs of a table for the different cases.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

To help students consolidate their new insights, consider connecting them to what they already know about the graphs representing linear functions. Discuss questions such as:

- "How is the constant term in a quadratic equation in standard form—the c in $y = ax^2 + bx + c$—like or unlike the constant term in a linear equation in slope-intercept form—the b in $y = mx + b$?" (They both tell us about the y-intercept. Increasing the value of each constant term moves the graph up, and decreasing it moves the graph down.)
- "Is the coefficient of the squared term in a quadratic equation in standard form (the a in $y = ax^2 + bx + c$) like the coefficient of the linear term in slope-intercept form (the m in $y = mx + b$)? Why or why not?" (They both affect the behavior of the graph in similar ways:
    - In linear equations of the form $y = mx + b$, the greater m is, the steeper the line gets. In quadratic equations in standard form, the greater a is, the steeper or narrower the U-shaped graph gets.
    - A negative m results in a graph that is a downward-sloping line. A negative a gives a graph that opens downward.)

Tell students that we will look at the linear term b, in $y = ax^2 + bx + c$, in an upcoming lesson.

Encourage students to reflect on the advantages of using expressions in different forms to anticipate the graphs representing quadratic functions. Ask students questions such as:

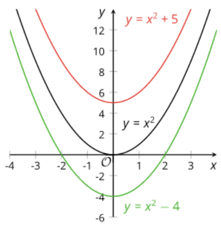
- "What information about the graph can you easily obtain from an expression in standard form?" (whether the graph opens up or down, the y-intercept)
- "What information can we easily obtain from the factored form?" (the x-intercepts, the x-coordinate of the vertex)

PLANNING NOTES

Student Lesson Summary and Glossary

Remember that the graph representing any quadratic function is a shape called a *parabola*. People often say that a parabola "opens upward" when the lowest point on the graph is the vertex (where the graph changes direction), and "opens downward" when the highest point on the graph is the vertex. Each coefficient in a quadratic expression written in standard form $ax^2 + bx + c$ tells us something important about the graph that represents it.

The graph of $y = x^2$ is a parabola opening upward with vertex at $(0, 0)$. Adding a constant term 5 gives $y = x^2 + 5$ and raises the graph by 5 units. Subtracting 4 from x^2 gives $y = x^2 - 4$ and moves the graph 4 units down.



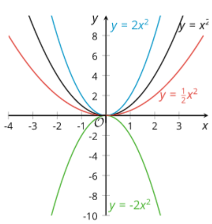
x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$x^2 + 5$	14	9	6	5	6	9	14
$x^2 - 4$	5	0	-3	-4	-3	0	5

A table of values can help us see that adding 5 to x^2 increases all the output values of $y = x^2$ by 5, which explains why the graph moves up 5 units. Subtracting 4 from x^2 decreases all the output values of $y = x^2$ by 4, which explains why the graph shifts down by 4 units.

In general, the constant term of a quadratic expression in standard form influences the vertical position of the graph. An expression with no constant term (such as x^2 or $x^2 + 9x$) means that the constant term is 0, so the y-intercept of the graph is on the x-axis. It's not shifted up or down relative to the x-axis.

The coefficient of the squared term in a quadratic function also tells us something about its graph. The coefficient of the squared term in $y = x^2$ is 1. Its graph is a parabola that opens upward.

- Multiplying x^2 by a number greater than 1 makes the graph steeper, so the parabola is narrower than that representing x^2.
- Multiplying x^2 by a number less than 1 but greater than 0 makes the graph less steep, so the parabola is wider than that representing x^2.
- Multiplying x^2 by a number less than 0 makes the parabola open downward.



x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$2x^2$	18	8	2	0	2	8	18
$-2x^2$	-18	-4	-2	0	-2	-8	-18

If we compare the output values of $2x^2$ and $-2x^2$, we see that they are opposites, which suggests that one graph would be a reflection of the other across the x-axis.

Cool-down: Matching Equations and Graphs (*5 minutes*)

Addressing: NC.M1.F-IF.7

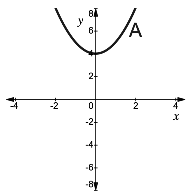
Cool-down Guidance: Press Pause
If students are still struggling with finding x- and y-intercepts of the graph, address misconceptions in the cool-downs and practice problems prior to starting Lesson 12.

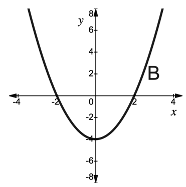
Since this activity was designed to be completed without technology, ask students to put away any devices.

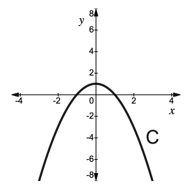
Cool-down

Here are graphs that represent three quadratic functions, defined by:

- $f(x) = x^2 - 4$
- $g(x) = 1 - x^2$
- $h(x) = x^2 + 4$







1. Match each equation to a graph that represents it. Explain how you know.
2. Which part of the equation for graph C tells us that the graph opens downward?

Student Reflection:

Today I got excited or felt confident doing my math work when...

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

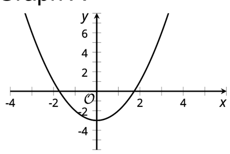
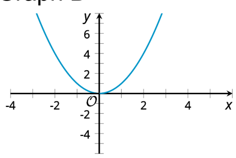
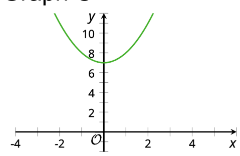
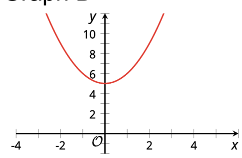
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What have you noticed happening with student math identities? Would you say they are strengthening? If so, how will you continue to foster that? If not, what will you do differently to help develop stronger math identities?

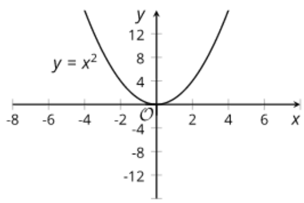
Practice Problems

1. Here are four graphs. Match each graph with a quadratic equation that it represents.

Quadratic equations	Graph A	Graph B	Graph C	Graph D
a. $y=x^2$ b. $y=x^2+5$ c. $y=x^2+7$ d. $y=x^2-3$				

2. The two equations $y=(x+2)(x+3)$ and $y=x^2+5x+6$ are equivalent.
   a. Which equation helps find the x-intercepts most efficiently?
   b. Which equation helps find the y-intercept most efficiently?

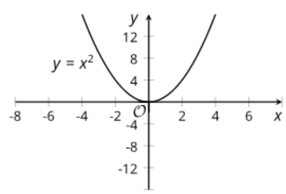
3. Here is a graph that represents $y=x^2$.



   On the same coordinate plane, sketch and label the graph that represents each equation:
   a. $y=x^2-4$
   b. $y=-x^2+5$

4. Select **all** equations whose graphs have a y-intercept with a positive y-coordinate.
   a. $y=x^2+3x-2$
   b. $y=x^2-10x$
   c. $y=(x-1)^2$
   d. $y=5x^2-3x-5$
   e. $y=(x+1)(x+2)$

5. Here is a graph that represents $y=x^2$.



   a. Describe what would happen to the graph if the original equation were modified as follows:
      i. $y=-x^2$
      ii. $y=3x^2$
      iii. $y=x^2+6$
   b. Sketch the graph of the equation $y=-3x^2+6$ on the same coordinate plane as $y=x^2$.

6.
   a. What are the x-intercepts of the graph that represents $y=(x+1)(x+5)$? Explain how you know.
   b. What is the x-coordinate of the vertex of the graph that represents $y=(x+1)(x+5)$? Explain how you know.
   c. Find the y-coordinate of the vertex. Show your reasoning.
   d. Sketch a graph of $y=(x+1)(x+5)$.

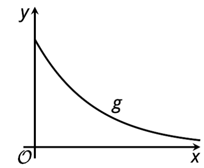
   (From Unit 7, Lesson 10)

7. Determine the x-intercepts, the vertex, and the y-intercept of the graph of each equation.

Equation	x-intercepts	Vertex	y-intercept
$y=(x-5)(x-3)$			
$y=2x(8-x)$			

(From Unit 7, Lesson 10)

8. Here is a graph of the function g given by $g(x)=a\cdot b^x$.



What can you say about the value of b? Explain how you know.

(From Unit 6)

9. Equal amounts of money were invested in stock A and stock B. In the first year, stock A increased in value by 20%, and stock B decreased by 20%. In the second year, stock A decreased in value by 20%, and stock B increased by 20%.

Was one stock a better investment than the other? Explain your reasoning.

(From Unit 6)

10. The area of a square can be determined by the function $A=s^2$, where A represents the area of the square and s represents the length of one side. Jada is buying square picture frames to hang her high school graduation pictures in her college dorm room.

    a. If the length of one side of the picture frame is s inches, what expression represents the area of the frame?

    b. If Jada decides that she actually needs four picture frames, what expression represents the total area?

    c. If Jada decides to buy one picture frame with a side length 4 times longer than the first, what expression represents its area?

    d. If the length of one side of the original picture frame was 8 inches, what was the area of:

        i. The original picture frame?

        ii. Four of the original picture frames?

        iii. One picture frame with a side length 4 times as long?

(Addressing NC.8.EE.1)

Lesson 12: Graphing the Standard Form (Part Two)

PREPARATION

Lesson Goals	Learning Targets
• Describe (orally and in writing) how the values of a and b in $y=ax^2+bx+c$ affect the graph. • Understand why the expression $\frac{-b}{2a}$ (or its verbal equivalent) represents the x-coordinate of the vertex of the graph of $y=ax^2+bx+c$.	• I can explain how the values of a and b in $y=ax^2+bx+c$ affect the graph of the equation. • I can use patterns to find a formula for the x-coordinate of the vertex of the graph of $y=ax^2+bx$.

Lesson Narrative

In this lesson, students investigate graphs of equations in the form $y=x^2+bx$ and then $y=ax^2+bx$, using the factored form to help them find the vertex. This leads them to conjecture a formula for the x-coordinate of the vertex of these parabolas, which might be the familiar $x=\frac{-b}{2a}$ or might be expressed in words. In the lesson debrief, students see how vertical translations can be used to generalize this formula to all equations of the form $y=ax^2+bx+c$.

What do you hope to learn mathematically from this lesson?

Focus and Coherence

Building On	Addressing
NC.7.EE.1: Apply properties of operations as strategies to: • Add, subtract, and expand linear expressions with rational coefficients. • Factor linear expression with an integer GCF.	**NC.M1.F-IF.8a:** Use equivalent expressions to reveal and explain different properties of a function. **a.** Rewrite a quadratic function to reveal and explain different key features of the function. **NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Agenda, Materials, and Preparation

Technology is required for this lesson for Activities 1 and 2: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L12 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.7.EE.1

The purpose of this bridge is to support students with the distributive property and factoring out the greatest common factor (GCF) that can be used to write equivalent expressions. These skills help students develop fluency and will be helpful later in this lesson when students will need to rewrite quadratic expressions with no constant term in both standard and factored forms.

Student Task Statement

In each row, write the equivalent expression. The first row has been done for you as an example.

Factored	Expanded
$-3(5-2y)$	$-15+6y$
$5(a-6)$	
	$6a-2b$
$-4(2w-5z)$	
$-(2x-3y)$	

DO THE MATH

PLANNING NOTES

Warm-up: Equivalent Expressions *(5 minutes)*

Building Towards: NC.M1.F-IF.8a

In this warm-up, students practice applying the distributive property to write equivalent quadratic expressions in standard and factored forms. They also notice that when a quadratic expression has no constant term (that is, is in the form of $x^2 + bx$), its factors are a variable and a sum (or a difference). Their awareness of this structure (MP7) prepares them to think about finding the vertex of the graph of $y = x^2 + bx$ later in the lesson.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping. They will remain in these groups for the entire lesson.
- Give students a moment of quiet think time and then time to briefly share their responses with their partner.

Student Task Statement

1. Complete each row with an equivalent expression in standard form or factored form.
2. What do the quadratic expressions in each column have in common (besides the fact that everything in the left column is in standard form and everything in the right column is in factored form)? Be prepared to share your observations.

Standard form	Factored form
x^2	
	$x(x+9)$
$x^2 - 18x$	
	$x(6-x)$
$-x^2 + 10x$	
	$-x(x+2.75)$

Step 2

- Consider displaying the completed table for all to see. If needed, discuss only the last few expressions, which can be written in a few different but equivalent ways.
- Then, focus the discussion on the second question. Ask students:
  - "Look at all the quadratic expressions in standard form."
    - "What do they have in common?" (They have a squared term and a linear term. They do not have a constant term.)
    - "From these expressions, what can we predict, if anything, about the features of the graphs that represent them?" (Their y-intercept is $(0, 0)$.) "Can you tell where the x-intercepts or the vertex will be?" (Not easily.)
  - "Look at all the expressions in factored form."
    - "What do they have in common?" (They are all in the form of x or $-x$ times a sum or a difference.)
    - "From these expressions, what can we predict, if anything, about the features of the graphs?" (We can "see" their x-intercepts and the vertex.)

DO THE MATH	PLANNING NOTES

Activity 1: What about the Linear Term? *(10 minutes)*

Instructional Routines: Graph It; Discussion Supports (MLR8); Collect and Display (MLR2)
Addressing: NC.M1.F-IF.8a; NC.M1.F-IF.7

In this *Graph It* activity, students use graphing technology to experiment with the linear term in quadratic expressions of the form $ax^2 + bx$, where a is either positive or negative 1. They notice that adding a linear term to x^2 or $-x^2$ shifts the graph both horizontally and vertically, moving the vertex away from the y-axis. They also notice that the x-intercepts are related to the values of a and b. Students make sense of the values of these x-intercepts by recognizing, for instance, that $x^2 + 20x$ is equivalent to $x(x + 20)$, and that x values of 0 and -20 give the expression a value of 0. Students then transition to finding the x-intercepts of the graph using the distributive property, and then finding the vertex by choosing the x-coordinate halfway in between those of the x-intercepts.

Based on their work, students are asked to make a generalization about the x-coordinate of the vertex of a graph with equation $y = ax^2 + bx$. Based on the examples here, they are likely to conclude incorrectly that the x-coordinate of the vertex is $\frac{-b}{2}$. This generalization will be refined in the next activity.

As they change the parameters of an expression on the graph, study the effects on the graph, and articulate their observations, students practice looking for and describing regularity through repeated reasoning (MP8).

Step 1

- Provide access to devices that can run Desmos.
- For the first question, ask one partner to operate the graphing technology and the other to record the group's observations, and then to switch roles halfway.
- Instruct students that creating a slider, in Desmos, to experiment with linear terms might be a useful tool in this activity.
- Use *Discussion Supports* to encourage students' use of mathematical language to communicate understanding. After a student shares a response, invite them to elaborate on their reasoning using everyday language. Consider inviting remaining students to rephrase the reasoning using mathematical language relevant to the lesson. to provide additional opportunities for all students to produce language and solidify understanding. For example, say, "Can someone say that again, using the phrase . . .?"

Student Task Statement

1. Using graphing technology:

    a. Graph $y = x^2$ and then experiment with adding different linear terms (for example, $x^2 + 4x$, $x^2 + 20x$, $x^2 - 50x$). Record your observations.

    b. Graph $y = -x^2$ and then experiment with adding different linear terms. Record your observations.

2. In the previous question, you should have seen that adding a linear term shifts the vertex both vertically and horizontally from (0,0). Use the structure of each equation to find the coordinates of the vertex without graphing.

Equation	Factored form	x-intercepts	Vertex (x, y)
$y = x^2 + 6x$			
$y = x^2 - 10x$			
$y = -x^2 + 50x$			
$y = -x^2 - 36x$			

3. Based on what you observed in question 1 and your findings in question 2, make a generalization about how the form $y = ax^2 + bx$ and the x-intercepts and vertex are related.

Step 2

- Invite students to share their observations and generalizations. Use the *Collect and Display* routine to record students' observations and generalizations on the board to refer back to in the whole-group discussion of Activity 2.
    - Some students may generalize that the vertex always has an x-value that is half of the opposite of b, or $\frac{-b}{2}$, because it represents the halfway point between the two x-intercepts of $(0, 0)$ and $(-b, 0)$. Acknowledge that this is certainly the pattern for the first two cases in Activity 1, all of which had a equal to 1 and $c = 0$. Encourage students to revisit and revise this generalization, if needed, as they consider the last two cases of this activity and other cases going forward.
- Emphasize that the factored form allows us to see the zeros of a quadratic function and the x-intercepts of the graph representing the function. Writing the expression $x^2 - 20x$ as $x(x - 20)$ lets us see that the zeros are 0 and 20 (because $0(0 - 20)$ and $20(20 - 20)$ both give an output of 0) and the x-intercepts are $(0, 0)$ and $(20, 0)$. Likewise, $x^2 + 50x$ can be written as $x(x + 50)$, so the x-intercepts of $(-50, 0)$ and $(0, 0)$ make sense.
- Discuss with students:
    - "What if the squared term has a negative coefficient, as in $y = -x^2 - 36x$? How can we use the equivalent expression in factored form to make sense of the x-intercepts?" (We could rewrite the expression in factored form. Using the distributive property, $-x^2 - 36x$ is equivalent to $-x(x + 36)$, so the x-intercepts are at $x = 0$ and $x = -36$, because $0(0 + 36)$ and $-36(-36 + 36)$ both give an output of 0.)
    - "Can we still use our generalization to predict the x-coordinate of the vertex of a graph if the coefficient of the squared term is negative? Why or why not?" (Yes. When there are two x-intercepts, the vertex is still halfway between the two x-intercepts.)

- By the end of this discussion, students are likely to have concluded that the x-coordinate of the vertex of the graph of $y = ax^2 + bx$ is half of b if a is negative, and half of the opposite of b if a is positive. Tell students they will continue to make sense of quadratic equations and their graphs in the rest of this unit.

DO THE MATH	PLANNING NOTES

Activity 2: Graphing $ax^2 + bx$ (*15 minutes*)

Addressing: NC.M1.F-IF.7

In the previous activity, students made a conjecture about the x-coordinate of the vertex of the graph of $y = x^2 + bx$. This activity aims to provide students with time to develop and expand their initial generalizations from Activity 1 and confirm their thinking using technology to check their reasoning, engaging in MP5, MP7, and MP8.

Step 1

- Ask students to decide who in their groups will be Partner A and who will be Partner B.
- Give students 8 minutes to independently complete rows 1–5 of each of their three columns, without the aid of technology. Remind students that they are graphing sketches. This means students should focus on the vertex, intercepts, and direction of opening.
- After 6 minutes, ask students to check the graphs by graphing either of the equations in Desmos, noting any inconsistencies between the Desmos graph and the sketch in row 6. You may need to remind students to adjust the windows of their Desmos graphs to facilitate comparison.
- As inconsistencies are noted, particularly in questions where $a \neq 1$ or -1, encourage partners to discuss why these inconsistencies may have occurred and to brainstorm a different way to arrive at a more accurate sketch of the graph. Encourage students to revise their generalizations made in Activity 1 based on their experiences in this activity and then be prepared to share with the whole class.
- As Partner A completes column B and Partner B completes column C, watch for how they determine the x-value of the vertex.

Monitoring Tip: Monitor for students who predict that the x-value of the vertex is actually $\frac{-b}{2a}$, or verbalize this idea, let those students know that you may ask them to share out during the whole-group discussion that follows.

***Advancing Student Thinking*:** When given the factored form of a quadratic equation, some students may determine the standard form equation first, and then apply their generalization made in Activity 1 to determine the x-value of the vertex. Students who follow this pathway are likely to find that their graph sketch is not correct, especially if they generalized the formula $\frac{-b}{2}$ for the x-value of the vertex during Activity 1. To advance their thinking, ask these students, "In general, where is the vertex in relation to the x-intercepts? How might we use this idea to find the x-value of the vertex?"

(*continued*)

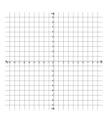
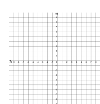
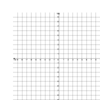
Other students may determine the x-intercepts first and then find the halfway point between them to figure out the x-value of the vertex. Encourage these students to try to find a relationship between the vertex x-value and the values they see in the standard form equation and to share this with their partner during the partner discussion.

In column C, Partner A may think the equation is $y = x(x - 10)$ since that equation captures the fact that the x-intercepts are $(0, 0)$ and $(10, 0)$. Ask these students to substitute 5, the x-coordinate of the vertex, into the equation and see if they find that $y = 25$. If not, what do they need to do to change the equation? If a further hint is needed, ask students to make a sketch of the parabola with the given x-intercepts and vertex. Then ask what they remember about the equations of parabolas that are “upside down.”

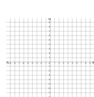
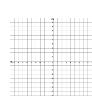
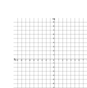
Student Task Statement

Decide with your partner who will complete the table labeled “Partner A” and who will complete the table labeled “Partner B.” Complete each column in your table, without technology, based on the information given to you. After sketching each graph, check your graph in Desmos. If correct, place a check in row 6. If incorrect, use row 6 to name how your graph is different from the graph Desmos provides and revise the generalization you made in Activity 1, or to record a question to ask your partner during the discussion that follows.

Partner A

	Column A	Column B	Column C
Standard form	$y = x^2 + 4x$		
Factored form		$y = -2x(x + 6)$	
x-intercepts			$(0, 0)$ and $(10, 0)$
Vertex (x, y)			$(5, 25)$
Sketch the graph			
Desmos check			

Partner B

	Column A	Column B	Column C
Standard form	$y = -x^2 + 4x$		
Factored form			$y = -5x(x - 2)$
x-intercepts		$(0, 0)$ and $(12, 0)$	
Vertex (x, y)		$(6, -36)$	
Sketch the graph			
Desmos check			

Step 2

- Display the generalizations shared during the whole-group discussion in Activity 1. Invite students to share with the whole group any revisions they made with their partner to their previous generalizations from Activity 1.
- Ask the class to consider the general form $y = ax^2 + bx$, then lead them through this derivation:
  - What is the factored form of the equation? ($y = x(ax + b)$)
  - What are the x-intercepts? ($(0, 0)$ and $(\frac{-b}{a}, 0)$)
  - What is the x-coordinate of the vertex? (The x-coordinate of the vertex is halfway between 0 and $\frac{-b}{2a}$. So $x = (\frac{(0+\frac{-b}{a})}{2} = \frac{-b}{2a}$.
  - How would you find the y-coordinate of the vertex? (Substitute $\frac{-b}{2a}$ for x.)
- Ask students to select one of the standard forms from the activity and verify that this formula does give the x-coordinate of the vertex.

DO THE MATH	PLANNING NOTES

Lesson Debrief *(5 minutes)*

The purpose of this lesson is to explore how the linear term shifts the graph both horizontally and vertically away from the origin, culminating in finding a general method for determining the x-value of the vertex directly from the standard form of the equation.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

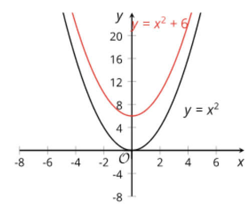
Use these questions to review the effect of a, b, and c on the graphs of quadratic functions expressed in standard form and to generalize the method for finding the x-value of the vertex to all quadratic functions in standard form. Consider using Desmos to illustrate the various transformations.

- Ask questions such as:
  - "How does the graph of $y = -2x^2$ differ from the graph of $y = x^2$?" (The parabola is upside down, and the y-coordinates of the points on the parabola are all twice as far away from the x-axis, stretching the parabola vertically and making it appear narrower)
  - "How does the graph of $y = x^2 + 5$ differ from the graph of $y = x^2$?" (It is shifted up 5 units.)
  - "How does the graph of $y = x^2 + 3x + 5$ differ from the graph of $y = x^2 + 3x$?" (It is still shifted up 5 units)
  - "How does the graph of $y = x^2 + 8x$ differ from the graph of $y = x^2$?" (Its x-intercepts are (-8,0) and (0,0), so it is shifted down and to the left. The vertex also moves down and to the left. We can tell the x-coordinate of the vertex must be -4 because that is halfway between -8 and 0, and using the equation we can find the y-coordinate, -16)

PLANNING NOTES

- To generalize the $\frac{-b}{2a}$ formula to all quadratic functions written in standard form, ask:
  - "Does the formula $\frac{-b}{2a}$ work to find the x-coordinate of the vertex of $y = x^2 + 8x$?" (Yes.)
  - "Do you think it would work to find the x-coordinate of the vertex of $y = x^2 + 8x + 3$?" (Yes, because adding 3 shifts the parabola vertically but does not affect the x-coordinate of the vertex.)
  - "Will the formula $\frac{-b}{2a}$ work to find the x-coordinate of the vertex of any quadratic function of the form $y = ax^2 + bx + c$?" (Yes, because adding c just results in a vertical translation of the graph of $y = ax^2 + bx$; the x-coordinate of the vertex doesn't change.
- Consider displaying the equation $y = x^2 + 8x + c$ in Desmos, adding a slider for c. Show students that when the value of c changes, the parabola moves up and down, but the x-coordinate of the vertex always changes.

Student Lesson Summary and Glossary



In an earlier lesson, we saw that a quadratic function written in standard form $ax^2 + bx + c$ can tell us some things about the graph that represents it. The coefficient a can tell us whether the graph of the function opens upward or downward, and also gives us information about whether it is narrow or wide. The constant term c can tell us about its vertical position.

Recall that the graph representing $y = x^2$ is an upward-opening parabola with the vertex at $(0, 0)$. The vertex is also the x-intercept and the y-intercept.

Suppose we add 6 to the squared term: $y = x^2 + 6$. Adding a 6 shifts the graph upwards, so the vertex is at $(0, 6)$. The vertex is the y-intercept and the graph is centered on the y-axis.

What can the linear term bx tell us about the graph representing a quadratic function? When we compare the graphs of $y = x^2$ and $y = x^2 + 6x$, we see that the vertex of the graph is no longer the y-intercept and is not centered on the y-axis. We can calculate the x-coordinate of the vertex exactly by:

- writing $x^2 + 6x$ in factored form: $x(x + 6)$
- finding the x-intercepts: $(-6, 0)$ and $(0, 0)$
- then calculating the number halfway between the x-coordinates: -3.

After we know the x-coordinate of the vertex, we can use the equation, $y = x^2 + 6x$, to find the y-coordinate. $y = (-3)^2 + 6(-3) = -9$. This means the vertex of the graph is $(-3, -9)$.

We can use the same process to find the x-coordinate of the vertex of any parabola with equation $y = ax^2 + bx$:

- factored form: $y = x(ax + b)$
- x-intercepts: $(0, 0)$ and $(\frac{-b}{a}, 0)$ (because if $ax + b = 0$, $x = \frac{-b}{a}$)
- halfway between 0 and $\frac{-b}{a}$ is half of $\frac{-b}{a}$, or $\frac{-b}{2a}$

This means that if we have a quadratic function with equation $y = ax^2 + bx$, we can find the x-coordinate of the vertex of its graph by calculating the value of $\frac{-b}{2a}$. In fact, this method works to find the vertex of any quadratic function with an equation written in standard form. Since the graph of $y = ax^2 + bx + c$ is just an upward or downward shift of the equation $y = ax^2 + bx$, the x -coordinates of the vertex of each graph should be the same.

Cool-down: Sketching Graphs *(5 minutes)*

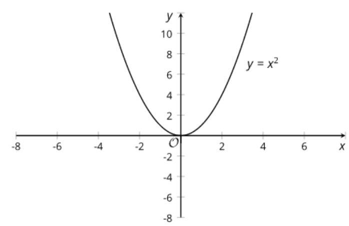
Addressing: NC.M1.F-IF.7; NC.M1.F-IF.8a
Cool-down Guidance: More Chances The warm-up and two main activities of the next lesson all involve finding the vertex of parabolas. Take these opportunities to remind students that they have two methods for finding the x-coordinate of the vertex: going halfway in between the two x-intercepts, or calculating $\frac{-b}{2a}$.

Ask students to put away devices for this cool-down.

Cool-down

1. Consider the quadratic equation $y = x^2 - 4x$. If we graph the equation, where are the x-intercepts located? What is the x-coordinate of the vertex?

2. Here is a graph of $y = x^2$. Sketch a graph of $y = x^2 + 5x$ on the same axes. Label the coordinates of the x-intercepts and the vertex. Briefly explain how you know where to sketch the graph.



Student Reflection:

While working with peers today in math class, what ideas did you get? How did your peers' ideas help you learn?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

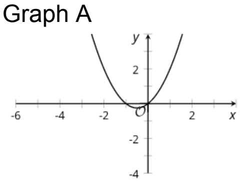
Reflect on times you heard students affirm or challenge one another's math ideas in class today. How will you continue to provide experiences and space for this to happen? Is there anything you might change?

Practice Problems

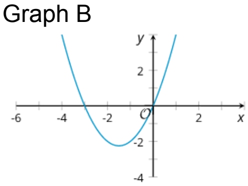
1. Here are four graphs. Match each graph with the quadratic equation that it represents.

a. $y = x^2 + x$
b. $y = -x^2 + 2$
c. $y = x^2 - x$
d. $y = x^2 + 3x$

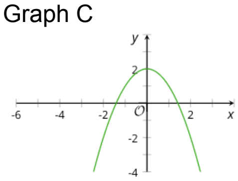
Graph A



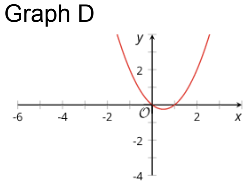
Graph B



Graph C



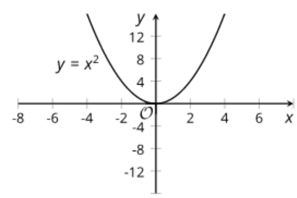
Graph D



2. Complete the table without graphing the equations.

Equation	x-intercepts	Vertex (x, y)
$y = x^2 + 12x$		
$y = x^2 - 3x$		
$y = -x^2 + 16x$		
$y = -x^2 - 24x$		

3. Here is a graph that represents $y = x^2$.



   a. Describe what would happen to the graph if the original equation were changed to $y = x^2 - 6x$. Predict the x- and y-intercepts of the graph and the quadrant where the vertex is located.

   b. Sketch the graph of the equation $y = x^2 - 6x$ on the same coordinate plane as $y = x^2$.

4. Select **all** equations whose graph opens upward.

   a. $y = -x^2 + 9x$
   b. $y = 10x - 5x^2$
   c. $y = (2x - 1)^2$
   d. $y = (1 - x)(2 + x)$
   e. $y = x^2 - 8x - 7$

5. Match each quadratic expression that is written as a product with an equivalent expression that is expanded.

   a. $(x + 3)(x + 4)$
   b. $(x + 3)(x + 7)$
   c. $(3x + 4)(x + 3)$
   d. $(x + 7)(3x + 1)$

   1. $x^2 + 10x + 21$
   2. $3x^2 + 13x + 12$
   3. $3x^2 + 22x + 7$
   4. $x^2 + 7x + 12$

   (From Unit 7, Lesson 7)

6. A bank loans $4,000 to a customer at a $9\frac{1}{2}\%$ annual interest rate.

   Write an expression to represent how much the customer will owe, in dollars, after 5 years without payment.

   (From Unit 6)

7. Determine whether each expression is equivalent to $\frac{1}{x^{-6}}$.

   a. $(x^2)^{-3}$ b. $\frac{x^9}{x^3}$ c. $x \cdot x^2 \cdot x^3$

   (From Unit 6)

8. Which ordered pair is a solution to this system of equations? $\begin{cases} 7x + 5y = 59 \\ 3x - 9y = 159 \end{cases}$

   a. $(-17, -12)$
   b. $(-17, 12)$
   c. $(17, -12)$
   d. $(17, 12)$

   (From Unit 3)

9. The density of an object can be found by dividing its mass by its volume. Write an equation to represent the relationship between the three quantities (density, mass, and volume) in each situation. Let the density, D, be measured in grams/cubic centimeters (or g/cm^3).

   a. The mass is 500 grams, and the volume is 40 cubic centimeters.
   b. The mass is 125 grams, and the volume is v cubic centimeters.
   c. The volume is 1.4 cubic centimeters, and the density is 80 grams per cubic centimeter.
   d. The mass is m grams, and the volume is v cubic centimeters.

   (From Unit 2)

10. Complete the equation with numbers that make the expression on the right side of the equal sign equivalent to the expression on the left side.[1]

    $75a + 25b = \underline{\quad\quad}(\underline{\quad\quad\quad} a + b)$

    (Addressing NC.7.EE.1)

Lesson 13: Graphs That Represent Situations

PREPARATION

Lesson Goal	Learning Target
• Interpret (orally and in writing) the graph and equation representing a function in terms of the context.	• I can explain how a quadratic equation and its graph relate to a situation.

Lesson Narrative

By now, students have seen how the parameters of a quadratic expression in standard form and in factored form relate to the graph representing the function. In the past few lessons, students worked with decontextualized quadratic functions. In this lesson, they transfer what they learned about the graphs to make sense of quadratic functions that model concrete contexts.

Students interpret equations and graphs of quadratic functions in terms of the situations they represent. They use their analyses to solve problems and to compare quadratic functions given in different representations. Along the way, they practice reasoning quantitatively and abstractly (MP2).

What math language will you want to support your students with in this lesson? How will you do that?

Focus and Coherence

Building On	Addressing
NC.8.F.4: Analyze functions that model linear relationships. • Understand that a linear relationship can be generalized by $y = mx + b$. • Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. • Construct a graph of a linear relationship given an equation in slope-intercept form. • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values.	**NC.M1.A-SSE.1a:** Interpret expressions that represent a quantity in terms of its context. **a.** Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents. **NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities. **NC.M1.F-IF.2:** Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior. **NC.M1.F-IF.9:** Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Agenda, Materials, and Preparation

Technology is required for this lesson (with the exception of the Warm-up): Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
  - A Catapulted Pumpkin video: https://bit.ly/PumpkinToss
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L13 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.F.4

This lesson asks students to relate features of quadratic equations to real-world situations, and this bridge will connect the features of linear equations to real-world situations by highlighting the slopes and y-intercepts of the equations.

Student Task Statement

From 6 p.m. to midnight, the equation $t=-1.7h+5.5$ represents the temperature in degrees Celsius, t, after a given number of hours, h, on a cold night in the North Carolina mountains. From 6 a.m. to noon, the equation $C=1.1h-2.3$ represents the temperature in degrees Celsius, C, after a given number of hours, h, as the sun rises.

1. Describe what is happening to the temperature based on the slope of each equation.
2. Describe what you know about the temperatures based on the y-intercepts of each equation.
3. Sketch a graph of each equation. How are they the same, and how are they different?

DO THE MATH	PLANNING NOTES

Warm-up: More Pharmaceutical Profiting (*5 minutes*)

Addressing: NC.M1.F-IF.2; NC.M1.F-IF.7

In this warm-up, students begin to apply their new understandings about graphs to reason about quadratic functions contextually. They evaluate a simple quadratic function, find its maximum, and interpret these values in context. The input values to be evaluated produce an output of 0, reminding students of the meaning of the zeros of a function and their connection to the horizontal intercepts (x-intercepts) of the graph. Students may also reason directly from the values of a and b in this equation of the form $p(v) = av^2 + 150v$. The last question in Step 2 is appropriate for these students only.

Monitoring Tip: To find the maximum value of the function, students could graph the function, but applying what they learned about the connection between the horizontal intercepts and the vertex (without graphing) would be more efficient. If at least one student uses technology to find the maximum, let them know you will invite them to share how they utilized this tool. Also identify students, if any, who who find the x-coordinate of the vertex by taking the middle value between 0 and 500, as well as those who use -b/2a. Ask these students to share their thinking at the appropriate times in the discussion as well.

Step 1

- Remind students that they have seen this pharmaceutical profit function before. Tell students they can use technology for this task but they may find using the patterns developed in Lesson 12 to be more efficient.
- Provide students with quiet work time.

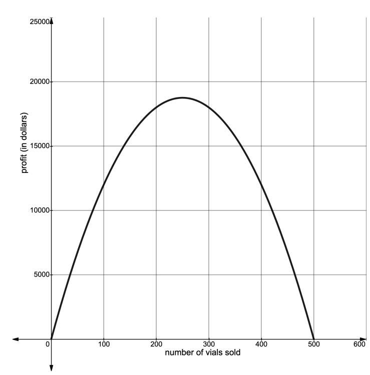
Student Task Statement

The profit of a small pharmaceutical company's insulin, within one week, is modeled by the equation $p(v) = -0.3v^2 + 150v$, where p represents profit in dollars, and v represents the number of vials of insulin sold.

1. Find $p(0)$ and $p(500)$. What do these values mean in terms of the company's profit?
2. How many vials of insulin would the company need to sell to earn the maximum profit? Explain how you know.

Step 2

- Select students to share their responses and explanations. Even though students are not asked to graph the function, make sure that they begin to connect the quadratic expression that defines the function to the features of the graph representing that function. Create a display that includes multiple ways to identify the maximum on the graph. Leave this display visible for the remainder of the lesson.
- Ask students, "If we graph the equation, what would the graph look like? Where would the intercepts be? Would the graph open up or down? Where would the vertex be?"
- Display the graph and highlight the following points:
  - The graph intersects the horizontal axis at $t = 0$ and $t = 500$ because $p(0)$ and $p(500)$ both have a value of 0.
  - $p(250)$ is the maximum profit since $v = 250$ is halfway between the horizontal intercepts. If we were to graph it, $(250, p(250))$ would be the vertex.
  - The graph opens downward because the coefficient of the squared term is a negative number (-0.3).



- Ask students, "Did anyone use the values of a and b from the equation given to find the maximum profit?"

DO THE MATH	PLANNING NOTES

Activity 1: A Catapulted Pumpkin (*10 minutes*)

Instructional Routines: Graph It; Co-Craft Questions (MLR5)	
Addressing: NC.M1.A-SSE.1a; NC.M1.A-CED.2; NC.M1.F-IF.7	

Earlier in this unit, students read off of a diagram the position of an object dropped from the top of a building and then observed that the data is modeled by a quadratic function. This *Graph It* activity continues to develop this idea with an additional layer of complexity, appropriate for this stage in the unit.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Display the first 15 seconds of this video: https://bit.ly/PumpkinToss.
- Use the *Co-Craft Questions* routine to spark curiosity about pumpkin catapulting and connection to mathematics.
  - Before students record possible mathematical questions that could be asked about the situation, invite them to share any questions they may have about the context.
  - Invite students to propose mathematical questions that could be asked, and to compare their questions, before revealing the remainder of the task.
  - Listen for and amplify any questions involving interpretations of features of the quadratic graph.
- Display the equation $d = 10 + 406t - 16t^2$ for all to see. Remind students that earlier in the unit, we saw this equation used to model the height of a cannonball that is shot up in the air as a function of time in seconds. The height was measured in feet. Ask students to recall what each term in the equation represents and briefly discuss their thinking.
- Next, give students a few minutes of quiet think time to read the first question (all four parts) to themselves and think about their responses. Ask them to share their thoughts with a partner before proceeding to the graphing questions. Clarify if needed that "horizontal intercept" and "vertical intercept" are a more general way to refer to x- and y-intercepts when the equation that defines a function uses variables other than x and y.
- Provide access to devices that can run Desmos. If needed, remind students how to use Desmos to identify the coordinates of points on a graph.

RESPONSIVE STRATEGIES

Invite students to follow along as you read the task statement and the questions from the first problem aloud. Students who both listen to and read the information will benefit from extra processing time. Pause for think time and then give students 2–3 minutes to discuss their initial thoughts with a partner before moving on.

Supports accessibility for: Language; Conceptual processing

Student Task Statement

The equation $h = 2 + 23.7t - 4.9t^2$ represents the height of a pumpkin that is catapulted up in the air as a function of time, t, in seconds. The height is measured in meters above ground. The pumpkin is shot up at a vertical velocity of 23.7 meters per second.

1. Without writing anything down, consider these questions:
   a. What do you think the 2 in the equation tells us in this situation? What about the $-4.9t^2$?
   b. If we graph the equation, will the graph open upward or downward? Why?
   c. Where do you think the vertical intercept would be?
   d. What about the horizontal intercepts?
   e. Over what interval do you think the graph will increase? Over what interval do you think the graph will decrease?
2. Graph the equation using Desmos.
3. Identify the vertical and horizontal intercepts, the vertex of the graph, and where the graph is increasing and decreasing. Explain what each feature of the graph means in this situation.

Are You Ready For More?

What approximate vertical velocity would this pumpkin need for it stay in the air for about 10 seconds? (Assume that it is still shot from 2 meters in the air and that the effect of gravity pulling it down is the same.)

Step 2

- Invite students to share their graph and interpretations of the features of the graph. Discuss with students:
  - “The graph shows two horizontal intercepts, one with a positive t-coordinate and the other with a negative t-coordinate. How do we make sense of the negative t-coordinate in this situation?” (Only the positive one has any meaning in this situation, since a negative value of time—the time before the pumpkin was launched—is not relevant in this model.)
  - “How would you estimate the coordinates of the vertex without using a graph? Suppose you know both of the horizontal intercepts.” (We can find the halfway point of the two horizontal intercepts and evaluate the function at that value of t.)
  - “How might you find the intervals over which the graph increases and decreases? (Since we know that the graph opens downwards based on the a value or the context, we know that the graph has to increase to the left of the vertex and decrease to the right of the vertex. Keeping in mind the relevant domain of the function, we can estimate that the graph will be increasing over the domain $0 < x < 2.418$ and decreasing over the domain $2.418 < x < 4.92$, where 2.418 is the x-value of the vertex.)

DO THE MATH

PLANNING NOTES

Activity 2: Flight of Two Baseballs (*15 minutes*)

Instructional Routine: Collect and Display (MLR2)

Addressing: NC.M1.F-IF.9

This activity prompts students to analyze two quadratic functions—one represented by a graph and the other by an equation—and to solve a problem. Graphs can make it easier to compare functions, but because students are asked not to graph the second function, they need to rely on their knowledge of the connections between equations and their graphs to make the comparison. To answer the questions, students have to interpret various pieces of the given information (the graph, the numbers in the equation, the descriptions in the questions, etc.), allowing them to reason quantitatively and abstractly (MP2).

Step 1

- Give students a moment to read the task statement. Make sure students understand what it means for a baseball to "stay in flight."
- Consider keeping students in pairs and asking them to think quietly before discussing their thinking with a partner.

Monitoring Tip: As students work, look for students who reason in the following ways so they can share their thinking in the whole-group discussion later.

- To determine which baseball stayed in flight longer, compare where the graphs intersect the horizontal axis or the zeros of the functions, which signals when the baseball hits the ground.
- For Player B's (function g) maximum height, approximate the halfway point between the horizontal intercepts, which is around $t = 2$, and then find $g(2)$.
- The vertical intercept (y-intercept) represents the height at which the baseball was hit. For function g, expand the $(-16t - 1)(t - 4)$ and write an equivalent expression in standard form, or find $g(0)$.

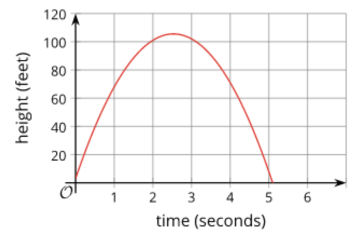
Advancing Student Thinking: Some students may find it challenging to find the zeros of function g because of the -16 in $-16t - 1$. Support these students reasoning about the equation $-16t - 1 = 0$ by asking whether this equation will have a positive or negative solution and whether it will have a solution close to 0 or not. Another challenge students may face is estimating the location of the vertex. Point out the graph of h and ask them how the vertex relates to the horizontal intercepts. These students should understand that the vertex is located halfway between the horizontal intercepts. If they are unsure of how to find the halfway point, encourage them to average the x-coordinates. Encourage struggling students to use technology to graph g to check their thinking.

Step 2

- As students discuss their explanations with a partner, use the *Collect and Display* routine to capture student language that reflects a variety of ways to describe the different representations of quadratic functions. Write the students' words on a visual display of the graph.
  - Capture any student language that will help to emphasize connections between "constant term" and "y-intercept," between "starting height" and "$t = 0$," and between "x-intercepts" and "when the ball hits the ground."

Student Task Statement

Here is a graph that represents the height of a baseball, h, in feet as a function of time, t, in seconds after it was hit by Player A.



The function g defined by $g(t) = (-16t - 1)(t - 4)$ represents the height in feet of a baseball t seconds after it was hit by Player B. Without graphing function g, answer the following questions and explain or show how you know.

1. Which player's baseball stayed in flight longer?
2. Which player's baseball reached a greater maximum height?
3. How can you find the height at which each baseball was hit?

Step 3

Invite students to share their responses and reasoning. Remind students to borrow language from the display as needed. If not mentioned in students' explanations, highlight the following points:

- To determine which baseball stayed in flight longer, a comparison can be made between where the graphs intersect the horizontal axis or the zeros of the functions, which tell us when the baseball hits the ground.
  - The graph of function h intersects the horizontal axis around 5.
  - Function g is given in factored form, so the zeros will be a very small negative number (about $-\frac{1}{16}$), and the other will be 4.
- The maximum height of Player A's baseball (function h) is shown in the graph. For Player B (function g), we can approximate the halfway point between the horizontal intercepts, which is around $t = 2$, and then find $g(2)$.
- The baseball was hit at $t = 0$, so to find the height we would look at the vertical intercept (y-intercept).
  - For function h, this point is shown on the graph.
  - For function g, it is not immediately apparent because the equation is in factored form. In standard form, the constant term is the vertical intercept. We can either expand the $(-16t - 1)(t - 4)$ and write an equivalent expression in standard form, or find $g(0)$ directly from the factored form.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

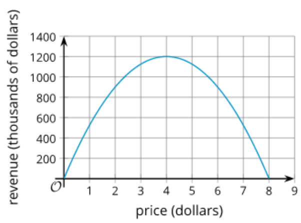
The purpose of this lesson is to interpret the parts of an expression defining a quadratic function and the features of the graph representing it in the context of projectile motion. Emphasize that the graphs that represent quadratic functions that model other kinds of situations can also be interpreted in context.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

Use the following example to illustrate interpreting quadratic functions in other situations and contexts.

In an earlier lesson we saw a quadratic function defined by $r(d) = d(600 - 75d)$. It described the revenue in thousands of dollars collected from ticket sales as a function of the price of one ticket.

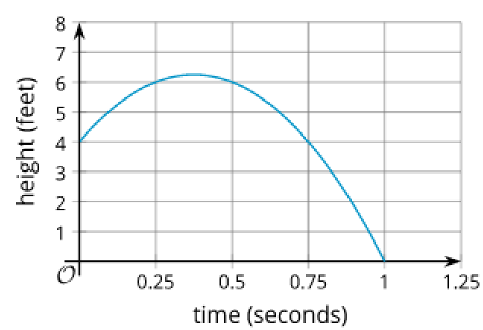
- "Without calculating, what can you tell about the ticket sales and revenue from the equation?" (If the ticket is sold for 0 dollars, or if $d = 0$, the revenue will be 0.)
- "In what form is the quadratic expression defining r written?" (factored form)
- "If we rewrite it in standard form, what new information would we gain?" (In standard form, the equation is $r(d) = 60d - 75d^2$. It gives us the vertical intercept, but that is not new information. We saw that one of the horizontal intercepts is at $(0, 0)$, so the vertical intercept is known.)
- "Here is the graph representing the same function. What information do the horizontal intercepts, the vertical intercept, and the vertex of the graph give us?" (The horizontal intercepts tell us that no revenue will be collected if the tickets are sold at $0 and $8. The vertical intercept gives the same information: that if the ticket is free, there will be no revenue. The vertex tells us that the greatest revenue will be collected when the ticket is sold at $4 each, and that amount is $1,200.)



PLANNING NOTES

Student Lesson Summary and Glossary

Let's say a tennis ball is hit straight up in the air, and its height in feet above the ground is modeled by the equation $f(t) = 4 + 12t - 16t^2$. Here is a graph that represents the function, from the time the tennis ball was hit until the time it reached the ground.



In the graph, we can see some information we already know, and some new information:

- The 4 in the equation means the graph of the function intersects the vertical axis at 4. It shows that the tennis ball was 4 feet off the ground at $t = 0$, when it was hit.
- The horizontal intercept is $(1, 0)$. It tells us that the tennis ball hits the ground 1 second after it was hit.
- The vertex of the graph is at approximately $(0.4, 6.3)$. This means that about 0.4 second after the ball was hit, it reached the maximum height of about 6.3 feet.

The equation can be written in factored form as $f(t) = (-16t - 4)(t - 1)$. From this form, we can see that the zeros of the function are $t = 1$ and $t = -\frac{1}{4}$. The negative zero, $-\frac{1}{4}$, is not meaningful in this situation, because the time before the ball was hit is irrelevant.

Cool-down: Beach Ball Trajectory (*5 minutes*)

Addressing: NC.M1.F-IF.7
Cool-down Guidance: Press Pause If students struggle to interpret functions in context, plan to spend extra time on this topic before the Mid-Unit assessment by reviewing activities and practice problems.

Provide continued access to graphing technology, in case students choose to use it.

Cool-down

The equation $y = (-16t - 2)(t - 1)$ represents the height in feet of a beach ball thrown by a child as a function of time, t, in seconds.

1. Find the zeros of the function. Explain or show your reasoning.
2. What do the zeros tell us in this situation? Are both zeros meaningful?
3. From what height is the beach ball thrown? Explain or show your reasoning.

Student Reflection:

If you were asked how to find horizontal and vertical intercepts, vertices, and increasing and decreasing intervals for a quadratic function, how confident would you be to answer?

a. Very confident b. Somewhat confident c. Not confident at all

If you answered "b" or "c," what support do you need to feel very confident in finding horizontal and vertical intercepts, vertices, and increasing and decreasing intervals?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

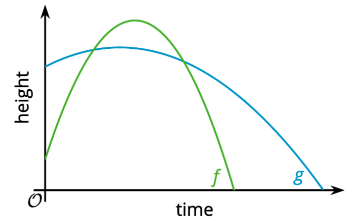
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What evidence have students given that they understand horizontal and vertical intercepts, vertices, and increasing and decreasing intervals? What language do they use or associate with these features?

Practice Problems

1. Here are graphs of functions f and g.



   Each represents the height of an object being launched into the air as a function of time.

   a. Which object was launched from a higher point?

   b. Which object reached a higher point?

   c. Which object was launched with the greater upward velocity?

   d. Which object landed last?

2. (*Technology required.*) The function h given by $h(t) = (1 - t)(8 + 16t)$ models the height of a ball in feet, t seconds after it was thrown.

   a. Find the zeros of the function. Show or explain your reasoning.

   b. What do the zeros tell us in this situation? Are both zeros meaningful?

   c. From what height is the ball thrown? Explain your reasoning.

   d. About when does the ball reach its highest point, and about how high does the ball go? Show or explain your reasoning.

3. The height in feet of a thrown football is modeled by the equation $f(t) = 6 + 30t - 16t^2$, where time, t, is measured in seconds.

   a. What does the constant 6 mean in this situation?

   b. What does the $30t$ mean in this situation?

   c. How do you think the squared term $-16t^2$ affects the value of the function f? What does this term reveal about the situation?

4. The height in feet of an arrow is modeled by the equation $h(t) = (1 + 2t)(18 - 8t)$, where t is time in seconds after the arrow is shot.

   a. When does the arrow hit the ground? Explain or show your reasoning.

   b. From what height is the arrow shot? Explain or show your reasoning.

5. The height in feet of a soccer ball is modeled by the equation $g(t) = 2 + 50t - 16t^2$, where t is time measured in seconds after it was kicked.

   a. How far above the ground was the ball when kicked?

   b. What was the initial upward velocity of the ball?

   c. Why is the coefficient of the squared term negative?

6. Two objects are launched into the air. In both functions, t is time in seconds after launch.

   - The height, in feet, of object A is given by the equation $f(t) = 4 + 32t - 16t^2$
   - The height, in feet, of the object B is given by the equation $g(t) = 2.5 + 40t - 16t^2$.

   a. Which object was launched from a greater height? Explain how you know.

   b. Which object was launched with a greater upward velocity? Explain how you know.

7. (*Technology required.*) Consider the following questions:

   a. Predict the x- and y-intercepts of the graph of the quadratic function defined by the expression $(x+6)(x-6)$. Explain how you made your predictions.

   b. Check your predictions by graphing $y=(x+6)(x-6)$.

   (From Unit 7, Lesson 9)

8. (*Technology required.*) The functions f and g are given by $f(x)=13x+6$ and $g(x)=0.1\cdot(1.4)^x$.

   a. Which function eventually grows faster, f or g? Explain how you know.

   b. Use graphing technology to decide when the graphs of f and g meet.

   (From Unit 6)

9. (*Technology required.*) A student needs to get a loan of $12,000 for the first year of college. Bank A has an annual interest rate of 5.75%, bank B has an annual interest rate of 7.81%, and bank C has an annual rate of 4.45%.

   a. If we graph the amount owed for each loan as a function of years without payment, predict what the three graphs would look like. Describe or sketch your prediction.

   b. Use graphing technology to plot the graph of each loan balance.

   c. Based on your graph, how much would the student owe for each loan when they graduate from college in four years?

   d. Based on your graph, if no payments are made, how much would the student owe for each loan after 10 years?

   (From Unit 6)

10. According to the University of Michigan Department of Medicine, the average weight of a baby in its first 4 days of life can be modeled by the equation $w=-2d+120$, where w represents the weight in ounces and d represents the number of days after the baby is born. Then, the equation $w=0.7d+112$ models an average baby's weight for the next 30 days of its life, where w represents the weight in ounces and d represents the number of days after day 4 of the baby's life.

    a. Describe what is happening to the weight based on the slope of each equation.

    b. Describe what you know about the weights based on the y-intercepts of each equation.

    c. Describe the graphs of each equation in words. How are they the same, and how are they different?

    (Addressing NC.8.F.4)

Lessons 14 & 15: Checkpoint

PREPARATION

Lesson Goals	Learning Targets
• Add and subtract linear and quadratic expressions. • Learn and grow mathematically in course-level content. • Communicate and address mathematical areas of strength and areas of growth.	• I can add and subtract linear and quadratic expressions. • I can continue to grow as a mathematician and challenge myself. • I can share what I know mathematically.

Lesson Narrative

This is a Checkpoint day. Checkpoint days consist of two lessons (one full block) and are structured as four 20-minute stations that students rotate between. There are a total of eight stations students can engage with. Since students will not be able to participate in all eight stations, please note that Station A (Unit 8 Check Your Readiness) and Station B (Adding and Subtracting Quadratics) are required for all students.

This Checkpoint does not include an Are You Ready For More? Station; however, if students did not previously complete the Are You Ready For More? activities, an additional station option would be for students to do so today.

A. Unit 8 Check Your Readiness (*Required*)
B. Adding and Subtracting Quadratics (*Required*)
C. Teacher-led Small-group Instruction
D. Solving Number Riddles and Puzzles
E. Fair Housing Wage
F. Micro-Modeling
G. Reflection

What students will participate in the teacher-led small groups? What will be the focus of those small groups? How will you honor students in those groups?

Agenda, Materials, and Preparation

- **Station A** (*Required, 20 minutes*)
  - Unit 8 Check Your Readiness (print 1 copy per student)
- **Station B** (*Required, 20 minutes*)
  - Students will need access to technology for the following Desmos Activity: https://bit.ly/AddSubtractPolynomials
- **Station C** (*20 minutes*)
- **Station D** (*20 minutes*)
- **Station E** (*20 minutes*)
  - Fair Wage cooperative cards (1 set per group)
  - Red, green, yellow, blue, purple, and orange colored pencils or markers
  - Graph paper and/or chart paper for the group graph
- **Station F** (*20 minutes*)
- **Station G** (*20 minutes*)

STATIONS

Station A: Unit 8 Check Your Readiness (*Required, 20 minutes*)

Remind students that it is really important that their responses to these questions accurately represent what they know. Ask them to answer what they can to the best of their ability. If they get stuck, they should name what they don't know or understand.

Station B: Adding and Subtracting Quadratics (*Required, 20 minutes*)

Addressing: NC.M1.A-APR.1

Station B is a required station that extends adding and subtracting linear expressions to adding and subtracting quadratic expressions. Students may check their work in adding and subtracting by graphing the related functions for both the original expression and the simplified expression, checking to see that the graphs are one in the same.

Step 1

Prior to class, go to https://bit.ly/AddSubtractPolynomials to access the Activity Builder for this Desmos station. You will need to be signed into Desmos.

- Give students access to this station by clicking on "Assign" and choose either "Assign to Your Classes" or "Single Session Code."
  - A class must be created, and students added to it, in a teacher's Desmos account in order to use the "Assign to Your Classes" option. Students will see a "Start" button next to the activity title when logged in on the student.desmos.com page.
  - In order to do this activity without creating a class in Desmos, a "Single Session Code" can be generated to give to students. Instruct students to go to student.desmos.com and enter the single session code.

Step 2

- Student progress can be monitored by clicking "View Dashboard" underneath Activity Sessions on the Desmos Activity Builder page. From this dashboard, student pacing can be adjusted, the activity can be paused for students, and student names can be anonymized.
- Provide feedback to individual students by clicking the chat icon at the top of the student work window of a particular slide.

Station B

Follow your teacher's directions to access this station's Desmos activity. Use the available space below to show your work.

Station C: Teacher-led Small-group Instruction (*20 minutes*)

Use student Cool-down data, Check Your Readiness Unit 7 data, and informal formative assessment data from Unit 7 (Lessons 1–13) to provide targeted small-group instruction to students who demonstrate the need for further support or challenge on topics taught up to this point.

Potential topics:

- Key features of quadratic functions
- Comparing linear, exponential, and quadratic functions
- Graphing quadratic functions in factored form
- Factored form versus standard form of quadratic functions
- Topics from across the course using unused practice problems

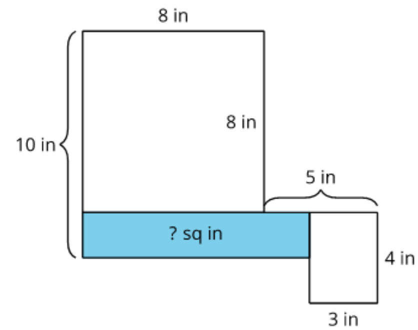
Station D: Solving Number Riddles and Puzzles

Building Towards: NC.M1.A-SSE.2

In this station, students consider factors of values and then examine those factors to find a particular sum. In upcoming lessons, students will factor quadratic expressions, and knowing pairs of values that have a certain product and sum will be important. It is not essential that students solve the challenge riddle, but it may present an interesting way to practice these skills.

Station D

1. List all the pairs of integers whose product is 12.
    a. Circle any pairs with a sum of 7.
    b. Draw a box around any pairs with a sum of 13.
2. Here is a riddle: “I have 2 dogs. The product of their ages is 12, and the sum of their ages is 8. How old are my dogs?"
3. Here is a harder riddle: “I have 3 daughters. The product of their ages is 24. The sum of their ages is the lowest number it could possibly be. How old are they?”
4. A mathematician threw a party. She told her guests, “I have a riddle for you. I have three daughters. The product of their ages is 72. The sum of their ages is the same as my house number. How old are my daughters?” The guests went outside to look at the house number. They thought for a few minutes, and then said, “This riddle can’t be solved!” The mathematician said, “Oh yes, I forgot to tell you the last clue. My youngest daughter prefers strawberry ice cream.” With this last clue, the guests could solve the riddle. How old are the mathematician’s daughters?
5. Can you find the missing area in the figure below? Share your reasoning.



DO THE MATH

PLANNING NOTES

Station E: Fair Living Wage[1] (*20 minutes*)

In this station, students will consider what a “fair wage” would be in a city by comparing housing costs and hourly wages. They will practice graphing and interpreting intercepts and points of intersection. Each group of students should receive a set of Fair Wage cooperative cards. The cards represent the housing costs and wages in Charlotte, NC, but could be revised for wherever students reside.

Station E

You will be given a set of Fair Wage cooperative cards highlighting multiple types of families in Charlotte. You will see their hourly wage information and more. Your goal is to use mathematics to decide whether or not you think six families in Charlotte are paid fair wages.

As a team, you will:

- Draw graphs showing the relationships between the number of hours work and the total wages for multiple families
- Use a different colored pencil or marker for each family
- Identify the dependent and independent variables
- Analyze the wage and rent data

Use this data in the table below on rental prices in Charlotte metro to complete your task:[2]

Studio	1BDR	2BDR	3BDR	4BDR
$653	$745	$864	$1,173	$1,469

1. Each member of the group should select a Fair Wage cooperative card.
2. Draw a graph and write an equation for each family’s earnings over time using a colored pencil or marker based on the color named on the card.
3. Determine the amount each family will need to work in a month to afford monthly rent for an apartment based on their family size (assume there are 4 weeks in a month).

After all members have graphed and solved an equation, answer the following questions:

4. Which family needs to work the **fewest** hours per month to pay rent? How do you know?

[1] Adapted from Frances Harper
[2] U.S. Department of Housing and Urban Development Office of Policy Development and Research. *Hud User*. https://www.huduser.gov

5. Which family needs to work the **most** hours per month to pay rent? How do you know?
6. Were there any lines that intersect? If so, what does that intersection mean in context of this problem?
7. Financial advisors recommend that you only use 30% of your monthly income to pay for rent. (The rest of your income goes to taxes, clothing, food, transportation, savings, and other expenses.) Does the number of hours needed to pay rent match 30% of the earnings for each family? If not, what kind of housing can the family afford with only 30% of their income?
8. According to the National Low Income Housing Coalition,[3] in 2021, a family in NC needs to make $18.46 per hour to afford a moderate two-bedroom home. How well does this match what you found? Explain your reasoning.

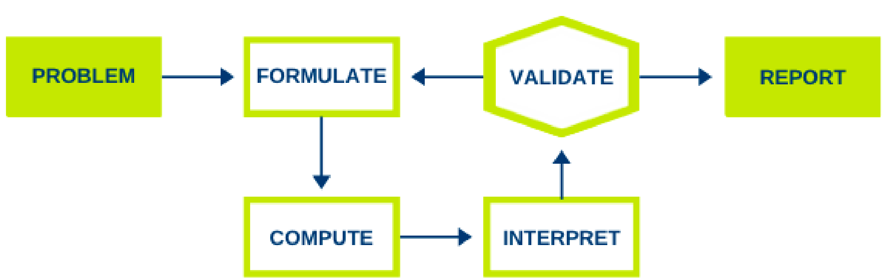
DO THE MATH	PLANNING NOTES

Station F: Micro-Modeling (*20 minutes*)

Instructional Routine: Aspects of Mathematical Modeling

Modeling is the link between the mathematics students learn in school and the problems they will face in college, career, and life. Time spent on modeling in Math 1 is crucial, as it prepares students to use math to handle technical subjects in their further studies, and problem solve and make decisions that adults regularly encounter in their lives.

ASPECTS OF MATHEMATICAL MODELING	***What Is This Routine?*** In activities tagged with this routine, students engage in scaled-back modeling scenarios, for which students only need to engage in a part of a full modeling cycle. For example, they may be selecting quantities of interest in a situation or choosing a model from a list. ***Why This Routine?*** Mathematical modeling is often new territory for both students and teachers. Activities tagged as *Aspects of Mathematical Modeling* offer opportunities to develop discrete skills in the supported environment of a classroom lesson to make success more likely when students engage in more open-ended modeling.



[3] https://reports.nlihc.org/oor

Station F

1. An academic team is going to a state mathematics competition. There are 30 people going on the trip. There are 5 people who can drive and 2 types of vehicles, vans and cars. A van seats 8 people, and a car seats 4 people, including drivers. How many vans and cars does the team need for the trip? Is more than one option available? Explain your reasoning.[4]

2. Mai has a new rabbit that she named Wascal. She wants to build Wascal a pen so that the rabbit has space to move around safely. Mai has purchased a 72-foot roll of fencing to build a rectangular pen.[5]

   a. If Mai uses the whole roll of fencing, what are some of the possible dimensions of the pen?

   b. If Mai wants a pen with the largest possible area, what dimensions should she use for the sides? Justify your answer.

   c. Write and display a model for the area of the rectangular pen in terms of the length of one side.

   d. What kind of function is this? How do you know?

   e. Bonus: If the rabbit pen does not need to be a rectangle, is there a way to get more area with 72 feet of fencing around the perimeter?

DO THE MATH	PLANNING NOTES

Station G: Reflection *(20 minutes)*

This station provides students an opportunity to reflect on their experiences in Math 1 through creative means.

Station G

Describe your year in mathematics so far. You can do this through any form of writing or drawing. Feel free to include your experiences with peers, math material, teachers, etc.

[4] Adapted from Achievethecore.org
[5] Adapted from Open Up Resources. Access the full curriculum, supporting tools, and educator communities at openupresources.org.

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What part of your identity do you see having the most effect on your interactions with students?

Lesson 16: Finding Unknown Inputs

PREPARATION

Lesson Goals	Learning Targets
• Explain (orally and in writing) the meaning of the solution to a quadratic equation in terms of a situation. • Write a quadratic equation that represents geometric constraints.	• I can explain the meaning of a solution to a quadratic equation in terms of a situation. • I can write a quadratic equation that represents a situation involving geometric measures.

Lesson Narrative

Earlier in this unit, students studied quadratic functions in some depth. They built quadratic expressions to represent situations and wrote equivalent expressions. They also graphed, analyzed, and evaluated quadratic functions to solve problems. In some cases, they investigated and interpreted the outputs of the functions. In others, they looked for the input values that produce certain outputs, and they found these values mainly by reasoning with graphs.

Students have not yet explored non-graphical methods for finding what input generates a particular output for quadratic functions. In this lesson students work to find values in a variety of ways including graphing, estimating and solving algebraically. They encounter a problem that cannot be easily solved by familiar strategies, which gives them a chance to persevere in problem solving (MP1). They write a quadratic equation and interpret what a solution means in the given situation. The work here motivates the need to solve quadratic equations. The formal definition of a quadratic equation will not be introduced until the next lesson, after students have seen some variations of such equations and worked with them in context.

What ways might this lesson give students opportunities to surprise you with their thinking or reasoning?

Focus and Coherence

Building On	Addressing
NC.6.G.1: Create geometric models to solve real-world and mathematical problems to: • Find the area of triangles by composing into rectangles and decomposing into right triangles. • Find the area of special quadrilaterals and polygons by decomposing into triangles or rectangles. (*continued*)	**NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

NC.M1.F-IF.4: Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums. **NC.M1.A-APR.1:** Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions.	**NC.M1.A-SEE.1b:** Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
  - A Trip to a Frame Shop framing material (print 1 copy per student)
  - Scissors (1 per student or enough for small groups of students to share)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L16 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.6.G.1

In this lesson, students are asked to persevere by experimenting with framing materials (a sheet of paper) to find the frame that is uniform in width and uses as much of the material as possible. This bridge orients students to a picture frame, its various dimensions and how the dimensions of the frame and picture are different yet dependent on each other.

Student Task Statement

The picture frame shown, made of tree bark, can display an 8" by 10" photo. The width of the bark is 1.5”. How many linear inches of tree bark are needed to create this frame?

DO THE MATH

PLANNING NOTES

Warm-up: What Goes Up Must Come Down (*5 minutes*)

Building On: NC.M1.F-IF.4

This warm-up reminds students that they can use a graph of a function to gain some information about the situation that the function models. The class discussion that follows will draw out the differences in exact answers using the function and estimated answers using the graph.

Step 1

- From earlier work in this unit, students should be familiar with projectiles. But, if needed, provide students with a brief orientation to the context. (Using compressed air, a great amount of force can be generated to launch a potato.)
- Provide students with 2 minutes of quiet work time to answer the questions. Move to the Step 2 discussion even if not all students have completed all questions.

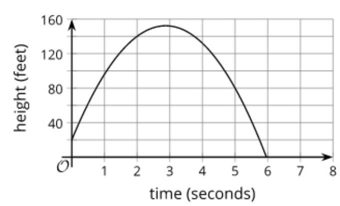
M **Monitoring Tip:** Watch for students who answer the first question by
- estimating the value from the graph
- using the function to evaluate the output for specific inputs (that is, evaluating the function for $h(1)$)

Student Task Statement

A mechanical device is used to launch a potato vertically into the air. The potato is launched from a platform 20 feet above the ground, with an initial vertical velocity of 92 feet per second.

The function $h(t) = -16t^2 + 92t + 20$ models the height of the potato over the ground, in feet, t seconds after launch.

Here is the graph representing the function.



For each question, be prepared to explain your reasoning.

1. What is the height of the potato 1 second after launch?
2. 8 seconds after launch, will the potato still be in the air?
3. Will the potato reach 120 feet? If so, when will it happen?
4. When will the potato hit the ground?

Step 2

- Display the function and graph for all students to see.
- Ask students who used the graph to arrive at answers to share their work. As they share, record their thoughts on the graph and encourage precise mathematical vocabulary specific to key features of the graph (x-intercepts, y-intercepts, maximum, minimums). Repeat this process for all four questions.
- Next, ask a student who found the answer to the first question by finding a value of $h(1)$ to share their work. Ask students, “Which answer—this approach or using the graph—is more exact? Why?” (The use of the function will yield an exact answer, while the answer from the graph is an estimation. The results should be close in value.)
- If time permits, ask students the following questions:
  - “Which questions in the activity could be answered by calculating?” (The first two questions—about where the potato is after some specified number of seconds—can be answered by calculating.)

- "Which questions were not as easy to figure out by calculation? (The questions about when the potato reaches certain heights are not as easy to figure out by calculation.)
- "How can we verify the time when the potato is 120 feet above the ground or when, exactly, it hits the ground?" (To find when the potato is exactly 120 feet about the ground, we would need to solve the equation $h(t) = 120$, or $-16t^2 + 92t + 20 = 120$. To find when the potato hits the ground exactly, we would need to find the zeros of h by solving the equation $h(t) = 0$.)

- In this lesson, we will investigate how answers to these questions could be calculated rather than estimated from a graph.

DO THE MATH	PLANNING NOTES

Activity 1: A Trip to the Frame Shop[1] (*15 minutes*)

Instructional Routine: Compare and Connect (MLR7) - Responsive Strategy
Building Towards: NC.M1.A-CED.1

The purpose of this activity is to motivate the need to write and solve a quadratic equation in order to solve a problem.

Students are asked to frame a picture by cutting up a rectangular piece of "framing material" into strips and arranging it around a picture such that they create a frame with a uniform thickness. The framing material has a different length-to-width ratio and is smaller than the picture, so students cannot simply center the framing material on the back of the picture.

Students are not expected to succeed in the task through trial and error. They are meant to struggle just enough to want to know a way to solve the problem that is better than by guessing and checking. Some students may try writing an equation to help them figure out the right measurement for the frame, but they don't yet have the knowledge to solve the quadratic equation. (The solutions to this equation are irrational, so it is also unlikely for students to find them by chance.)

RESPONSIVE STRATEGIES

Display and read the directions aloud. Students who both listen to and read the information will benefit from additional processing time. Check for understanding by inviting students to rephrase directions in their own words.

Supports accessibility for: Language; Memory

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Orient students to the task by asking if anyone in the class has ever had an artwork or a picture framed at a frame shop. Explain that it is very expensive—often hundreds of dollars for one piece. The framing materials, which are cut to exact specifications, can be costly. The time and labor needed to properly frame a picture further increase the cost.

[1] This activity was inspired by the post "When I Got Them to Beg" by Fawn Nguyen on http://fawnnguyen.com/got-beg/ and used with permission.

- Tell students that they are now going to frame a picture, using a sheet of paper to model their framing material. The sheet is to be cut such that:
  - All of it is used. (Framing material is expensive!)
  - The framing material does not overlap.
  - The resulting frame has uniform thickness all the way around.
- Distribute scissors, along with the pictures and the A Trip to the Frame Shop framing material (copies of the blackline master).
- Provide students time to work with their partner to create their frames.

Student Task Statement

Your teacher will give you a picture that is 7 inches by 4 inches, a piece of framing material measuring 4 inches by 2.5 inches, and a pair of scissors.

Cut the framing material to create a rectangular frame for the picture. The frame should have the same thickness all the way around and have no overlaps. All of the framing material should be used (with no leftover pieces). Framing material is very expensive!

You get three copies of the framing material in case you make mistakes and need to recut.

Are You Ready For More?

Han says, "The perimeter of the picture is 22 inches. If I cut the framing material into 9 pieces, each one being 2.5 inches by $\frac{4}{9}$ inch, I'll have more than enough material to surround the picture because those pieces would mean 22.5 inches for the frame."

Do you agree with Han? Explain your reasoning.

Step 2

- Facilitate a class discussion of the activity by asking questions such as:
  - "How did you decide how thick the frame should be?" (I tried a couple of different thicknesses to see if they would work. I first tried $\frac{1}{2}$ inch, but that didn't give enough to frame the entire picture. Then, I tried $\frac{1}{4}$ inch, but that was too thin.)
  - "How did you know what thickness would be too large or too small?" (It is hard to tell, but I know that the framing needs to have a linear measurement of at least 22 inches, enough for the entire perimeter of the picture, plus some more length for the corners. If the frame is too thin, there will be extra material. If it is too thick, there won't be enough to get the 22-plus-some-inch length.)
  - "How did you determine the thickness such that no framing materials were left unused? Is it possible to determine this?" (I tried to find a way to evenly spread the area of the framing material around the picture, but I wasn't quite sure how to do that.)
  - "What frame thickness did you end up with?"
  - "Were all the strips or pieces the same size?"
- Students are likely to share the challenges they encountered along the way. Tell students that, in this unit, they will learn strategies that are more effective than trial and error or solving problems such as this one.

RESPONSIVE STRATEGY

Use this routine to prepare students for the whole-class discussion. After students have had time to work independently and share their strategies with a partner, invite students to quietly circulate and examine at least one other "frame" in the room. Give students time to talk to their partner to consider what is the same and what is different about the constructed frames. Listen for and amplify observations that include connections across approaches, challenges faced identifying dimensions of framing material, and the need for more sophisticated strategies for solving this problem using quadratic equations.

Compare and Connect (MLR7)

DO THE MATH	PLANNING NOTES

Activity 2: Representing the Framing Problem (*10 minutes*)

Instructional Routines: Aspects of Mathematical Modeling; Three Reads (MLR6)	
Building On: NC.M1.A-APR.1	**Addressing:** NC.M1.A-CED.1; NC.M1.A.SSE.1b

This activity allows students to engage in *Aspects of Mathematical Modeling* by formulating a mathematical model around the framing task they saw earlier (MP4). Students are prompted to write an equation but are not expected to solve it at this point. In writing an equation and interpreting the solution in context, students practice reasoning quantitatively and abstractly (MP2).

RESPONSIVE STRATEGIES

Represent the same information through different modalities by using diagrams. Provide students with graph paper and suggest that they draw and label their diagrams with everything they know so far. Invite students to suggest types of diagrams that might be helpful to draw, such as the strips of framing material, and the picture both with and without the frame.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Step 1

- Students remain with their same partner.
- Share with students that they are now going to explore how the quantities of the framing problem could be represented with an equation.
- Use the *Three Reads* routine to get students started on this task.
  - First Read: Read the problem aloud to the class: “Here is a diagram that shows the picture with a frame that is the same thickness all the way around. The picture is 7 inches by 4 inches. The frame is created from 10 square inches of framing material (in the form of a rectangle measuring 4 inches by 2.5 inches).”
    - Ask students: “What is this situation about? What is going on here?”
    - Spend less than 1 minute scribing student ideas. Let students know the focus is just on the situation, not on the numbers and making connections to the frame of Activity 1.
  - Second Read: Display the situation and ask a student volunteer to read it aloud to the class again.
    - Then ask: “What are the quantities in this situation? A quantity is something that can be counted or measured.”
    - Again, spend less than a minute scribing student responses. Listen for, and amplify, the important quantities in this situation: the dimensions of the picture, the length of framing materials, the length and width of the picture and of the frame.
  - Third Read: Invite students to read the situation again to themselves, or ask another student volunteer to read it aloud. After the third read, reveal the first question that follows.

- Ask, “ How might we approach the question being asked? What is the first thing you will do?” (The advancing student thinking questions may be useful.)
- Spend 1–2 minutes scribing student ideas as they brainstorm possible starting points. Be sure to stop any students who begin to share a complete solution; the goal is to crowdsource some starting points. (Again, the questions and ideas in the advancing student thinking question may be useful.)

Step 2

- Provide students 1–2 minutes of quiet think time to work on the task and then another 1–2 minutes for partners to share their thinking with each other.

M

Monitoring Tip: The prompt to write an equation is left relatively open to allow for different approaches. As students work, monitor for different strategies students use, such as thinking in terms of:

- The length and width of the framed picture. The length is 7 inches plus the thickness of the frame, x, on either side, which gives $7+2x$. The height is 4 inches plus the same thickness, x, for the top and bottom, which gives $4+2x$. The total area is $7 \cdot 4 + 4 \cdot (2.5)$ or 38 square inches, so the equation is $(7+2x)(4+2x)=38$.
- The area of the picture plus the area of the frame. The area of the picture is 28 square inches. The area of the frame, in square inches, can be found by decomposing it into four squares (in the corners) that are x by x or x^2 each, two rectangles that are $7x$ each (top and bottom), and two rectangles that are $4x$ each (left and right). The equation is $4x^2+2(7x)+2(4x)+28=38$ or $4x^2+22x+28=38$.

Prepare students with contrasting approaches that they may be asked to share later.

Advancing Student Thinking: If students have trouble getting started, suggest that they start by labeling the diagram with relevant lengths. Then, if needed, use scaffolding questions such as:

- “How could we show that the frame is the same thickness all the way around?” (Use the same variable, for example x, to label the thickness of the strip of framing on all four sides.)
- “What is the combined area of the picture and the framing material?” (38 square inches, because $7 \cdot 4 + 4 \cdot (2.5) = 38$.)
- “How could we express the width of the framed picture?” ($7+2x$) “What about the height of the framed picture?” ($4+2x$)
- “Once the framing material is cut up and arranged around the picture, how could we express its area?” (Two rectangles with area $x(7+2x)$ and two rectangles with area $4x$, or $2x(7+2x)+2(4x)$.)

Some students may struggle to express the overall length and width of the framed picture because of trouble combining numbers and variables. Consider drawing a segment composed of 3 pieces of length 1.5 inches, 10 inches, and 1.5 inches, and prompting students to find the length of the entire segment. Then, change each 1.5 to an x and ask for an expression for the length of the entire segment $(10+2x)$.

If students mistake the result of adding x and x as x^2, consider drawing a square with side length x and ask students to write expressions for the perimeter and area. Then, ask them to point out the difference between multiplying to find area and adding to find total length. If needed, draw a few more segments that are decomposed into parts, with each part labeled with a number or a variable expression, and ask students to write expressions for the total length.

Student Task Statement

Here is a diagram that shows the picture with a frame that is the same thickness all the way around. The picture is 7 inches by 4 inches. The frame is created from 10 square inches of framing material (in the form of a rectangle measuring 4 inches by 2.5 inches).

1. Write an equation to represent the relationship between the outer measurements of the picture and of the frame, and the area of the framed picture. Be prepared to explain what each part of your equation represents.
2. What would a solution to this equation mean in this situation?

Step 3

- Select students to present their equations and reasoning. If no students bring up one of the strategies shown in the Monitoring Tip, bring it up.
- If students share equations that are equivalent, ask students if they are equivalent and, if so, how we know. (For example, are $(7+2x)(4+2x)=38$ and $4x^2+22x+28=38$ equivalent?) Discuss what a solution to these equations represents. Make sure students understand that a solution reveals the thickness of the frame when all of the framing material (10 square inches) is used. Solicit some ideas on how they might go about finding the solution(s). Students may mention:
  - graphing the equation $y=(7+2x)(4+2x)$ and finding a point on the graph with a y-coordinate of 38
  - evaluating the expression on one side of the equation at different values of x until it has a value of 38
  - using a spreadsheet to evaluate the expression at different values of x and see what values produce an output of 38
- We can see that the equations $(7+2x)(4+2x)=38$ and $4x^2+22x+28=38$ each include a quadratic expression. Tell students that these are examples of quadratic equations. They will learn several techniques for using algebra to solve equations like this in future lessons.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to introduce students to quadratic equations and to motivate the need to solve them. Students may have informal or partial understandings of quadratic equations at this point, and that is ok. Students will build techniques for solving quadratic equations throughout the remainder of this unit.

Choose which of these questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- "In this lesson, we saw a few examples where we knew the output of a function and were interested in finding the input that produced it. In the launched potato situation, what outputs were known?" (the potato's heights or distances from the ground) "What inputs did we want to know?" (the times at which the potato was at certain heights)
- "In the framing problem, what was the output that we cared about?" (the total area of the framed picture, which was 38 square inches) "What information was the input?" (the thickness of the frame that would produce that total area)

PLANNING NOTES

- "Why might it be helpful to write an equation to represent a problem such as the one about framing?" (An equation helps us see the relationship between the quantities in the problem, including those whose values are unknown. We can solve for those values.)
- "In the framing situation, if x represents the thickness of the frame, in inches, what would a solution to the equation $(2x+7)(2x+4)=50$ mean?" (the thickness of the frame that would produce a total area of 50 square inches)
- "Equations such as $(2x+7)(2x+4)=50$ and $4x^2+22x+28=38$ are called quadratic equations. Why do you think equations like these are described as quadratic?" (Each equation has a quadratic expression in it. Each one relates quantities in a quadratic function.)

Student Lesson Summary and Glossary

The height of a softball, in feet, t seconds after someone throws it straight up, can be defined by $f(t)=-16t^2+32t+5$. The input of function f is time, and the output is height.

We can find the output of this function at any given input. For instance:

- At the beginning of the softball's journey, when $t=0$, its height is given by $f(0)$.
- Two seconds later, when $t=2$, its height is given by $f(2)$.

The values of $f(0)$ and $f(2)$ can be found using a graph or by evaluating the expression $-16t^2+32t+5$ at those values of t. What if we know the output of the function and want to find the inputs? For example:

- When does the softball hit the ground?

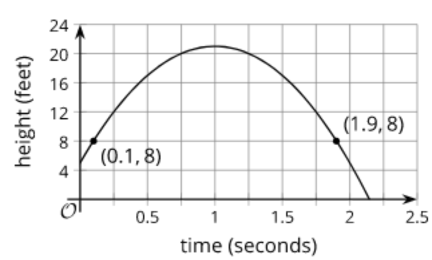
  Answering this question means finding the values of t that make $f(t)=0$, or solving $-16t^2+32t+5=0$.
- How long will it take the ball to reach 8 feet?

  This means finding one or more values of t that make $f(t)=8$, or solving the equation $-16t^2+32t+5=8$.

The equations $-16t^2+32t+5=0$ and $-16t^2+32t+5=8$ are quadratic equations. One way to solve these equations is by graphing $y=f(t)$.

- To answer the first question, we can look for the horizontal intercepts of the graph, where the vertical coordinate is 0.
- To answer the second question, we can look for the horizontal coordinates that correspond to a vertical coordinate of 8.

We can see that there are two solutions to the equation $-16t^2+32t+5=8$.



The softball has a height of 8 feet twice, when going up and when coming down, and these occur when t is about 0.1 or 1.9.

Often, when we are modeling a situation mathematically, an approximate solution is good enough. Sometimes, however, we would like to know exact solutions, and it may not be possible to find them using a graph.

In this unit, we will learn more about quadratic equations and how to solve them exactly using algebraic techniques.

Cool-down: Interpreting a Solution (*5 minutes*)

Building Towards: NC.M1.A-SSE.1b

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

A framed picture has a total area y, in square inches. The thickness of the frame is represented by x, in inches. The equation $y = (8 + 2x)(10 + 2x)$ relates these two variables.

1. What are the length and width of the picture without the frame?
2. What would a solution to the equation $100 = (8 + 2x)(10 + 2x)$ mean in this situation?

Student Reflection:

Today my participation was (*high, medium, low*) _________ because…

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Reflect on who participated in math class today. What assumptions are you making about those who did not participate? How can you leverage each of your students' ideas to support them in being seen and heard in tomorrow's math class?

Practice Problems

1. Jada throws a paper airplane from her treehouse. The height of the plane is a function of time and can be modeled by the equation $h(t) = 25 + 2.5t - \frac{1}{2}t^2$. Height is measured in feet and time is measured in seconds.

    a. Evaluate $h(0)$ and explain what this value means in this situation.

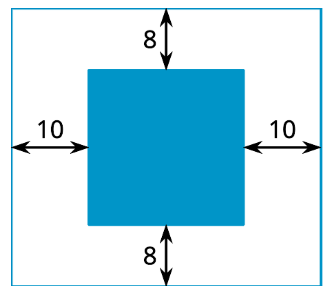
    b. What would a solution to $h(t) = 0$ mean in this situation?

    c. What does the equation $h(9) = 7$ mean?

    d. What does the model say about the airplane 2.5 seconds after Jada throws it if each of these statements is true?

    $h(2) = 28$ $h(2.5) = 28.125$ $h(3) = 28$

2. A garden designer designed a square decorative pool. The pool is surrounded by a walkway.

    On two opposite sides of the pool, the walkway is 8 feet. On the other two opposite sides, the walkway is 10 feet. Here is a diagram of the design.

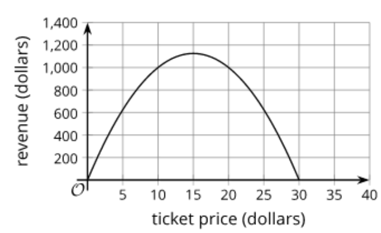
    The final design for the pool and walkway covers a total area of 1,440 square feet.

    a. The side length of the square pool is x. Write an expression that represents:

    - the total length of the rectangle (including the pool and walkway)
    - the total width of the rectangle (including the pool and walkway)
    - the total area of the pool and walkway

    b. Write an equation of the form: $\textit{your expression} = 1,440$. What does a solution to the equation mean in this situation?

3. The revenue from a youth league baseball game depends on the price of per ticket, x.

    Here is a graph that represents the revenue function, R.

    Select **all** the true statements.

    a. $R(5)$ is a little more than 600.
    b. $R(600)$ is a little less than 5.
    c. The maximum possible ticket price is \$15.
    d. The maximum possible revenue is about \$1,125.
    e. If tickets cost \$10, the predicted revenue is \$1,000.
    f. If tickets cost \$20, the predicted revenue is \$1,000.

4. A square picture has a frame that is 3 inches thick all the way around. The total side length of the picture and frame is x inches.

    Which expression represents the area of the square picture, without the frame? If you get stuck, try sketching a diagram.

    a. $(2x + 3)(2x + 3)$
    b. $(x + 6)(x + 6)$
    c. $(2x - 3)(2x - 3)$
    d. $(x - 6)(x - 6)$

5. Add or subtract:

   a. $(7x^2 - 3x + 2.4) + (-4x^2 - 5x - 4)$

   b. $(8x^2 + 7) - (3x^2 - 5.5x + 11.5)$

   c. $(4x^2 - 2x) + (9x^2 + 5) - (8.2x - 4.5)$

   (From Unit 7, Lessons 14 and 15)

6. Two rocks are launched straight up in the air. The height of rock A is given by the function f, where $f(t) = 4 + 30t - 16t^2$. The height of rock B is given by g, where $g(t) = 5 + 20t - 16t^2$. In both functions, t is measured in seconds, and height is measured in feet.

   a. Which rock is launched from a higher point?

   b. Which rock is launched with greater velocity?

   (From Unit 7, Lesson 5)

7. A bacteria population is 10,000 when it is first measured and then doubles each day.

   a. Use this information to complete the table.

   b. Which is the first day, after the population was originally measured, that the bacteria population is more than 1,000,000?

   c. Write an equation relating p, the bacteria population, to d, the number of days since it was first measured.

   (From Unit 6)

d, time (days)	p, population (thousands)
0	
1	
2	
5	
10	
d	

8. An American traveler who is heading to Europe is exchanging some U.S. dollars for European euros. At the time of his travel, 1 dollar can be exchanged for 0.91 euros.

   a. Find the amount of money in euros that the American traveler would get if he exchanged 100 dollars.

   b. What if he exchanged 500 dollars?

   c. Write an equation that gives the amount of money in euros, e, as a function of the dollar amount being exchanged, d.

   d. Upon returning to America, the traveler has 42 euros to exchange back into U.S. dollars. How many dollars would he get if the exchange rate is still the same?

   e. Write an equation that gives the amount of money in dollars, d, as a function of the euro amount being exchanged, e.

   (From Unit 5)

9. Suppose m and c each represent the position number of a letter in the alphabet, but m represents the letters in the original message and c represents the letters in a secret code. The equation $c = m + 2$ is used to encode a message.

   a. Write an equation that can be used to decode the secret code into the original message.

   b. What does this code say: "OCVJ KU HWP!"?

   (From Unit 5)

10. Give a value for r, the correlation coefficient, that would indicate that a line of best fit has a positive slope and models the data well.

    (From Unit 4)

Lesson 17: When and Why Do We Write Quadratic Equations?

PREPARATION

Lesson Goals	Learning Targets
• Recognize the limitations of certain strategies used to solve a quadratic equation. • Understand that the factored form of a quadratic expression can help us find the zeros of a quadratic function and solve a quadratic equation. • Write quadratic equations and reason about their solutions in terms of their situation.	• I can use a graph to find the solutions to a quadratic equation but also know its limitations. • I can recognize the factored form of a quadratic expression and know when it can be useful for solving problems. • I can write quadratic equations and reason about their solutions in terms of their situation.

Lesson Narrative

In this lesson, students revisit some situations that can be modeled with quadratic functions. They analyze and interpret given equations, write equations to represent relationships and constraints (MP4), and work to solve these equations. In doing so, students see that sometimes solutions to quadratic equations cannot be easily or precisely found by graphing or reasoning.

Earlier, students saw that when a function is defined by a quadratic expression in factored form, the zeros of the function could be easily identified. Here, they notice that when a quadratic equation is written as $factored\ form = 0$ solving the equation is also relatively simple. This revelation motivates upcoming work on rewriting quadratic expressions in factored form.

Share some ways this lesson connects to previous lessons. What connections will you want to make explicit?

Focus and Coherence

Building On	Addressing	Building Towards
NC.8.F.4: Analyze functions that model linear relationships. • Understand that a linear relationship can be generalized by $y = mx + b$. • Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. • Construct a graph of a linear relationship given an equation in slope-intercept form. • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values. (*continued*)	**NC.M1.A-REI.4:** Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.	**NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

NC.M1.A-REI.1: Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning. **NC.M1.A.REI.3:** Solve linear equations and inequalities in one variable.		

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L17 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.F.4

This bridge connects the inputs and outputs of a function with the x-and y-intercepts of a graph using linear functions, a familiar function type. Students will use these concepts with quadratic functions in this lesson.

Student Task Statement

Use the function $f(x) = -5x - 10$ to answer the questions.

1. What is the y-intercept of the graph of $y = f(x)$?
2. What is the x-intercept of the graph of $y = f(x)$?
3. Solve the equation $0 = -5x - 10$. Which intercept does this reveal? Explain.
4. Find $f(0)$. What intercept does this reveal? Explain.

DO THE MATH	PLANNING NOTES

Warm-up: How Many Tickets? (*5 minutes*)

Building On: NC.M1.A-REI.3

This warm-up activates what students know about interpreting equations in context and about solving for a variable. The given equation is linear and is relatively straightforward. The work prepares students to reason about quadratic equations in the lesson.

Step 1

- Give students 2 minutes to work independently.

Student Task Statement

The expression $12t + 2.50$ represents the cost to purchase tickets for a play, where t is the number of tickets. Be prepared to explain your response to each question.

1. A family paid $62.50 for tickets. How many tickets were bought?
2. A teacher paid $278.50 for tickets for her students. How many tickets were bought?

Step 2

- Ask students to share their responses and reasoning. Highlight the different strategies used to answer the questions. Those strategies may include:
  - trying different values of t until they find one that yields the specified value of c
  - reasoning backwards (subtract 2.5 from 62.50 and then divide the result by 12) without writing out the steps
  - solving $62.5 = 12t + 2.5$ and $278.5 = 12t + 2.5$ by performing the same operations to each side to isolate t
- If no students thought of the situation in terms of a function and wrote an equation, point out that we can think about cost as being a function of the number of tickets. In answering the questions, we were looking for the inputs that produce different outputs. This can be done by solving equations. In the case of linear equations, we can "do the same thing to each side" to isolate the variable.

DO THE MATH	PLANNING NOTES

Activity 1: The Flying Potato Again (*10 minutes*)

Instructional Routines: Take Turns; Discussion Supports (MLR8); Poll the Class	
Building On: NC.M1.A-REI.1	**Building Towards:** NC.M1.A-CED.1; NC.M1.A-REI.4

This task prompts students to try different ways to solve a quadratic equation. They are familiar with solving equations by performing the same operations to each side of an equation, but here they see that this is not really a workable strategy. Because they do not yet know an efficient way to use algebra to solve $-16t^2 + 80t + 64 = 0$, they need to try different strategies and persevere in problem solving (MP1).

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Provide a few minutes of quiet think time and then time to collaborate with partners on solving the equation.
- As partners work, they should *Take Turns* describing what steps they think should be taken next to find the solution and explaining the reasoning behind those steps. Use *Discussion Supports* and display the following sentence frames for all to see:
  - "We should do _____ next because….."
  - "I noticed _____, so I think…"
- Encourage students to challenge each other when they disagree. This will help students clarify their reasoning when persevering in problem solving quadratic equations.

Monitoring Tip: As partners are working, keep track of partners who use the following strategies to solve the problem.

- Isolating the variable. (This will not result in a solution pathway, but students may try.)
- Guessing and checking by substituting different inputs trying to arrive at a given output.
- Graphing.

If all students choose graphing as their strategy, stop the class and have the graphing discussion in Step 2 quickly. Then provide students with the challenge to find this same solution without using the graph. Provide students a few minutes to approach this challenge and then proceed to the rest of the class discussion.

Student Task Statement

Earlier in this unit, you saw an equation that defines the height of a potato as a function of time after it was launched from a mechanical device. Here is a different function modeling the height of a potato, in feet, t seconds after being fired from a different device:

$$f(t) = -16t^2 + 80t + 64$$

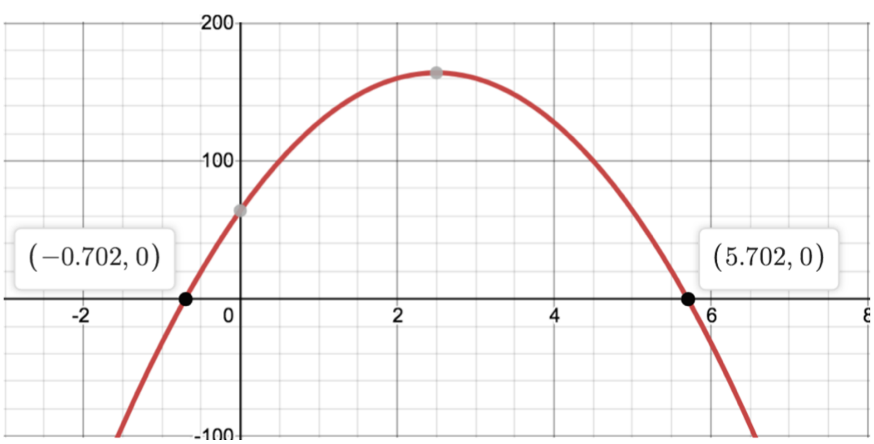
1. What equation would we solve to find the time at which the potato hits the ground?
2. Use any method to find a solution to this equation.

Step 2

- *Poll the Class* to collect, record, and display the solution methods for all to see. Ask some students to share their strategies. Make sure to address each of the three strategies mentioned in the Monitoring Tip:
    - Isolating the variable: If we try to solve by performing the same operations to each side of the equation, we quickly get stuck.

$$\begin{aligned} -16t^2+80t+64&=0 && \textit{Original equation}\\ -16t^2+80t&=-64 && \textit{Subtract 64 from each side}\\ t^2-5t&=4 && \textit{Divide each side by } -16 \end{aligned}$$

      But then what? If we add $5t$ to each side, we now have a variable on both sides and cannot combine any like terms. We could multiply or divide each side by any constant we wish, but we are no closer to isolating t.
    - Guessing and checking: We can evaluate the quadratic expression at different values of t until the expression has a value that is 0 or close to 0. For example, when t is 4, the expression has a value of 128. At $t=5$, it has a value of 64, and at $t=6$, it has a value of -32. That means t is between 5 and 6, so we need to try different decimal values in that range. This process is laborious, and may not get us to a precise solution.
    - Graphing: Students may suggest that a graph would allow them to solve the problem much more quickly. Use graphing technology to demonstrate that if we graph the equation $y=f(t)$, an approximate solution given is 5.702, as shown in the image.



  If we evaluate $f(5.702)$, however, we get roughly -0.044864, rather than exactly 0.

  A graph is useful for approximating values, but it isn't always possible to use it to find exact values.

- Tell students that in this lesson and the next few lessons, they will learn some efficient strategies for solving equations like these.

RESPONSIVE STRATEGIES

Use color and annotations to illustrate student thinking. As students share their reasoning about methods to find a solution, scribe their thinking on a visible display. Display sentence frames for students to use during the discussion, such as: "This method works/doesn't work because . . . ." and "Another strategy would be ____ because . . . ."

Supports accessibility for: Visual-spatial processing; Conceptual processing

DO THE MATH	PLANNING NOTES

Activity 2: Revenue from Ticket Sales (*15 minutes*)

Instructional Routine: Collect and Display (MLR2)	
Addressing: NC.M1.A-REI.4	**Building Towards:** NC.M1.A-CED.1

This activity aims to show that it is relatively easy to solve a quadratic equation when one side of the equation is zero and the other side is a quadratic expression in factored form, and that it may be a little tricky to solve the equation otherwise.

The activity prompts students to recall what they learned in an earlier unit, that the zeros of a function correspond to the horizontal intercepts of the graph representing that function, and that the zeros are the solutions to an equation of the form $quadratic\ expression = 0$.

As students make sense of the equations and ways to use them to solve a contextual problem, they practice reasoning quantitatively and abstractly (MP2).

Step 1

- Keep students in their pairs from the previous activity.
- Read the opening paragraph in the task statement. Give students a moment to think about how much the school would collect if they sell the tickets at $5 each. (It would collect $875.) Briefly survey how they found the answer. (Students are likely to have used the factored form because it lends itself to simpler calculations.)
- Ask students to recall the form in which each quadratic expression is written. Then, give them a minute to talk to a partner and recall at least two things about each form and what the form might tell us about the graph of the function that the expression defines.
- Use the *Collect and Display* routine to record responses to two things about each form for all to see. Language can be collected during partner discussions or class discussions. Listen for and collect the language students use, including: vertex, y-intercept, zeros, etc. Connect the language on a visual display and continue to come back to, and perhaps add to the visual display, throughout the lesson. If no students mentioned the connection between either of the forms to the horizontal intercepts of the graph or the zeros of the function, ask them about it.

Advancing Student Thinking: If students struggle to connect the expressions that define the function to the questions, ask them what the input and output of the function represent. If students struggle with the first question, ask them what values of p would yield a value of 0 for the expression.

Student Task Statement

The expressions $p(200-5p)$ and $-5p^2+200p$ define the same function. The function models the revenue a school would earn from selling raffle tickets at p dollars each.

1. At what price or prices would the school collect \$0 revenue from raffle sales? Explain or show your reasoning.
2. The school staff noticed that there are two ticket prices that would both result in a revenue of \$500. How would you find out what those two prices are?

Are You Ready For More?

Can you find the following prices without graphing?

1. If the school charges \$10, it will collect \$1,500 in revenue. Find another price that would generate \$1,500 in revenue.
2. If the school charges \$28, it will collect \$1,680 in revenue. Find another price that would generate \$1,680 in revenue.
3. Find the price that would produce the maximum possible revenue. Explain your reasoning.

Step 2

- Invite students to share their responses and strategies, and to refer back to any useful language collected and displayed as they share.
- Make sure students see that the first question can be represented by solving the equation $p(200-5p)=0$, and that the second question can be represented by solving either $p(200-5p)=500$ or $-5p^2+200p=500$.
- Although students have not yet been formally introduced to the zero product property, they do have experience with finding the zeros of a quadratic function when given an expression in factored form. This prior knowledge enables them to reason about the solutions to the equation $p(200-5p)=0$. For instance, noticing the factor p, students are likely to say that one zero of the function is 0.

RESPONSIVE STRATEGIES

Use color and annotations to illustrate student thinking. As students share their reasoning about finding the solutions for which the ticket sales would equal zero, scribe their thinking on a visible display. Consider highlighting each factor separately and annotating to show that when one factor equals zero, the entire expression equals zero regardless of the value of the other factor.

Supports accessibility for: Visual-spatial processing; Conceptual processing

- Ask students:
  - "How can you show, without graphing, that $p=0$ will produce no revenue, or that it is a solution to the equation $p(200-5p)=0$?" (When $p=0$, the factor p is 0, and therefore the entire expression equals 0.)
  - "How can you show that $p=40$ will also produce no revenue, or that it is also a solution to the same equation?" (When $p=40$, the factor $(200-5p)$ is 0, and likewise, the entire expression is 0.)
  - "Can we use the same reasoning to find the solutions to $p(200-5p)=500$ or $-5p^2+200p=500$? Why or why not?" (No, not easily. Neither show the zeros of a function. There are many pairs of factors that have 500 as a product.)
  - "Did anyone find a solution to $p(200-5p)=500$ using a graph?" (There are two approaches to using a graph. First, students could make the equation equal to zero by subtracting 500 from both sides and then finding the x-intercepts. Alternatively, students could graph $p(200-5p)$ and 500 as two different functions and estimate the intersection of the two functions.)

- Make sure students see that it is fairly straightforward to find the solutions to equations such as $p(200-5p)=0$, but the same cannot be said about equations such as $p(200-5p)=500$ or $-5p^2+200p=500$. However, graphing technology does allow us to estimate solutions for equations like this.
- Highlight that all the equations in this activity are quadratic equations. Explain that a quadratic equation is one that can be written in the form of $ax^2+bx+c=0$, where a is not 0.
- If time permits, ask students to show how all of the equations seen here can be written in this form.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to explore the idea of solving quadratic equations—that is, finding inputs that will yield specific outputs. While some strategies from other function types may work, students are presented with limitations when solving quadratics.

Help students to reflect on the key ideas of the lesson by discussing, as a class or with a partner, questions such as:

- "What are some limitations of solving $-2x^2-2x+40=10$ by guessing and checking? What about by graphing?" (Guessing and checking can be time-consuming and may be difficult to find exact solutions. Graphing is a great way to approximate solutions but may not find exact solutions.)
- "Which equation do you think is easier to solve: $-2x^2-2x+40=0$ or $(8-2x)(x+5)=0$? Why?" (The second equation, in factored form, is easier to solve than the first equation, in standard form. When an equation is in factored form and is equal to 0, if either of the factors have a value of 0, then the product of the factors will be 0.)
- "Which is easier to solve: $(8-2x)(x+5)=10$ or $-2x^2-2x+40=10$? Why?" (Both of these are difficult to solve because they are equal to 10. If they were equal to 0, the factored form would be helpful, but we cannot easily tell what values of x would make $(8-2x)(x+5)$ equal to 10.)

PLANNING NOTES

Student Lesson Summary and Glossary

The height of a potato that is launched from a mechanical device can be modeled by a function g. Here are two expressions that are equivalent and both define function g.

$$-16x^2+80x+96$$

$$-16(x-6)(x+1)$$

Notice that one expression is in standard form and the other is in factored form.

Suppose we wish to know, without graphing the function, the time when the potato will hit the ground. We know that the value of the function at that time is 0, so we can write:

$$-16x^2+80x+96=0$$

$$-16(x-6)(x+1)=0$$

Let's try solving $-16x^2+80x+96=0$, using some familiar moves. For example:

- Subtract 96 from each side:

  $$-16x^2+80x=-96$$

- Apply the distributive property to rewrite the expression on the left:

  $$-16(x^2-5x)=-96$$

- Divide both sides by -16:

  $$x^2-5x=6$$

- Apply the distributive property to rewrite the expression on the left:

  $$x(x-5)=6$$

These steps don't seem to get us any closer to a solution. We need some new moves!

What if we use the other equation? Can we find the solutions to $-16(x-6)(x+1)=0$?

Earlier, we learned that the *zeros* of a quadratic function can be identified when the expression defining the function is in factored form. The solutions to $-16(x-6)(x+1)=0$ are the zeros of function g, so this form may be more helpful! We can reason that:

- If x is 6, then the value of $x-6$ is 0, so the entire expression has a value of 0.
- If x is -1, then the value of $x+1$ is 0, so the entire expression also has a value of 0.

This tells us that 6 and -1 are solutions to the equation, and that the potato hits the ground after 6 seconds. (A negative value of time is not meaningful, so we can disregard the -1.)

Both equations we see here are **quadratic equations**. In general, a quadratic equation is an equation that can be expressed as $ax^2+bx+c=0$.

In upcoming lessons, we will learn how to rewrite quadratic equations into forms that make the solutions easy to see.

Quadratic equation: An equation that is equivalent to one of the form ax^2+bx+c, where a, b, and c are constants and $a \neq 0$.

Cool-down: The Movie Theatre (*5 minutes*)

Addressing: NC.M1.A-REI.4

Cool-down Guidance: More Chances
If students do not yet recognize that the factored form helps find the zeros of a quadratic function, carefully plan the introduction and debrief of Activity 2 in Lesson 19 in order to address misconceptions.

Cool-down

A movie theater models the revenue from ticket sales in one day as a function of the ticket price, p. Here are two expressions defining the same revenue function.

$$p(120 - 4p)$$

$$120p - 4p^2$$

1. According to this model, how high would the ticket price have to be for the theater to make $0 in revenue, assuming the theater charges anything at all? Explain your reasoning.
2. What equation can you write to find out what ticket price(s) would allow the theater to make $600 in revenue?

Student Reflection:

Now that you have worked with solving quadratic equations, what is the most important part you want to remember as you continue to work in this unit?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What evidence have students given that they understand the concept of solving a quadratic equation (not procedures for solving)? What language do they use or associate with solving quadratic equations?

Practice Problems

1. Select **all** values of x that are solutions to the equation $(x-5)(7x-21)=0$.

    a. -7
    b. -5
    c. -3
    d. 0
    e. 3
    f. 5
    g. 7

2. The expressions $30x^2-105x-60$ and $(5x-20)(6x+3)$ define the same function, f.

    a. Which expression makes it easier to find $f(0)$? Explain your reasoning.

    b. Find $f(0)$.

    c. Which expression makes it easier to find the values of x that make the equation $f(x)=0$ true? Explain or show your reasoning.

    d. Find the values of x that make $f(x)=0$.

3. A band is traveling to a new city to perform a concert. The revenue from their ticket sales is a function of the ticket price, x, and can be modeled with $(x-6)(250-5x)$.

    What are the ticket prices at which the band would make no money at all?

4. Here are a few pairs of positive numbers whose sums are 15. The pair of numbers that has a sum of 15 and will produce the largest possible product is **not** shown.

    Find this pair of numbers.

First number	Second number	Product
1	14	14
3	12	36
5	10	50
7	8	56

5. Two students built a small rocket from a kit and attached an altimeter (a device for recording altitude or height) to the rocket. They use the table to record the height of the rocket over time since it is launched, based on the data from the altimeter.

Time (seconds)	0	1	3	4	7	8
Height (meters)	0	110.25	236.25	252	110.25	0

    Function h gives the height in meters as a function of time in seconds, t.

    a. What is the value of $h(3)$?

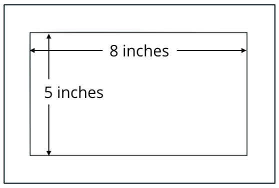
    b. What value of t gives $h(t)=252$?

    c. Explain why $h(0)=h(8)$.

    d. Based on the data, which equation about the function could be true: $h(2)=189$ or $h(189)=2$? Explain your reasoning.

    (From Unit 7, Lesson 16)

6. The screen of a tablet has dimensions 8 inches by 5 inches. The border around the screen has thickness x.



    a. Write an expression for the total area of the tablet, including the frame.

    b. Write an equation for which your expression is equal to 50.3125. Explain what a solution to this equation means in this situation.

    c. Try to find the solution to the equation. If you get stuck, try guessing and checking. It may help to think about tablets that you have seen.

    (From Unit 7, Lesson 16)

7. (*Technology required*.) Two objects are launched upward. Each function gives the distance from the ground in meters as a function of time, t, in seconds.

   Object A: $f(t) = 25 + 20t - 5t^2$ Object B: $g(t) = 30 + 10t - 5t^2$

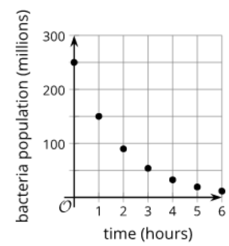
   Use graphing technology to graph each function.

   a. Which object reaches the ground first? Explain how you know.

   b. What is the maximum height of each object?

   (From Unit 7, Lesson 5)

8. The graph shows a bacteria population decreasing exponentially over time.

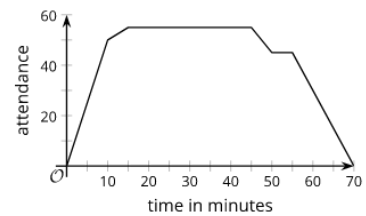


   The equation $p = 100,000,000 \cdot (\frac{2}{3})^h$ gives the size of a second population of bacteria, where h is the number of hours since it was measured at 100 million.

   Which bacterial population decays more quickly? Explain how you know.

   (From Unit 6)

9. The graph shows the attendance at a sports game as a function of time in minutes.



   a. Describe how attendance changed over time.

   b. Describe the domain.

   c. Describe the range.

   (From Unit 5)

10. A set of kitchen containers can be stacked to save space. The height of the stack is given by the expression $1.5c + 7.6$, where c is the number of containers.

    a. Find the height of a stack made of eight containers.

    b. A tower made of all the containers is 40.6 cm tall. How many containers are in the set?

    c. Noah looks at the equation and says, "7.6 must be the height of a single container." Do you agree with Noah? Explain your reasoning.

    (From Unit 2)

11. Use the function $f(x) = -2x + 5$ to answer the questions.

    a. What is the y-intercept of the graph of $y = f(x)$?

    b. What is the x-intercept of the graph of $y = f(x)$?

    c. Solve the equation $0 = -2x + 5$. Which intercept does this reveal? Explain.

    d. Find $f(0)$. What intercept does this reveal? Explain.

    (Building On NC.8.F.4)

Lesson 18: Solving Quadratic Equations by Reasoning

PREPARATION

Lesson Goals	Learning Targets
• Find the solutions to simple quadratic equations and justify (orally) the reasoning that leads to the solutions. • Understand that a quadratic equation may have two solutions.	• I can find solutions to quadratic equations by reasoning about the values that make the equation true. • I know that quadratic equations may have two solutions.

Lesson Narrative

In this lesson, students begin to solve quadratic equations by reasoning about what values would make the equations true and by using structure in the equations. The idea that some quadratic equations have two solutions is also made explicit. Students may begin to record their reasoning process as steps for solving, but this is not critical at this point as it will be emphasized in a later lesson.

As students reason about an equation, they may intuitively perform the same operation on each side of the equal sign to get closer to the solution(s). When they reach equations of the form $x^2 = \textit{some number}$, it is important to focus on reasoning about values that would make the equation true (MP2).

For example, to solve $4x^2 = 100$, they could divide each side by 4 and get $x^2 = 25$. Encourage students to interpret this equation as, "Some number being squared gives 25" and to reason that "there are two different values that can be squared to get 25: -5 and 5."

Reasoning this way helps to curb two common misconceptions:

- Misconception 1: The only solution to an equation such as $x^2 = 25$ is $\sqrt{25}$. Each positive number has two square roots, one positive and the other negative. By convention, the radical symbol $\sqrt{3}$ refers to the positive square root. So the number $\sqrt{25}$ refers only to the positive square root of 25 and does not capture the negative square root.
- Misconception 2: Squaring is invertible. The inverse of an operation undoes that operation. Suppose we multiply a number by 8. Dividing the product by 8 takes us back to the original number, so we say that division by 8 is the inverse operation of multiplication by 8, and that multiplication by 8 is invertible.

  Suppose we square -3, which gives 9. The operation of taking the square root using a radical symbol takes 9 to $\sqrt{9}$, which is positive 3, not the original number. Because there are two possible numbers whose square is 9, we don't consider squaring to be invertible. Avoiding the misconception that squaring is invertible supports future work with inverse functions, specifically quadratic and square root functions, and restricted domains in NC Math 3.

Once students understand the reasoning for why a quadratic equation can have two solutions, and notice that the solutions are related to taking the square root of both sides, they are more likely to remember to understand and use the notation $\pm\sqrt{}$ when expressing the solutions to a quadratic equation.

When solving an equation such as $x^2 = 49$, these notations are commonly used to express the solutions:

- $x = 7$ *or* $x = -7$
- $x = 7$ *and* $x = -7$

The use of “or” is really a shorthand for: "If x is a number such that $x^2 = 49$, then $x = 7$ or $x = -7$.” The use of “and” is a shorthand for: “Both $x = 7$ and $x = -7$ are values that make the equation $x^2 = 49$ true." Either notation can be appropriate, depending on how the question is stated.

Solving the problems in the lesson gives students many opportunities to engage in sense making, perseverance, and abstract reasoning (MP1, MP2).

How is the approach of this lesson similar and different from other ways you have taught these concepts or procedures?

Focus and Coherence

Building On	Addressing
NC.8.EE.2: Use square root and cube root symbols to: • Represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. • Evaluate square roots of perfect squares and cube roots of perfect cubes for positive numbers less than or equal to 400.	**NC.M1.A-REI.4:** Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring. **NC.M1.A-REI.11:** Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, and/or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using graphing technology or successive approximations with a table of values.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
  - Access to Desmos for the Employing Square Roots activity: https://bit.ly/EmployingSqRts
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L18 Cool-down (print 1 copy per student)

LESSON

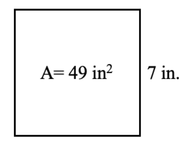
Bridge (*Optional*, *5 minutes*)

Building On: NC.8.EE.2

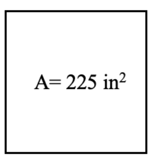
This bridge will link a visual representation of square roots to the algebraic notation, preparing students to solve equations with exponents of 2 in the warm-up and solve quadratic equations by taking square roots in the lesson activities. Students should not be expected to simplify radicals when they represent irrational numbers in this course. The discussion should focus on the relationship between the square root (and notation), squared number, side length, and area.

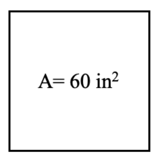
Student Task Statement

Square A has an area of 49 square inches, and each side has a length of 7 inches, because the square root of 49, written $\sqrt{49} = 7$, and $7^2 = 49$.



Use square root notation to write the length of one side of each square below:

1. 

2. 

DO THE MATH

PLANNING NOTES

Warm-up: How Many Solutions? (*5 minutes*)

Addressing: NC.M1.A-REI.4

Previously, students saw that a function could have two input values that give the same output value (which could be 0). The input values were primarily interpreted in terms of a situation. In this warm-up, students begin to think more abstractly about this process—in terms of finding the solutions to an equation. They recognize that some quadratic equations have one solution and others have two.

All of the equations can be solved by reasoning and do not require formal knowledge of algebraic methods such as rewriting into factored form or completing the square. For example, for $x^2 - 1 = 3$, students can reason that x^2 must be 4 because that is the only number that, when subtracted by 1, gives 3.

Finding the solutions of these equations, especially the last few equations, requires perseverance in making sense of problems and of representations (MP1).

Step 1

- Ask students to evaluate $4 \cdot 4$ and $-4 \cdot (-4)$. Make sure they recall that both products are positive 16.

Advancing Student Thinking: When solving $2x^2 = 50$, some students may confuse $2x^2$ with $(2x)^2$ and conclude that the solutions are $\frac{\sqrt{50}}{2}$ and $-\frac{\sqrt{50}}{2}$. Clarify that $2x^2$ means 2 times x^2, and that the only quantity being squared is the x. If both the 2 and x are squared, a pair of parentheses is used to group the 2 and the x so that we know both are being squared.

Student Task Statement

How many solutions does each equation have? What are the solution(s)? Be prepared to explain how you know.

1. $x^2 = 9$
2. $x^2 = 0$
3. $x^2 - 1 = 3$
4. $2x^2 = 50$
5. $(x+1)(x+1) = 0$
6. $x(x-6) = 0$
7. $(x-1)(x-1) = 4$

Step 2

- Ask students to share their responses and reasoning. After each student explains, ask the class if they agree or disagree and discuss any disagreements.
- Make sure students see that in cases such as $x(x-6) = 0$ and $(x-1)(x-1) = 4$, the solutions to each equation may not necessarily be opposites, as was the case in the preceding equations. For example, in the last question, we want to find a number that produces 4 when it is squared. That number can be 2 or -2. If the number is 2, then x is 3. If the number is -2, then x is -1.
- If time permits, discuss questions such as:
  - "The equation $(x+1)(x+1) = 0$ has only one solution, while $(x-1)(x-1) = 4$ has two. Why is that?" (The former has only one solution because the only number that equals 0 when squared is 0 itself. The latter has two solutions because there are two numbers that, when squared, equal 4.)
  - "In an equation like $x(x-6) = 0$, how can we tell that there are two solutions?" (There are two factors here, either of which could make the product 0.)

DO THE MATH	PLANNING NOTES

Activity 1: Finding Pairs of Solutions (*15 minutes*)

Instructional Routine: Compare and Connect (MLR7)	
Addressing: NC.M1.A-REI.4	

In this activity, students encounter quadratic equations that are slightly more elaborate than those in the warm-up but that can still be solved by reasoning in various ways.

Students who use Desmos to solve the equations engage in choosing tools strategically (MP5). Those who solve by analyzing and taking advantage of the composition of the equations practice making use of structure (MP7).

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Give students quiet work time and then time to share their work with a partner.
- Give students access to Desmos, if requested.

RESPONSIVE STRATEGIES

Chunk this task into manageable parts for students who benefit from support with organizational skills in problem solving. For example, cut problems into four separate slips and have students obtain the new problem only on finishing the last. Provide students four copies of a graphic organizer with two sections—one for work, and another for notes/explanations. Invite students to record their work for each problem on its own paper. When presenting multiple strategies and approaches, encourage students to use the space for notes/explanations to record explanations and alternate strategies.

Supports accessibility for: Memory; Organization

M

Monitoring Tip: Students' approaches likely vary in efficiency and effectiveness. Monitor for students who:

- Substitute different values for n until hitting on the ones that work.
- Use Desmos to make a table and look for the target value.
- Use Desmos to graph $y = left\ side\ of\ equation$ and $y = right\ side\ of\ equation$ and find the intersection.
- Reason about and make use of the structure in the equations. For example, seeing $432 = 3n^2$ as "432 is 3 times something squared" will lead to 144 as the "something squared" and 12 and -12 as the "something." Seeing $(n-5)^2 = 100$ as "something squared is 100" enables them to arrive at 10 and -10 for the value of $n-5$, and then reason that n must be 15 or -5.
- Solve algebraically, by performing the same operation to each side of the equation, and when arriving at an equation of the form $n^2 = some\ number$, reasoning that the solutions (the values of n) are the positive and negative square roots of that number.

Identify students who use these strategies and ask them to share during the classroom discussion. Alternatively, consider arranging for students who use the same strategy to discuss and then prepare to share their approach.

Student Task Statement

Each of these equations has two solutions. What are they? Explain or show your reasoning.

1. $n^2 + 4 = 404$
2. $432 = 3n^2$
3. $144 = (n+1)^2$
4. $(n-5)^2 - 30 = 70$

Are You Ready For More?

1. How many solutions does the equation $(x-3)(x+1)(x+5) = 0$ have? What are the solutions?
2. How many solutions does the equation $(x-2)(x-7)(x-2) = 0$ have? What are the solutions?
3. Write a new equation that has 10 solutions.

Step 2

- Use the *Compare and Connect* routine to prepare students for the whole-class discussion.
  - At the appropriate time, invite student pairs to create a visual display of their strategies for solving the quadratic equations, with enough detail for other students to follow their reasoning.
  - Allow students time to quietly circulate and analyze the strategies in at least two other visual displays in the room to consider what is the same and what is different about their solution strategies.
  - Give students a short time to return to their partner and discuss what they noticed.

- Select previously identified students to present their strategies in order of their efficiency, as listed in the Monitoring Tip.
  - Where appropriate, help students to make connections between the different strategies. For example, ask students how graphing the expression on each side of the equation and finding the intersection is similar to substituting values for n until they find one that works.
  - As students discuss, listen for and amplify observations that highlight advantages and disadvantages to each method. This will help students make connections between different strategies for solving quadratic equations.
  - If no students reasoned about the solutions algebraically, be sure to demonstrate these methods and to record the reasoning process for all to see. For example:
    - We can interpret $n^2+4=404$ as "something plus 4 is 404." That "something" must be 400, so we can write $n^2=400$. This equation means "something times itself is 400." That "something" must be 20 or -20 (the positive and negative square roots of 400), because they each give 400 when squared. The two solutions are therefore 20 and -20.
    - Point out that the reasoning that took us from $n^2+4=404$ to $n^2=400$ gave the same equation as subtracting 4 from each side of the original equation.
    - We can see $144=(n+1)^2$ as "144 is something squared," so the "something" is either 12 or -12 (the positive and negative square roots of 144). We can represent this with $n+1=12$ and $n+1=-12$. The solutions are 11 and -13.

DO THE MATH	PLANNING NOTES

Activity 2: Employing Square Roots (*10 minutes*)

Instructional Routine: Critique, Correct, Clarify (MLR3)	
Addressing: NC.M1.A-REI.4; NC.M1.A-REI.11	

In this activity, students narrow their focus on the algebraic approach of using square roots to solve quadratic equations. Students practice identifying the steps to arrive at solutions of given quadratic equations through a virtual activity using Desmos (MP7), which is then followed by analyzing and correcting a student's incorrect work (MP3). Additionally, students encounter two situations in which the solutions of a quadratic equation are irrational numbers.

Step 1

- Prior to class, go to https://bit.ly/EmployingSqRts to access this Desmos activity. You will need to be logged in to your Desmos account.
- Give students access to this activity by clicking the arrow on the side of the "Assign" button and choose either "Assign to Your Classes" or "Single Session Code."
  - A class must be created, and students added to it, in a teacher's Desmos account in order to use the "Assign to Your Classes" option. Students will see a "Start" button next to the activity title when logged in on the student.desmos.com page.
  - In order to do this activity without creating a class in Desmos, a "Single Session Code" can be generated to give to students. Instruct students to go to student.desmos.com and enter the single session code.

Provide continued access to Desmos for simple calculations in a separate tab, if requested.

Step 2

- Monitor student progress by clicking "View Dashboard" underneath Activity Sessions on the Desmos Activity Builder page. From this dashboard, student pacing can be adjusted, the activity can be paused for students, and student names can be anonymized.
- Provide feedback to individual students by clicking the chat icon at the top of the student work window of a particular slide.

Advancing Student Thinking: This is the first time that students have solved quadratic equations with irrational solutions. Students may have questions about notation like $x + 5 = -\sqrt{75}$. For example, they may be unsure about
- what $-\sqrt{75}$ means (take the square root of 75, then make it negative),
- how to isolate x (just like solving any equation of this form, we subtract 5 from each to isolate the variable, but to avoid confusion about what goes under the square root sign we write the -5before the $-\sqrt{75}$), and
- whether an expression like $x = -5 - \sqrt{75}$ can be simplified (you can put it into your calculator to express it as a decimal, but there is no way to simplify the expression *exactly* because an integer minus an irrational number can only be another irrational number).

Student Task Statement

Follow your teacher's directions to access the Desmos activity. Use the available space below to show your work.

Step 3

- On slides 4 and 5 of the Desmos activity, use the *Critique, Correct, Clarify* routine to help students justify why the given quadratic equation must have two solutions, even when taking a square root to solve the equation.
- On slide 4, students will have an opportunity to pinpoint Lin's mistake and explain why it is a mistake. Once students submit their reasoning, they will see a sample of their classmates' reasoning. Because students are unable to see all classmates' responses, consider pausing the activity and selecting students who submitted either of the following reasoning approaches to share with the whole class.
  - Use Desmos to graph $y = left\ side\ of\ equation$ and $y = right\ side\ of\ equation$ and see there are two points of intersection.
  - Reason about and make use of the structure in the equations. Specifically, $(x+3)^2 = 8$ can be described as "something squared is 8," enabling them to arrive at $\sqrt{8}$ and $-\sqrt{8}$ for the value of $x+3$.
- Unpause the activity to allow students to proceed with correcting Lin's work on slide 5.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to have students brainstorm different strategies that could be used to solve quadratic equations and to begin to use square roots to facilitate solving quadratic equations algebraically.

<table>
<tr><td>
Choose whether students should first have an opportunity to reflect in their workbooks or talk through these questions with a partner.

To help students generalize their reasoning and attend more closely to the structure of quadratic equations, consider revisiting the equations from the lesson and prompting students to articulate what the equations might reveal about the solutions. For example:

• Display the equations from the warm-up. Ask students if they can tell (without referring to the solutions they found earlier and without solving again) if an equation would have 0, 1, or 2 solutions.

• Display the equations from the first activity. Ask students if they can tell (without referring to the solutions they found earlier) which equations will have two solutions that are opposites (such as 5 and -5) and which will have two solutions that are not opposites (such as 3 and 7).

• Display the equation $(x+5)^2 = 75$ from the Desmos activity. Ask students how they can see without solving that the equation will have two solutions.
</td><td>PLANNING NOTES</td></tr>
</table>

Student Lesson Summary and Glossary

Some quadratic equations can be solved by performing the same operations to each side of the equal sign and reasoning about values of the variable would make the equation true.

Suppose we wanted to solve $7x^2 = 112$. We can proceed like this:

- Divide each side by 7: $x^2 = 16$
- What number can be squared to get 16? $?^2 = 16$
- Using the fact that $\sqrt{16} = 4$, we know that there are two numbers that when squared result in 16, 4 and -4: $4^2 = 16$ and $(-4)^2 = 16$
- Therefore, $x = 4$ and $x = -4$

Suppose we wanted to solve $3(x+1)^2 - 75 = 0$. We can proceed like this:

- Add 75 to each side: $3(x+1)^2 = 75$
- Divide each side by 3: $(x+1)^2 = 25$
- What number can be squared to get 25? $?^2 = 25$
- Using the fact that $\sqrt{25} = 5$, we know that there are two numbers that when squared result in 25, 5 and -5: $5^2 = 25$ and $(-5)^2 = 25$
- If $x + 1 = 5$, then $x = 4$.
- If $x + 1 = -5$, then $x = -6$.

This means that both $x = 4$ and $x = -6$ make the equation true and are solutions to the equation.

Cool-down: Find Both Solutions (*5 minutes*)

Addressing: NC.M1.A-REI.4

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Provide continued access to Desmos, if requested.

Cool-down

Find both solutions to the equation $100 + (n-2)^2 = 149$. Explain or show your reasoning.

Student Reflection:

I feel like my math abilities have:

a. Increased greatly this year
b. Increased a bit this year
c. Stayed the same this year

My joy for mathematics has:

a. Increased this year
b. Stayed the same this year
c. Decreased this year

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What connections did students make between the different strategies shared? What questions did you ask to help make the connections more visible?

Practice Problems

1. Consider the equation $x^2 = 9$.
   a. Show that 3 and -3 are each a solution to the equation.
   b. Show that 9 and $\sqrt{3}$ are each **not** a solution to the equation.
2. Solve $(x-1)^2 = 16$. Explain or show your reasoning.
3. Here is one way to solve the equation $\frac{5}{9}y^2 = 5$. Explain what is done in each step.

$$\begin{array}{ll} \frac{5}{9}y^2 = 5 & \textit{Original equation} \\ 5y^2 = 45 & \textit{Step 1} \\ y^2 = 9 & \textit{Step 2} \\ y = 3 \textit{ or } y = -3 & \textit{Step 3} \end{array}$$

4. Diego and Jada are working together to solve the quadratic equation $(x-2)^2 = 100$.

   Diego solves the equation by dividing each side of the equation by 2 and then adding 2 to each side. He writes:

   $(x-2) = 50$
   $x = 52$

   Jada asks Diego why he divides each side by 2 and he says, “I want to find a number that equals 100 when multiplied by itself. That number is half of 100.”

   a. What mistake is Diego making?
   b. If you were Jada, what could you say to Diego to help him realize his mistake?
5. As part of a publicity stunt (an event designed to draw attention), a TV host drops a watermelon from the top of a tall building. The height of the watermelon t seconds after it is dropped is given by the function $h(t) = 850 - 16t^2$, where h is in feet.
   a. Find $h(4)$. Explain what this value means in this situation.
   b. Find $h(0)$. What does this value tell us about the situation?
   c. Is the watermelon still in the air 8 seconds after it is dropped? Explain how you know.

   (From Unit 7, Lesson 16)

6. Add or subtract:

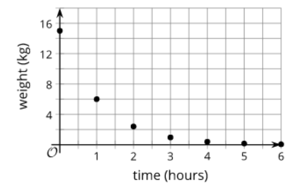
    a. $(-16t^2 - 32t + 14) + (t^2 + 14t - 32)$

    b. $(-5x - 6) - (-0.2x^2 - 4x + 11)$

    c. $(25x^2 - 16) + (16x^2 - 9)$

    (From Unit 7, Lessons 14 and 15)

7. The graph shows the weight of snow as it melts. The weight decreases exponentially.



    a. By what factor does the weight of the snow decrease each hour? Explain how you know.

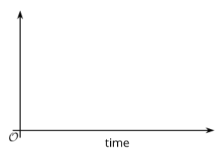
    b. Does the graph predict that the weight of the snow will reach 0? Explain your reasoning.

    c. Will the weight of the actual snow, represented by the graph, reach 0? Explain how you know.

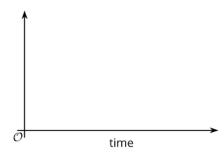
    (From Unit 6)

8. Sketch a graph to represent each quantity described as a function of time. Be sure to label the vertical axis.

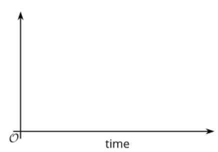
    a. Swing: The height of your feet above the ground while swinging on a swing on a swing at a playground



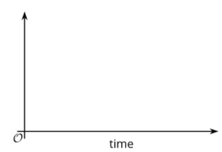
    b. Slide: The height of your shoes above ground as you walk to a slide, go up a ladder, and then go down a slide



    c. Merry-go-round: Your distance from the center of a merry-go round as you ride the merry-go-round



    d. Merry-go-round, again: Your distance from your friend, who is standing next to the merry-go-round as you go around



    (From Unit 5)

9. A zoo offers unlimited drink refills to visitors who purchase its souvenir cup. The cup and the first fill cost \$10, and refills after that are \$2 each. The expression $10 + 2r$ represents the total cost of the cup and r refills.

    a. A family visited the zoo several times over a summer. That summer, they paid \$30 for one cup and multiple refills. How many refills did they buy?

    b. A visitor has \$18 to spend on drinks at the zoo today and buys a souvenir cup. How many refills can they afford during the visit?

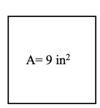
    c. Another visitor spent \$10 on this deal. Did they buy any refills? Explain how you know.

    (From Unit 2)

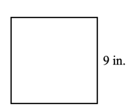
10. Solve the equation: $\frac{2}{3}(12x - 30) = 5x + 2$

    (From Unit 2)

11.



    a. What is the length of one side of the square?



    b. What is the area of the square?

    c. Explain how square roots and powers of 2 helped you figure out the side length and area.

    (Addressing NC.8.EE.2)

Lesson 19: Solving Quadratic Equations with the Zero Product Property

PREPARATION

Lesson Goals	Learning Targets
• Given quadratic equations where one side is a product of factors and the other is zero, find the solution(s) and explain (orally and in writing) why the solutions make the equation true. • Understand that the “zero product property” (in written and spoken language) means that if the product of two numbers is 0 then one of the factors must also be 0.	• I can find solutions to quadratic equations when one side is a product of factors and the other side is zero. • I can explain the meaning of the “zero product property.”

Lesson Narrative

In this lesson, students learn about the **zero product property**. They use it to reason about solutions to quadratic equations that have a quadratic expression in factored form on one side and 0 on the other side. They see that when an expression is a product of two or more factors and that product is 0, one of the factors must be 0. This fact enables us to find unknown values in the factored expression.

Students also continue to make connections to their earlier work on quadratic functions. They have seen that sometimes we want to find the input values of a function when the output is zero. They also learned that the factored form can help us identify the zeros of a quadratic function and the x-intercepts of its graph. They have not investigated how or why this form enables us to do so, however. Here, students make use of the structure of a quadratic expression in factored form and the zero product property to understand the connections between the numbers in the form and the x-intercepts of its graph (MP7).

What math language will you want to support your students with in this lesson? How will you do that?

Focus and Coherence

Building On	Addressing	Building Towards
NC.M1.A-REI.3: Solve linear equations and inequalities in one variable.	**NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems. **NC.M1.A-REI.1:** Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning. **NC.M1.A-REI.4:** Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.	**NC.M1.A-SSE.3:** Write an equivalent form of a quadratic expression $ax^2 + bx + c$, where a is an integer, by factoring to reveal the solutions of the equation or the zeros of the function the expression defines.

Agenda, Materials, and Preparation

- **Warm-up** (*10 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
  - Technology is required for this activity: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L19 Cool-down (print 1 copy per student)

LESSON

Warm-up: Solve These Equations (*10 minutes*)

Instructional Routines: Math Talk; Discussion Supports (MLR8)	
Building Towards: NC.M1.A-REI.1	

This *Math Talk* introduces students to the zero product property and prepares them to use it to solve quadratic equations. It reminds students that if two numbers are multiplied and the result is 0, then one of the numbers has to be 0. Answering the questions mentally prompts students to notice and make use of structure (MP7).

- Display one problem at a time and ask students to respond without writing anything down.
- Give students quiet think time for each problem and ask them to give a signal when they have an answer and a strategy.
- Keep all problems displayed throughout the talk.
- Ask students to share their strategies for each problem.
  - Record and display their responses for all to see.
  - Use *Discussion Supports* to involve more students in the conversation. Display sentence frames to support students when they explain their strategy. For example, “First, I _____ because . . .” or “I noticed _____ so I . . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
  - Consider asking:
    - “Who can restate ___’s reasoning in a different way?”
    - “Did anyone have the same strategy but would explain it differently?”
    - “Did anyone solve the problem in a different way?”
    - “Does anyone want to add on to ___’s strategy?”
    - “Do you agree or disagree? Why?”
- Highlight explanations that state that any number multiplied by 0 is 0. Then, introduce the zero product property, which states that if the product of two numbers is 0, then at least one of the numbers is 0.

RESPONSIVE STRATEGY

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

What values of the variables make each equation true?

1. $6 + 2a = 0$
2. $7b = 0$
3. $7(c - 5) = 0$
4. $g \cdot h = 0$

DO THE MATH

PLANNING NOTES

Activity 1: Take the Zero Product Property Out for a Spin (*15 minutes*)

Building On: NC.M1.A-REI.3	**Addressing:** NC.M1.A-REI.1; NC.M1.A-REI.4

In this activity, students solve equations of increasing complexity and do so by reasoning. They begin with linear equations, and move toward a series of quadratic expressions in factored form. The progression prompts students to reason about the parts and structure of the expressions (MP7), rather than to memorize steps for solving without understanding, and to notice regularity through repeated reasoning (MP8).

Step 1

- Ask students to arrange in pairs or use visibly random grouping.
- Tell students to work quietly and answer at least half of the questions before discussing their thinking with a partner.
- If needed, remind students that some equations have more than one solution. In this activity, the goal is for students to use reasoning and the structure of the equations to develop their solutions, rather than relying on Desmos.

RESPONSIVE STRATEGIES

To support development of organizational skills, check in with students within the first 2–3 minutes of work time, especially when students arrive at problems with two solutions. Look for students who quickly recognize that they will need to solve two equations and are organizing their work to account for the appropriate number of solutions for each problem. To support students in recognizing examples with two solutions, demonstrate annotating the problem by writing two equations separately, or drawing arrows from each factor to indicate two solutions. Encourage students to prepare their explanations by recording how they recognized the number of solutions as they work.

Supports accessibility for: Memory; Organization

Monitoring Tip: As students discuss their reasoning with their partner, listen for those who describe the zero product property, either by name or conceptually, to explain how the last three equations could be solved, as well as those who notice a pattern in how the equations could be solved. Let them know that they may be asked to share later.

Advancing Student Thinking: Students may incorrectly think that x can represent a different value in each factor in an equation. For example, upon finding -11 and 3 as solutions to $(x-3)(x+11)=0$, they think that one solution is for the x in $(x-3)$ and the other for the x in $(x-11)$.

Remind students that solving the equation $(x-3)(x+11)=0$ is like finding the zeros of the function defined by $(x-3)(x-11)$. Although there may be two values of x that lead to 0 for the value of $(x-3)(x-11)$, only one input can be entered into the function at a time. Ask students to substitute the solutions into the equations and check if the expression is equal to 0 each time.

- When $x=-11$, the value of the expression is $(-11-3)(-11+11)$ or $(-14)(0)$, which is 0.
- When $x=3$, the value of the expression is $(3-3)(3+11)$ or $(0)(14)$, which is 0.

Student Task Statement

For each equation, find its solution or solutions. Be prepared to explain your reasoning.

1. $x-3=0$
2. $x+11=0$
3. $2x+11=0$
4. $x(2x+11)=0$
5. $(x-3)(x+11)=0$
6. $(x-3)(2x+11)=0$

Are You Ready For More?

1. Use factors of 48 to find as many whole-number solutions as you can to the equation $(x-3)(x+5)=48$.
2. Once you think you have all the solutions, explain why these must be the only solutions.

Step 2

- Invite students to share their strategies for solving the non-linear equations. As they explain, record and organize each step of their reasoning process and display for all to see.
- For example, the equation $(x-3)(2x+11)=0$ tells us that, if the product of $(x-3)$ and $(2x+11)$ is 0, then either $x-3$ is equal to 0, or $2x+11$ is equal to 0. We can then organize the rest of the solving process as:

  If $x-3$ is equal to 0, then x is 3.
  $x-3=0$
  $x=3$

  If $2x+11$ is equal to 0, then $x=-\frac{11}{2}$
  $2x+11=0$
  $2x=-11$
  $x=-\frac{11}{2}$

  The equation is true when $x=3$ and when $x=-\frac{11}{2}$.

- Emphasize that because at least one of the factors must be 0 for the product to be 0, we can set each expression that is a factor equal to 0 and solve each of these equations separately.
- Remind students that we can check our solutions by substituting each one back into the equation and see if the equation remains true. Although the two factors, $(x-3)$ and $(2x-11)$, won't be 0 simultaneously when 3 or $-\frac{11}{2}$ is substituted for x, the expression on the left side of the equation will have a value of 0 because one of the factors is 0.
  - When x is 3, the expression is $(3-3)(2(3)+11)$ or $(0)(66)$, which is 0.
  - When x is $-\frac{11}{2}$, the expression is $(-\frac{11}{2}-3)(2(-\frac{11}{2})+11)$ or $(-\frac{17}{2})(0)$, which is 0.

DO THE MATH	PLANNING NOTES

Activity 2: Revisiting a Projectile (*10 minutes*)

Instructional Routine: Discussion Supports (MLR8)	
Addressing: NC.M1.A-CED.1; NC.M1.A-REI.4	**Building Towards:** NC.M1.A-SSE.3

This activity enables students to apply the zero product property to solve a contextual problem and reinforces the idea of solving quadratic equations as a way to reason about quadratic functions.

Previously, students have encountered two equivalent quadratic expressions that define the same quadratic function. Here, they work to show that two quadratic expressions—one in standard form and the other in factored form—really do define the same function. There are several ways to do this, but an efficient and definitive way to show equivalence would be to use the distributive property to expand quadratic expressions in factored form.

Next, they consider which of the two forms helps them find the zeros of the function and then use it to find the zeros without graphing. The work here reiterates the connections between finding the zeros of a quadratic function and solving a quadratic equation where a quadratic expression that defines a function has a value of 0. Prepare access to Desmos, in case it's requested.

Step 1

- Have students remain in groups from the previous activity.
- Display the two equations that define h for all to see.
  - Tell students that the two equations define the same function.
  - Ask students how they could show that the two equations indeed define the same function.
- Give students a moment of quiet time to think of a strategy and test it, and then time to discuss with a partner, if possible. Then, discuss their responses. Some likely strategies:
  - Graph both equations on the same coordinate plane and show that they coincide.
  - Inspect a table of values of both equations and show that the same output results for any input.
  - Use the distributive property to multiply the expression in factored form to show that $(-5t-3)(t-6)=-5t^2+27t+18$. (Only this reasoning is really a "proof," but the other methods supply a lot of evidence that they are the same function.)
- Once students see some evidence, ask students to proceed to the activity.

RESPONSIVE STRATEGIES

Represent the same information through different modalities. Display a sketch of a graph of a projectile, and label the y-intercept, vertex, and positive x-intercepts. Keep the display visible for the duration of the activity and refer to it when students discuss why the negative solution is not viable.

Supports accessibility for: Conceptual processing; Language

Student Task Statement

We have seen quadratic functions modeling the height of a projectile as a function of time.

Here are two ways to define the same function that approximates the height of a projectile in meters, t seconds after launch:

$h(t) = -5t^2 + 27t + 18$ $\qquad$ $h(t) = (-5t - 3)(t - 6)$

1. Which way of defining the function allows us to use the zero product property to find out when the height of the object is 0 meters?
2. Without graphing, determine at what time the height of the object is 0 meters. Show your reasoning.

Step 2

- Facilitate a whole-class discussion by asking students to share their responses and reasoning and to discuss questions such as:
  - "Why is the factored form more helpful for finding the time when the object has a height of 0 meters?" (To find the input values when the output has a value of 0 is to solve the equation $\textit{quadratic expression} = 0$. When the expression is in factored form, we can use the zero product property to find the unknown inputs.)
  - "What if we tried to solve the equation in standard form by performing the same operation to each side?" (We would get stuck. For instance, we could add or subtract terms from each side, but then there are no like terms to combine on either side, so we are no closer to isolating the variable.)
  - Use *Discussion Supports* to support whole-class discussion.
    - After each student shares, provide the class with the following sentence frames to help them respond: "I agree because . . ." or "I disagree because . . . ."
    - If necessary, revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students. For example, a statement such as, "The first one is more helpful" can be restated as a question such as, "Do you agree that the factored form is more helpful for finding when the object has a height of 0 meters?"
  - If no students related solving equations in factored form to using the factored form to find the horizontal intercepts of a graph of a quadratic function, discuss that connection:
    - "In an earlier lesson, we saw that the factored form of a quadratic expression such as $(x-5)(x+9)$ allows us to see the x-intercepts of its graph. Can you explain why it does?" (The x-intercepts have a y-value of 0, which means the quadratic function is 0 at those x-values: $(x-5)(x+9)=0$. If multiplying two numbers gives 0, one of them must be 0. So either $x-5=0$ or $x+9=0$. If $x-5=0$, then x is 5. If $x+9=0$, then x is -9.)

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to introduce the zero product property and to show students that the factored form of a quadratic equation not only is connected to the zeros on the graph of the related quadratic function, but also can also be used to solve quadratic equations in which one side is a quadratic expression in factored form and the other side is 0.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

To help students consolidate the ideas in the lesson, discuss questions such as:

- "How does the zero product property help us find the solutions to $(x-3)(x+4)=0$?" (It tells us that either $x-3=0$ or $x+4=0$," and each of these equations can be solved easily.)
- "Can you explain why the solutions to $(x-3)(x+4)=8$ are *not* 3 and -4?" (The zero product property only works when the product of the factors is zero. When the product is any other number, we can't conclude that each factor is that number.)
- "The expression x^2-x-12 is equivalent to $(x+3)(x-4)$. Can we apply the zero product property to solve $x^2-x-12=0$?" (Only if we rewrite the expression on the left in factored form first. We can't use the zero product property when the expression is not a product of factors.)
- "Can we solve $x^2-x-12=0$ by performing the same operations to each side of the equation?" (No, doing that doesn't help us isolate the variable.)

PLANNING NOTES

Student Lesson Summary and Glossary

The **zero product property** says that if the product of two numbers is 0, then one of the numbers must be 0. In other words, if $a \cdot b=0$, then either $a=0$ or $b=0$. This property is handy when an equation we want to solve states that the product of two factors is 0.

Suppose we want to solve $m(m+9)=0$. This equation says that the product of m and $(m+9)$ is 0. For this to be true, either $m=0$ or $m+9=0$, so both 0 and -9 are solutions.

Here is another equation: $(u-2.345)(14u+2)=0$. The equation says the product of $(u-2.345)$ and $(14u+2)$ is 0, so we can use the zero product property to help us find the values of u. For the equation to be true, one of the factors must be 0.

- For $u-2.345=0$ to be true, u would have to be 2.345.
- For $14u+2=0$ or $14u=-2$ to be true, u would have to be $-\frac{2}{14}$ or $-\frac{1}{7}$.

The solutions are 2.345 and $-\frac{1}{7}$.

In general, when a quadratic expression in factored form is on one side of an equation and 0 is on the other side, we can use the zero product property to find its solutions.

Zero product property: If the product of two numbers is 0, then one of the numbers must be 0.

Cool-down: Solve This Equation! (*5 minutes*)

Addressing: NC.M1.A-REI.4

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Encourage students to complete this cool-down without technology.

Cool-down

Find all solutions to $(x+5)(2x-3)=0$. Explain or show your reasoning.

Student Reflection:

In what ways does your math teacher make you feel supported in class? Is there anything you would change or add?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

In upcoming lessons, students will solve quadratic equations by graphing and will begin to strategize about which approach (reasoning, taking square roots, factoring, and graphing) is most efficient in each case. Which understandings are necessary for students to have in order to weigh different approaches to solving quadratic equations?

Practice Problems

1. If the equation $(x+10)x=0$ is true, which statement is also true according to the zero product property?

   a. only $x=0$
   b. either $x=0$ or $x+10=0$
   c. either $x^2=0$ or $10x=0$
   d. only $x+10=0$

2. What are the solutions to the equation $(10-x)(3x-9)=0$?

   a. -10 and 3
   b. -10 and 9
   c. 10 and 3
   d. 10 and 9

3. Solve each equation.

   a. $(x-6)(x+5)=0$

   b. $(x-3)(\frac{2}{3}x-6)=0$

   c. $(-3x-15)(x+7)=0$

4. Consider the expressions $(x-4)(3x-6)$ and $3x^2-18x+24$.

   Show that the two expressions define the same function.

5. Kiran saw that if the equation $(x+2)(x-4)=0$ is true, then, by the zero product property, either $x+2$ is 0 or $x-4$ is 0. He then reasoned that, if $(x+2)(x-4)=72$ is true, then either $x+2$ is equal to 72 or $x-4$ is equal to 72.

   Explain why Kiran's conclusion is incorrect.

6. Andre wants to solve the equation $5x^2-4x-18=20$. He uses a graphing calculator to graph $y=5x^2-4x-18$ and $y=20$ and finds that the graphs cross at the points $(-2.39, 20)$ and $(3.19, 20)$.

   a. Substitute each x-value Andre found into the expression $5x^2-4x-18$. Then evaluate the expression.

   b. Why did neither solution make $5x^2-4x-18$ equal exactly 20?

   (From Unit 7, Lesson 17)

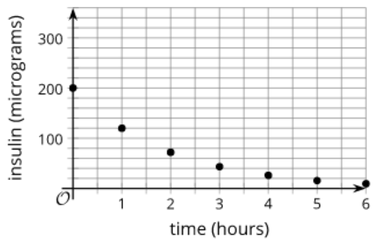
7. Select **all** the solutions to the equation $7x^2=343$.

   a. 49
   b. $-\sqrt{7}$
   c. 7
   d. -7
   e. $\sqrt{49}$
   f. $\sqrt{-49}$
   g. $-\sqrt{49}$

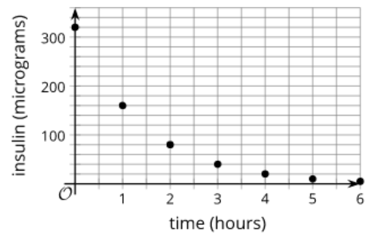
   (From Unit 7, Lesson 18)

8. Here are two graphs that correspond to two patients, A and B. Each graph shows the amount of insulin, in micrograms (mcg) in a patient's body h hours after receiving an injection. The amount of insulin in each patient decreases exponentially.

Patient A



Patient B



Select **all** statements that are true about the insulin level of the two patients.

a. After the injection, the patients have the same amount of insulin in their bodies.

b. An equation for the micrograms of insulin, a, in Patient A's body h hours after the injection is $a = 200 \cdot (\frac{3}{5})^h$.

c. The insulin in Patient A is decaying at a faster rate than in Patient B.

d. After 3 hours, Patient A has more insulin in their body than Patient B.

e. At some time between 2 and 3 hours, the patients have the same insulin level.

(From Unit 6)

9. Scientists are trying to invent a new kind of milk container that will help the environment by decaying faster in landfills. One sample material decays at a rate represented by the function $w(m) = 2.3 \cdot 0.87^m$, where $w(m)$ represents the remaining weight of the milk container in ounces and m represents the number of months since the carton was manufactured.

a. What percentage of the container decays each month?

b. What was the initial weight of the milk container?

(From Unit 5)

10.

a. Write the equation of a line parallel to the line $x = 6$ through point $(-2, -3)$.

b. Write the equation of a line perpendicular to the line $x = 6$ through point $(-2, -3)$.

(From Unit 3)

Lesson 20: How Many Solutions?

PREPARATION

Lesson Goals	Learning Targets
• Make connections (orally) among graphs with no horizontal intercepts, quadratic functions with no (real) zeros, and quadratic equations with no (real) solutions. • Describe (orally and in writing) the relationship between the solutions to quadratic equations of the form $expression = 0$ and the horizontal intercepts of the graph of the related function. • Explain (orally and in writing) why dividing each side of a quadratic equation by a variable is not a reliable way to solve the equation.	• I know that quadratic equations can have no solutions and can explain why there are none. • I can describe the relationship between the solutions to quadratic equations and the graph of the related function. • I can explain why dividing by a variable to solve a quadratic equation is not a good strategy.

Lesson Narrative

The work in this lesson builds on the idea that both graphing and rewriting quadratic equations in the form of $expression = 0$ are useful strategies for solving equations. It also reinforces the ties between the zeros of a function and the horizontal intercepts of its graph, which students began exploring in an earlier unit.

Previously, to solve an equation such as $-16t^2 + 8t + 10 = 32$ by graphing, students would graph $y = -16t^2 + 8t + 10$ and $y = 32$ and inspect where the parabolic graph and the horizontal line intersect.

Here, students learn another way to use graphs to solve equations and to anticipate the number of solutions. Instead of graphing two separate equations—one quadratic and one linear—students learn that they can solve by rearranging the equation into the form $expression = 0$, graphing the equation $y = expression$, and finding the horizontal intercepts. Why does this make sense?

- The x-coordinate of those intercepts produces a y-coordinate of 0, so they are the solutions to the equation $expression = 0$.
- The number of horizontal intercepts tells us the instances when the y-coordinate is 0, which tells us the number of solutions to the equation.

Later in the lesson, students think about why a quadratic equation that has an expression in factored form on one side of the equal sign but does not have 0 on the other side cannot be solved the same way as when the equation is $expression = 0$. They also notice that dividing each side of a quadratic equation by a variable is not reliable because it eliminates one of the solutions. As they explain why certain maneuvers are acceptable and others are not, students practice constructing logical arguments (MP3).

Which Standards for Mathematical Practice do you anticipate students engaging in during this lesson? How will you support them?

Focus and Coherence

Building On	Addressing
NC.6.EE.5: Use substitution to determine whether a given number in a specified set makes an equation true. **NC.8.EE.2:** Use square root and cube root symbols to: ● Represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. ● Evaluate square roots of perfect squares and cube roots of perfect cubes for positive numbers less than or equal to 400.	**NC.M1.A-APR.3:** Understand the relationships among the factors of a quadratic expression, the solutions of a quadratic equation, and the zeros of a quadratic function. **NC.M1.A-REI.1:** Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning. **NC.M1.A-REI.4:** Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring. **NC.M1.A-REI.10:** Understand that the graph of a two-variable equation represents the set of all solutions to the equation.

Agenda, Materials, and Preparation

Technology is required for Activity 1 and Activity 2: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*Optional, 15 minutes*)
- **Activity 3** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L20 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.EE.2

This bridge will begin to reinforce the difference between the inverse operations associated with multiplication and exponents. While discussing the solutions, discuss the student ideas about question 3 so students can state the difference between multiplying a variable by a coefficient and raising a variable to a power.

Student Task Statement

Figure out what value of x makes each equation true, and explain how you figured out each value.

1. $3x = 27$
2. $x^3 = 27$
3. $3x = 24$
4. $x^3 = 24$
5. Why do you need different operations to determine each value?

DO THE MATH	PLANNING NOTES

Warm-up: Four Equations (*10 minutes*)

Instructional Routines: Math Talk; Discussion Supports (MLR8)	
Building On: NC.6.EE.5	**Building Towards:** NC.M1.A-REI.4

This warm-up reminds students of two facts: that in order to use the zero product property, the product of the factors must be 0, and that there is no number that can be squared to get a negative number. At this point, students don't yet know about complex numbers or that squaring a complex number produces a negative number. They will learn this in NC Math 2. Consequently, for now we can just say that there is no real number that can be squared to get a negative number.

During this *Math Talk*, students practice constructing logical arguments (MP3) as they explain why they think each statement is true or false.

Step 1

- Display one problem at a time. Give students quiet think time for each problem and ask them to give a signal when they have an answer and an explanation.
- Use the *Discussion Supports* of sentence frames as students explain their strategy. For example, "I remembered ______ , so I . . ." or "I recognized ______ so I thought. . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
- Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

RESPONSIVE STRATEGY

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Decide whether each statement is true or false. Explain your reasoning.

Statement	True or False?	Explanation
1. 3 is the only solution to $x^2 - 9 = 0$		
2. A solution to $x^2 + 25 = 0$ is -5		
3. $x(x-7) = 0$ has two solutions		
4. 5 and -7 are the solutions to $(x-5)(x+7) = 12$		

Step 2

- Ask students to share their response and explanation for each problem. Record and display their responses for all to see. After each explanation, give the class a chance to agree or disagree. To involve more students in the conversation, consider asking:
  - "Who can restate ___'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone solve the problem in a different way?"
  - "Does anyone want to add on to ___'s strategy?"
  - "Do you agree or disagree? Why?"
- Make sure students understand the rationale that makes each statement true or false, as shown in the student response.
- For the statement "a solution to $x^2 + 25 = 0$ is -5," students may explain why -5 does not satisfy the given equation. They then may speculate about other values of x, such as 5. After attempting to find another value for x that would satisfy the equation, they may conclude that there is no solution. Share with students that there are no real number solutions that will satisfy the equation.

DO THE MATH	PLANNING NOTES

Activity 1: Solving by Graphing (*10 minutes*)

Instructional Routine: Graph It	
Building On: NC.M1.A-REI.1	**Addressing:** NC.M1.A-APR.3; NC.M1.A-REI.10

By now, students recognize that when a quadratic equation is in the form of $expression = 0$ and the expression is in factored form, the equation can be solved using the zero product property. In this activity, they encounter equations in which one side of the equal sign is not 0. To make one side equal 0 requires rearrangement. For example, to solve $x(x+6) = 8$, the equation needs to be rearranged to $x(x+6) - 8 = 0$. Yet because the expression on the left is no longer in factored form, the zero product property won't help after all, and another strategy is needed.

In this *Graph It* activity, students recall that to solve a quadratic equation in the form of $expression = 0$ is essentially to find the zeros of a quadratic function defined by that expression, and that the zeros of a function correspond to the horizontal intercepts of its graph. In the case of $x(x+6) - 8 = 0$, the function whose zeros we want to find is defined by $x(x+6) - 8$. Graphing $y = x(x+6) - 8$ and examining the x-intercepts of the graph allow us to see the number of solutions and the values of those solutions.

Step 1

- Ask students to arrange in pairs or use visibly random grouping.
- Provide access to Desmos.
- Give students a moment to think quietly about the first question and then ask them to briefly discuss their response with their partner before continuing with the rest of the activity collaboratively.

RESPONSIVE STRATEGY

Support effective and efficient use of tools and assistive technologies. Some students may benefit from a demonstration or access to step-by-step instructions to use graphing technology.

Supports accessibility for: Organization; Memory; Attention

Advancing Student Thinking: If students enter the equation $(x-5)(x-3)=0$ into Desmos, they may see vertical lines. The lines will intersect the x-axis at the solutions, but they are clearly not graphs of a quadratic function. Emphasize that we want to graph the function defined by $y=(x-5)(x-3)$ and use its x-intercepts to find the solution to the related equation. *All* the points on the two vertical lines do represent solutions to the equation (because the points along each vertical line satisfy the equation regardless of the value chosen for y), but understanding this is beyond the expectations for students in this course.

Student Task Statement

Han is solving three equations by graphing.

$(x-5)(x-3)=0$ $\quad$ $(x-5)(x-3)=-1$ $\quad$ $(x-5)(x-3)=-4$

1. To solve the first equation, $(x-5)(x-3)=0$, he graphed $y=(x-5)(x-3)$ and then looked for the x-intercepts of the graph.
    a. Explain why the x-intercepts can be used to solve $(x-5)(x-3)=0$.
    b. What are the solutions?
2. To solve the second equation, Han rewrote it as $(x-5)(x-3)+1=0$. He then graphed $y=(x-5)(x-3)+1$.

    Use graphing technology to graph $y=(x-5)(x-3)+1$. Then, use the graph to solve the equation. Be prepared to explain how you use the graph for solving.
3. Solve the third equation using Han's strategy.
4. Think about the strategy you used and the solutions you found.
    a. Why might it be helpful to rearrange each equation to equal 0 on one side and then graph the expression on the non-zero side?
    b. How many solutions does each of the three equations have?

Are You Ready For More?

The equations $(x-3)(x-5)=-1$, $(x-3)(x-5)=0$, and $(x-3)(x-5)=3$ all have whole-number solutions.

1. Use graphing technology to graph each of the following pairs of equations on the same coordinate plane. Analyze the graphs and explain how each pair helps to solve the related equation.
   a. $y=(x-3)(x-5)$ and $y=-1$
   b. $y=(x-3)(x-5)$ and $y=0$
   c. $y=(x-3)(x-5)$ and $y=3$
2. Use the graphs to help you find a few other equations of the form $(x-3)(x-5)=z$ that have whole-number solutions.
3. Find a pattern in the values of z that give whole-number solutions.
4. Without solving, determine if $(x-5)(x-3)=120$ and $(x-5)(x-3)=399$ have whole-number solutions. Explain your reasoning.

Step 2

- Invite students to share their responses, graphs, and explanations of how they used the graphs to solve the equations.
- Discuss questions such as:
  - “Are the original equation $(x-5)(x-3)=-1$ and the rewritten one $(x-5)(x-3)+1=0$ equivalent?” (Yes, each pair of equations are equivalent. In that example, 1 is added to both sides of the original equation.)
  - "Why might it be helpful to rearrange the equation so that one side is an expression and the other side is 0?" (It allows us to find the zeros of the function defined by that expression. The zeros correspond to the x-intercepts of the graph.)
  - “What equation would you graph to solve this equation: $(x-4)(x-6)=15$?” ($y=(x-4)(x-6)-15$) “What about $(x+3)^2-1=5$?" ($y=(x+3)^2-6$)
- Make sure students understand that some quadratic functions have two zeros, some have one zero, and some have no zeros, so their respective graphs will have two, one, or no horizontal intercepts.
- Likewise, some quadratic equations have two solutions, some have one solution, and some have no real solutions.

DO THE MATH

PLANNING NOTES

Activity 2: Finding All the Solutions (*Optional, 15 minutes*)

Instructional Routine: Graph It	
Building On: NC.M1.A-REI.10	**Addressing:** NC.M1.A-REI.4

This optional *Graph It* activity gives students an opportunity to practice solving quadratic equations and deciding on an effective strategy. Some equations can be easily solved by reasoning. Others would require solving by graphing, because students have not yet learned the strategies to solve algebraically. Students who use graphing technology only when needed practice choosing tools strategically (MP5).

RESPONSIVE STRATEGY

Leverage choice around perceived challenge. Invite students to select 4 of the 6 equations to solve. Chunking this task into more manageable parts may also support students who benefit from additional processing time.

Supports accessibility for: Organization; Attention; Social-emotional skills

Step 1

- Continue to provide students access to Desmos.

Monitoring Tip: Monitor for students who reason without paper or graphing, students who use +/- square roots to "undo," and students who use graphing to solve the equations, either looking for intersection points or setting the expression =0 and finding horizontal intercepts.

Student Task Statement

Solve each equation. Be prepared to explain or show your reasoning.

1. $x^2 = 121$
2. $x^2 - 31 = 5$
3. $(x-4)(x-4) = 0$
4. $(x+3)(x-1) = 5$
5. $(x+1)^2 = -4$
6. $(x-4)(x-1) = 990$

Step 2

- Select students to share their solutions and strategies based on the strategies named in the Monitoring Tip. If not mentioned by students' explanations, highlight that:
  - The first three equations, as well as the equation $(x+1)^2 = -4$, can be solved by reasoning and that graphing is not necessary.
  - The last three equations can be solved by graphing. There are two ways to do so, as shown in a previous activity.
    - One way is graph each side of the equation separately: $y = expression\ on\ one\ side$ and $y = number\ on\ the\ other\ side$.
    - Another way is to rearrange the equation such that it is in the form of $expression = 0$, graph $y = expression$, and then find the x-intercepts.
  - The equation $(x+1)^2 = -4$ states that some number squared is -4. Because no number can be squared to get a negative number, we can reason that there are no real solutions. If this equation is solved by graphing $y = (x+1)^2 + 4$, the graph would show no x-intercepts. This also tells us that there are no real solutions.

DO THE MATH	PLANNING NOTES

Activity 3: Analyzing Errors in Equation Solving (*10 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1)
Addressing: NC.M1.A-REI.1

This activity aims to uncover some common misconceptions in solving quadratic equations and to reinforce that certain familiar moves for solving equations are not effective. Students critique several arguments about how to solve quadratic equations. In articulating why certain lines of reasoning are correct or incorrect, they practice constructing logical arguments (MP3).

Multiplying or dividing both sides of an equation by a variable expression can change the solution set of an equation, either by eliminating a solution (as shown in Diego's method) or introducing a new solution (for example, starting with the equation $x = 3$ and multiplying both sides by x to get $x^2 = 3x$ gives an equation that now has 2 solutions). Students will learn more about such moves in a later course.

Step 1

- Keep students in pairs and ask them to work quietly on both questions before discussing their responses with a partner.
- Use the *Stronger and Clearer Each Time* routine to help students improve their written responses to the first question by providing them with an opportunity to clarify their explanations through paired conversation. Give students 2 minutes to meet with a partner to take turns sharing their initial response to the first question about whether they agree or disagree with Priya. Provide listeners with prompts for feedback that will help their partner add detail to strengthen and clarify their ideas. For example:
  - "Your explanation tells me . . ."
  - "Can you say more about why you . . . ?"
  - "A detail (or word) you could add is _____, because . . . ."
- Give students 1–2 minutes to revise their initial draft based on feedback from their peers. This will help students evaluate the written mathematical arguments of others and improve their own written responses about solving quadratic equations.

Monitoring Tip: As students work, look for students who:

- explain both the error in Priya's argument and the validity of Mai's argument in terms of the zero product property
- notice that Diego's method disregards the second solution of the equation
- create and use a graph to verify their critique of Priya, Mai, or Diego's work (for example, graphing to show that $x^2 - 10x$ has two x-intercepts)

Let these students know that they should be prepared to share their explanations during the whole-group discussion.

Student Task Statement

1. Consider $(x-5)(x+1)=7$. Priya reasons that if this is true, then either $x-5=7$ or $x+1=7$. So, the solutions to the original equation are 12 and 6.

   Do you agree? If not, where was the mistake in Priya's reasoning?

2. Consider $x^2 - 10x = 0$. Diego says to solve we can just divide each side by x to get $x - 10 = 0$, so the solution is 10. Mai says, "I wrote the expression on the left in factored form, which gives $x(x-10)=0$, and ended up with two solutions: 0 and 10."

   Do you agree with either strategy? Explain your reasoning.

Step 2

- Facilitate a whole-class discussion by selecting previously identified students to share their responses and reasoning. Here are some key observations to highlight:
  - For the first question, make sure students understand that the zero product property only works when the product of the factors is 0. While substituting 6 for x in the expression does produce 7, substituting 12 does not give the same result.
  - For the second question, consider graphing the function $y = x^2 - 10x$ so students can see that the graph intersects the x-axis at two points, which means that there are two x-values that give a zero output: 0 and 10. Rewriting $x^2 - 10x$ into the factored form, writing it to equal 0, and solving $x(x-10)=0$ allow us to see that this is indeed the case.
  - Dividing each side by a variable (as Diego did) seems to enable us to isolate the remaining variable, but only one solution remains. Dividing each side of an equation by x is not a valid move because when x is 0, the expressions on each side become undefined.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to make connections between the solutions to a quadratic equation and the zeros on the graph that represents the equation. Students also explore the validity of common approaches to solving quadratic equations and articulate, with specific examples, why each approach is valid or invalid.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

To synthesize the work in this lesson and connect it to prior work, discuss questions such as:

- "How would you solve the equation $x(x-250)=0$?" (The simplest way would be to use the zero product property, which tells us that one of the factors must be 0, so $x=0$ and $x=250$.)
- "If you choose to solve by graphing, is it necessary to rearrange and rewrite the equation first? What equation would you graph and how would you use the graph to solve the equation?" (No. We can just graph $y=x(x-250)$ and look for the x-intercepts of the graph.)
- "Can we use the zero product property to solve $x(x-250)=100$? Why or why not?" (No, because the expression on the right does not equal 0.)
- "How would you solve $x(x-250)=100$?" (We can rewrite the equation as $x(x-250)-100=0$, graph $y=x(x-250)-100$, and see what the x-intercepts are.)
- "How can the graph tell us how many solutions there are?" (The number of x-intercepts reveals the number of solutions.)

Remind students that examining a graph is not a reliable way to get exact solutions to an equation. For example, the x-intercepts of the graph for $y=x(x-250)-100$ are $(-0.399,0)$ and $(250.399,0)$, and those x-coordinates are likely rounded results.

To solve equations exactly, we need to use algebraic means. In upcoming lessons, we'll learn more strategies for doing so.

PLANNING NOTES

Student Lesson Summary and Glossary

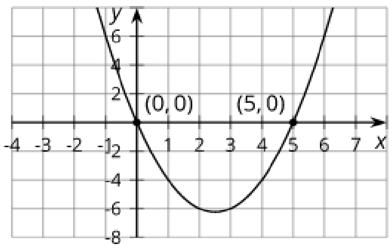
Quadratic equations can have two solutions, one solution, or no real solutions.

We can find out how many real solutions a quadratic equation has and what the solutions are by rearranging the equation into the form of $expression = 0$, graphing the function that the expression defines, and determining its zeros. Here are some examples.

- $x^2 = 5x$

  Let's first subtract $5x$ from each side and rewrite the equation as $x^2 - 5x = 0$. We can think of solving this equation as finding the zeros of a function defined by $x^2 - 5x$.

  If the output of this function is y, we can graph $y = x^2 - 5x$ and identify where the graph intersects the x-axis, where the y-coordinate is 0.

  From the graph, we can see that the x-intercepts are $(0, 0)$ and $(5, 0)$, so $x^2 - 5x$ equals 0 when x is 0 and when x is 5.

  The graph readily shows that there are two solutions to the equation.

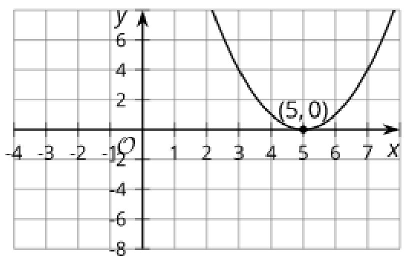
Note that the equation $x^2 = 5x$ can be solved without graphing, but we need to be careful *not* to divide both sides by x. Doing so will give us $x = 5$ but will show no trace of the other solution, $x = 0$!

Even though dividing both sides by the same value is usually acceptable for solving equations, we avoid dividing by the same variable because it may eliminate a solution.

- $(x - 6)(x - 4) = -1$

  Let's rewrite the equation as $(x - 6)(x - 4) + 1 = 0$ and consider it to represent a function defined by $(x - 6)(x - 4) + 1$ and whose output, y, is 0.

  Let's graph $y = (x - 6)(x - 4) + 1$ and identify the x-intercepts.

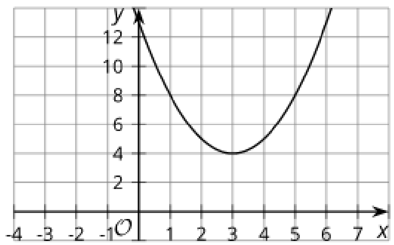
  The graph shows one x-intercept at $(5, 0)$. This tells us that the function defined by $(x - 6)(x - 4) + 1$ has only one zero.

  It also means that the equation $(x - 6)(x - 4) + 1 = 0$ is true only when $x = 5$. The value 5 is the only solution to the equation.

- $(x - 3)(x - 3) = -4$

  Rearranging the equation gives $(x - 3)(x - 3) + 4 = 0$.

  Let's graph $y = (x - 3)(x - 3) + 4$ and find the x-intercepts.

  The graph does not intersect the x-axis, so there are no x-intercepts.

  This means there are no real number x-values that can make the expression $(x - 3)(x - 3) + 4$ equal 0, so the function defined by $y = (x - 3)(x - 3) + 4$ has no zeros.

  The equation $(x - 3)(x - 3) = -4$ has no real number solutions.

  We can see that this is the case even without graphing. $(x - 3)(x - 3) = -4$ is equivalent to $(x - 3)^2 = -4$. Because no real number can be squared to get a negative value, the equation has no real number solutions.

Earlier you learned that graphing is not always reliable for showing precise solutions. This is still true here. The x-intercepts of a graph are not always whole-number values. While they can give us an idea of how many solutions there are and what the values may be (at least approximately), for exact solutions we still need to rely on algebraic ways of solving.

Cool-down: Two, One, or None? (*5 minutes*)

Addressing: NC.M1.A-REI.4

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Allow students to continue using Desmos if they choose to.

Cool-down

For each quadratic equation, decide whether it has two solutions, one solution, or no real number solutions. Explain how you know.

1. $x^2 = -16$
2. $x(x+2) = 0$
3. $(x-3)(x-3) = 0$

Student Reflection:

Over the last few lessons you have learned how to solve quadratic equations in multiple ways. What mathematical tools help you most? Are there any you wish you could have used or could have utilized more?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

How did students think of solving equations as they came into the lesson? In what ways did their understanding of solving equations change upon completing the lesson?

Practice Problems

1. Rewrite each equation so that the expression on one side could be graphed and the x-intercepts of the graph would show the solutions to the equation.

   a. $3x^2 = 81$

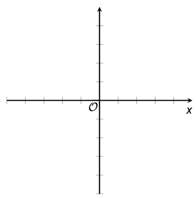
   b. $(x-1)(x+1) - 9 = 5x$

   c. $x^2 - 9x + 10 = 32$

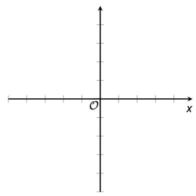
   d. $6x(x-8) = 29$

2.
   a. Here are equations that define quadratic functions f, g, and h. Sketch a graph, by hand or using technology, that represents each equation. Indicate the vertex, intercepts, and general shape and direction of the graph.

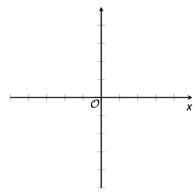
   $f(x) = x^2 + 4$

   $g(x) = x(x+3)$

   $h(x) = (x-1)^2$

   b. Determine how many real number solutions each $f(x) = 0$, $g(x) = 0$, and $h(x) = 0$ has. Explain how you know.

3. Mai is solving the equation $(x-5)^2 = 0$. She writes that the solutions are $x = 5$ and $x = -5$ Han looks at her work and disagrees. He says that only $x = 5$ is a solution. Who do you agree with? Explain your reasoning.

4. Decide if each equation has 0, 1, or 2 solutions and explain how you know.

   a. $x^2 - 144 = 0$

   b. $x^2 + 144 = 0$

   c. $x(x-5) = 0$

   d. $(x-8)^2 = 0$

   e. $(x+3)(x+7) = 0$

5. If the equation $(x-4)(x+6) = 0$ is true, which is also true according to the zero product property?

   a. only $x - 4 = 0$
   b. only $x + 6 = 0$
   c. $x - 4 = 0$ or $x + 6 = 0$
   d. $x = -4$ or $x = 6$

   (From Unit 7, Lesson 19)

6.
   a. Solve the equation $25 = 4z^2$.

   b. Show that your solution or solutions are correct.

   (From Unit 7, Lesson 18)

7. To solve the quadratic equation $3(x-4)^2=27$, Andre and Clare wrote the following:

Andre

$3(x-4)^2=27$
$(x-4)^2=9$
$x^2-4^2=9$
$x^2-16=9$
$x^2=25$
$x=5$ or $x=-5$

Clare

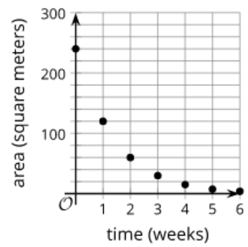
$3(x-4)^2=27$
$(x-4)^2=9$
$x-4=3$
$x=7$

a. Identify the mistake(s) each student made.

b. Solve the equation and show your reasoning.

(From Unit 7, Lesson 18)

8. The graph shows the number of square meters, A, covered by algae in a lake w weeks after it was first measured.



In a second lake, the number of square meters, B, covered by algae is defined by the equation $B=975\cdot(\frac{2}{5})^w$, where w is the number of weeks since it was first measured.

For which algae population is the area decreasing more rapidly? Explain how you know.

(From Unit 6)

9. At the end of each year, 10% interest is charged on a $500 loan. The interest applies to any unpaid balance on the loan, including previous interest.

Select **all** the expressions that represent the loan balance after two years if no payments are made.

a. $500+2\cdot(0.1)\cdot500$

b. $500\cdot(1.1)\cdot(1.1)$

c. $500+(0.1)+(0.1)$

d. $500\cdot(1.1)^2$

e. $(500+50)\cdot(1.1)$

(From Unit 6)

10. For the following pairs of equations, determine whether x is greater than, less than, or equal to y.

Fill in the blank with >, <, or =.

a. $9x=27$; $y^3=27$ $\quad x$ ____ y

b. $3x=50$; $y^3=50$ $\quad x$ ____ y

c. $5x=100$; $y^5=100$ $\quad x$ ____ y

(Addressing NC.8.EE.2)

Lesson 21: Rewriting Quadratic Expressions in Factored Form (Part One)

PREPARATION

Lesson Goals	Learning Targets
• Generalize the relationship between equivalent quadratic expressions in standard form and factored form, and use the generalization to transform expressions from one form to the other. • Use a diagram to represent quadratic expressions in different forms and explain (orally and in writing) how the numbers in the factors relate to the numbers in the product.	• I can explain how the numbers in a quadratic expression in factored form relate to the numbers in an equivalent expression in standard form. • When given quadratic expressions in factored form, I can rewrite them in standard form. • When given quadratic expressions in the form of $x^2 + bx + c$, I can rewrite them in factored form.

Lesson Narrative

Previously, students learned that a quadratic expression in factored form can be quite handy in revealing the zeros of a function and the x-intercepts of its graph. They also observed that the factored form can help us solve quadratic equations algebraically. In this lesson, students begin to rewrite quadratic expressions from standard to factored form. For this lesson, and the two subsequent lessons, *a* will have an assumed value of 1.

In an earlier unit, students learned to expand quadratic expressions in factored form and rewrite them in standard form. They did so by applying the distributive property to multiply out the factors—first by using diagrams for support, and then by relying on structure they observed in the process. The attention to structure continues in this lesson. Students relate the numbers in the factored form to the coefficients of the terms in standard form, looking for structure that can be used to go in reverse—from standard form to factored form (MP7).

This lesson only looks at expressions of the form $(x+m)(x+n)$ and $(x-m)(x-n)$ where m and n are positive. This is so that arithmetic doesn't get in the way of noticing the relationships between the numbers in factored form and the numbers in standard form. In the next lesson, students will encounter expressions of the form $(x+m)(x-n)$ and $(x-m)(x+n)$.

Note that this course includes only four lessons on transforming quadratic expressions from standard form to factored form. This is intentional. The goal is to help students see factored form conceptually, understand what it can tell us about the function, and use that knowledge in modeling problems, including problems where the zeros are not rational and algebraic factoring wouldn't help. Too much attention to the algebraic skill of factoring can obscure these underlying concepts.

Sometimes—including in later courses and beyond—expressions do need to be written in factored form. With the universal availability of computer algebra systems, however, there is less need to spend lots of time learning how to factor by hand.

What do you hope to learn mathematically from this lesson?

Focus and Coherence

Building On	Building Towards
NC.4.OA.4: Find all factor pairs for whole numbers up to and including 50 to: ● Recognize that a whole number is a multiple of each of its factors. ● Determine whether a given whole number is a multiple of a given one-digit number. ● Determine if the number is prime or composite. **NC.6.G.1:** Create geometric models to solve real-world and mathematical problems to: ● Find the area of triangles by composing into rectangles and decomposing into right triangles. ● Find the area of special quadrilaterals and polygons by decomposing into triangles or rectangles. **NC.7.NS.1:** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers, using the properties of operations, and describing real-world contexts using sums and differences. **NC.7.NS.2:** Apply and extend previous understandings of multiplication and division. **a.** Understand that a rational number is any number that can be written as a quotient of integers with a non-zero divisor. **b.** Apply properties of operations as strategies, including the standard algorithms, to multiply and divide rational numbers and describe the product and quotient in real-world contexts. **c.** Use division and previous understandings of fractions and decimals. ● Convert a fraction to a decimal using long division. ● Understand that the decimal form of a rational number terminates in 0s or eventually repeats. **NC.M1.A-APR.1:** Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions.	**NC.M1.A-SSE.3:** Write an equivalent form of a quadratic expression ax^2+bx+c, where a is an integer, by factoring to reveal the solutions of the equation or the zeros of the function the expression defines. **NC.M1.A-REI.4:** Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L21 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.4.OA.4; NC.7.NS.1; NC.7.NS.2

In this lesson, students will begin to factor trinomials. Understanding factors of integers is important background for this skill. This bridge presents a game to introduce the process of determining the factors and addends for given products and sums. This bridge aligns to Check Your Readiness question 10.

Student Task Statement

Example: Find two numbers that multiply to equal 21 and add to equal 10.

Solution: 7 and 3, because $7 \cdot 3 = 21$ and $7 + 3 = 10$

Now, try some as a game with your classmates and see who can figure out the most without a calculator! For each of the following questions, figure out two numbers that:

a. Multiply to equal 20 and add to equal 12

b. Multiply to equal -6 and add to equal 1

c. Multiply to equal 15 and add to equal -8

d. Multiply to equal 36 and add to equal 15

e. Multiply to equal 12 and add to equal 13

f. Multiply to equal -24 and add to equal 5

g. Multiply to equal 2 and add to equal -3

DO THE MATH

PLANNING NOTES

Warm-up: Puzzles of Rectangles (*5 minutes*)

Building On: NC.6.G.1

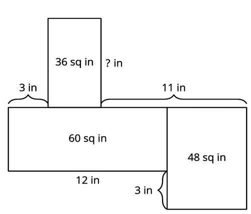
When they write expressions in factored form later in the lesson, students will need to reason about factors that yield certain products. This warm-up prompts students to find unknown factors in the context of area puzzles. Solving the puzzles involves reasoning about the measurements in multiple steps. Explaining these steps is an opportunity to practice constructing logical arguments (MP3).

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping. Students will be in these pairs for the remainder of the lesson.
- Give students a few minutes of quiet think time and then time to share their thinking with their partner. Follow with a whole-class discussion.

Student Task Statement

Here is a puzzle that involves side lengths and areas of rectangles. Can you find the missing length? Be prepared to explain your reasoning.



Step 2

- Display the images for all to see. Invite students to share their responses and how they reasoned about the missing values, using the diagrams to illustrate their thinking.
- After the solution to the second puzzle is presented, draw students' attention to the rectangle with area 36 sq in. Point out that, without reasoning about other parts of the puzzle, we cannot know which two numbers are multiplied to get 36 sq in. (The numbers may not be whole numbers.) But by reasoning about other parts, we can conclude what the missing length must be.
- Explain that in this lesson, they will also need to find two factors that yield a certain product and to reason logically about which numbers the factors can or must be.

DO THE MATH	PLANNING NOTES

Activity 1: Using Diagrams to Understand Equivalent Expressions (*15 minutes*)

Instructional Routine: Take Turns	
Building On: NC.M1.A-APR.1	**Building Towards:** NC.M1.A-SSE.3; NC.M1.A-REI.4

This activity prompts students to notice the structure that relates quadratic expressions in factored form and their equivalent counterparts in standard form. Students began this work earlier in the course. They applied the distributive property to expand an expression such as $(x+5)(x+4)$ into standard form, using rectangular diagrams to help organize and keep track of the partial products. Here, the focus is on using the structure to go in reverse: rewriting into factored form an expression given in standard form (MP7).

The expressions students encounter here are two sums or two differences, in the form of $(x+m)(x+n)$ or $(x-m)(x-n)$.

RESPONSIVE STRATEGIES

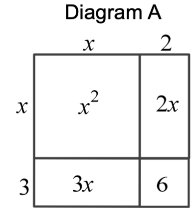
Use color coding and annotations to highlight connections between representations. Invite students to use color to identify the like terms in their diagrams and expressions. Demonstrate how students can use different colors to keep track of like terms as they use diagrams to show that each pair of expressions is equivalent.

Supports accessibility for:
Visual-spatial processing

Step 1

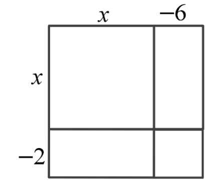
- Display diagram A for all to see and remind students that they have seen diagrams such as this one in a previous lesson on quadratic functions.
- Ask students what expression it represents (both $(x+2)(x+3)$ and x^2+5x+6).

Diagram A



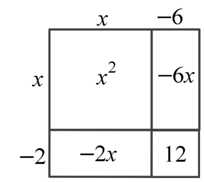
- Next, ask students what diagram B represents.
- Students might say $(x + -6)(x + -2)$ or $(x - 6)(x - 2)$. Whether they mention both or not, write both expressions for all to see and emphasize that they are equivalent.
- When we want to represent $(x - 6)(x - 2)$, it is convenient to think of it as $(x + -6)(x + -2)$ and label the diagram as such so that we keep track of what is positive and negative in the diagram.

Diagram B



- Then show diagram C. Ask students what expression or number goes in each blank rectangle. Make sure students see that the rectangles are used to organize the partial products of $(x - 6)(x - 2)$, the sum of which is $x^2 - 8x + 12$. Consider pointing out that this is an application of the distributive property, which students first studied in grade 6.

Diagram C



Step 2

- In the same groups from the warm-up, ask students to *Take Turns* (one partner doing 1a, the other doing 1b, and so on) showing that each pair of expressions is equivalent for question 1.
- Ask students to work collaboratively to answer question 2.

Student Task Statement

1. Use a diagram to show that each pair of expressions is equivalent. Use a diagram similar to the ones shown here to show your models.

   a. $x(x + 3)$ and $x^2 + 3x$

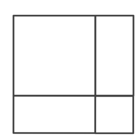
   b. $x(x + -6)$ and $x^2 - 6x$

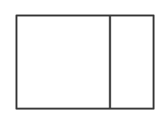
   c. $(x + 2)(x + 4)$ and $x^2 + 6x + 8$

   d. $(x + 4)(x + 10)$ and $x^2 + 14x + 40$

   e. $(x + -5)(x + -1)$ and $x^2 - 6x + 5$

   f. $(x - 1)(x - 7)$ and $x^2 - 8x + 7$





2. Observe the pairs of expressions from above that involve the product of two sums or two differences. How is each expression in factored form related to the equivalent expression in standard form?

Step 3

- Invite students to share their diagrams and observations.
- Make sure students notice that the linear term in the expression in standard form is the sum of the two numbers in the expression in factored form, and the constant term is the product of the two numbers in the expression in factored form. (For this explanation to be succinct, it requires rewriting any subtractions as adding the opposite.)

DO THE MATH	PLANNING NOTES

Activity 2: Let's Rewrite Some Expressions! (*10 minutes*)

Instructional Routine: Discussion Supports (MLR8) - Responsive Strategy
Building Towards: NC.M1.A-SSE.3; NC.M1.A-REI.4

This activity allows students to practice rewriting quadratic expressions in standard form by using the structure they observed in the earlier activity.

Step 1

- Ask students to work individually for 3–4 minutes to fill in as many expressions as they can on their own. Then arrange ask them to confer with their partner. Leave a few minutes for a whole-class discussion.
- Ask students to complete as many equivalent expressions as time permits while aiming to complete at least the first seven rows in the table. Then, ask them to generalize their observations in the last row.

Monitoring Tip: As students work, look for those who approach the work systematically: by looking for two factors of the constant term that add up to the coefficient of the linear term of the expression in standard form, listing the possible pairs of factors, and checking their chosen pair. Invite them to share their strategy during discussion.

***Advancing Student Thinking*:** Some students may struggle to remember how each term in standard form relates to the numbers in the equivalent expression in factored form. Encourage them to use a diagram (as in the earlier activity) to go from factored form to standard form, and then work backwards, thinking about the pattern they realized in the last activity.

Student Task Statement

Each row in the table contains a pair of equivalent expressions.

Complete the table with the missing expressions. If you get stuck, consider drawing a diagram.

Factored form	Standard form
$x(x+7)$	
	x^2+9x
	x^2-8x
$(x+6)(x+2)$	
	$x^2+13x+12$
$(x-6)(x-2)$	

Factored form	Standard form
	$x^2-7x+12$
	x^2+6x+9
	$x^2+10x+9$
	$x^2-10x+9$
	x^2-6x+9
	$x^2+(m+n)x+mn$

Step 2

- Focus first on the first three rows. Ask one or more students to share their equivalent expressions and any diagrams that they drew. Point out that this is an application of the distributive property.
- Select students to share how they transformed the remaining expressions from standard form to factored form, using specific examples in their explanations. For the example of $x^2+13x+12$, highlight that:
  - We are looking for two factors of 12.
  - We are looking for two numbers with a sum of 13.
  - One strategy is to list out all the factor pairs of 12.
  - Another strategy is to list out all pairs of numbers that add up to 13, but usually the list of factors is shorter.

RESPONSIVE STRATEGY

Use this routine to support whole-class discussion. At the appropriate time, give students, or pairs of students, 2–3 minutes to plan what they will say when they present how they transformed their expressions from standard form to factored form, using specific examples in their explanations. Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language.

Discussion Supports (MLR8)

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to examine the connection between expressions of the form $x^2 + bx + c$ and factored form.

Choose whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner. To help students summarize and generalize the reasoning involved in rewriting quadratic expressions from standard form to factored form, display a few expressions such as these:

- $x^2 + 8x + 15$
- $x^2 + 11x + 28$
- $x^2 + bx + c$

For each one, ask students to explain the process of transforming it into factored form by completing (in writing or by talking a partner) sentence starters such as these:

- To find the factors, I first try to find . . .
- Next, I think about .. . .
- The equivalent expression in factored form is . . .
- To check that the factors are correct, I can . . .

Make sure students see that for $x^2 + bx + c$, a helpful process goes something like this:

- First, we find all pairs of factors of c.
- Next, find a pair of factors of c that add up to equal b. (If the factors are m and n, then we want $m + n = b$.)
- The factors will be $(x + m)(x + n)$.
- We can check by expanding the factored form (by applying the distributive property) and see if we get the original expression as a result.

PLANNING NOTES

Student Lesson Summary and Glossary

Previously, you learned how to expand a quadratic expression in factored form and write it in standard form by applying the distributive property.

For example, to expand $(x + 4)(x + 5)$, we apply the distributive property to multiply x by $(x + 5)$ and 4 by $(x + 5)$. Then, we apply the property again to multiply x by x and x by 5, and multiply 4 by x and 4 by 5.

To keep track of all the products, we could make a diagram like this:

	x	4
x		
5		

Next, we could write the products of each pair inside the spaces:

	x	4
x	x^2	$4x$
5	$5x$	$4 \cdot 5$

The diagram helps us see that $(x + 4)(x + 5)$ is equivalent to $x^2 + 5x + 4x + 4 \cdot 5$, or in standard form, $x^2 + 9x + 20$.

- The linear term, $9x$, has a coefficient of 9, which is the sum of 5 and 4.
- The constant term, 20, is the product of 5 and 4.

We can use these observations to reason in the other direction: to start with an expression in standard form and write it in factored form.

For example, suppose we wish to write $x^2 - 11x + 24$ in factored form.

Let's start by creating a diagram and writing in the terms x^2 and 24.

We need to think of two numbers that multiply to make 24 and add up to -11.

	x	
x	x^2	
		24

After some thinking, we see that -8 and -3 meet these conditions.

The product of -8 and -3 is 24. The sum of -8 and -3 is -11.

	x	-8
x	x^2	$-8x$
-3	$-3x$	24

So, $x^2 - 11x + 24$ written in factored form is $(x-8)(x-3)$.

Cool-down: The Missing Numbers (*5 minutes*)

Building Towards: NC.M1.A-SSE.3, NC.M1.A-REI.4

Cool-down Guidance: More Chances
If students make computational errors, address those in the next lesson and emphasize the use of diagrams.

Cool-down

Here are pairs of equivalent expressions—one in standard form and the other in factored form. Find the missing numbers.

1. $x^2 + \boxed{}\,x + \boxed{}$ and $(x+1)(x+17)$
2. $x^2 + 9x + 14$ and $(x+2)(x+\boxed{})$
3. $x^2 - 7x + 10$ and $(x-2)(x-\boxed{})$
4. $x^2 - 9x + 20$ and $(x-\boxed{})(x-\boxed{})$

Student Reflection:

In prior lessons you have worked with standard and factored forms. How did it feel to work with them again today?

a. Very familiar
b. Somewhat familiar
c. I struggled to recall what I'd seen before.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Which students came up with an unexpected strategy in today's lesson? What are some ways you can be more open to the ideas of each and every student?

Practice Problems

1. Find two numbers that satisfy the requirements. If you get stuck, try listing all the factors of the first number.
    a. Find two numbers that multiply to 17 and add to 18.
    b. Find two numbers that multiply to 20 and add to 9.
    c. Find two numbers that multiply to 11 and add to -12.
    d. Find two numbers that multiply to 36 and add to -20.

2. Select **all** expressions that are equivalent to $x - 5$.

   a. $x + (-5)$
   b. $x - (-5)$
   c. $-5 + x$
   d. $-5 - x$
   e. $5 - x$
   f. $-5 - (-x)$
   g. $5 + x$

3. Use the diagram to show that:

   a. $(x + 4)(x + 2)$ is equivalent to $x^2 + 6x + 8$.

	x	2
x		
4		

   b. $(x - 10)(x - 3)$ is equivalent to $x^2 - 13x + 30$.

	x	-10
x		
-3		

4. Here are pairs of equivalent expressions—one in standard form and the other in factored form. Find the missing numbers.

   a. $x^2 + \boxed{}\,x + \boxed{}$ and $(x - 9)(x - 3)$
   b. $x^2 + 12x + 32$ and $(x + 4)(x + \boxed{})$
   c. $x^2 - 12x + 35$ and $(x - 4)(x + \boxed{})$
   d. $x^2 - 9x + 20$ and $(x - 4)(x + \boxed{})$

5. (*Technology required.*) When solving the equation $(2 - x)(x + 1) = 11$, Priya graphs $y = (2 - x)(x + 1) - 11$ and then looks to find where the graph crosses the x-axis.

   Tyler looks at Priya's work and says that graphing is unnecessary, and Priya can set up the equations $2 - x = 11$ and $x + 1 = 11$, so the solutions are $x = -9$ or $x = 10$.

   a. Do you agree with Tyler? If so, explain why. If not, where is the mistake in their reasoning?

   b. How many solutions does the equation have? Find out by graphing Priya's equation.

   (From Unit 7, Lesson 20)

6. Find all the values for the variable that make each equation true.

   a. $b(b - 4.5) = 0$

   b. $(7x + 14)(7x + 14) = 0$

   c. $(2x + 4)(x - 4) = 0$

   d. $(-2 + u)(3 - u) = 0$

   (From Unit 7, Lesson 19)

7. From 2005 to 2015, a population of p lions is modeled by the equation $p = 1,500 \cdot (0.98)^t$, where t is the number of years since 2005.

   a. About how many lions were there in 2005?

   b. Describe what is happening to the population of lions over this decade.

   c. About how many lions are there in 2015? Show your reasoning.

   (From Unit 6)

8. Lin charges $5.50 per hour to babysit. The amount of money earned, in dollars, is a function of the number of hours that she babysits.

   Which of the following inputs is impossible for this function?

   a. -1
   b. 2
   c. 5
   d. 8

   (From Unit 5)

9. Diego's goal is to walk more than 70,000 steps this week. The mean number of steps that Diego walked during the first 4 days of this week is 8,019.

   a. Write an inequality that expresses the mean number of steps that Diego needs to walk during the last 3 days of this week to walk more than 70,000 steps. Remember to define any variables that you use.

   b. If the mean number of steps Diego walks during the last 3 days of the week is 12,642, will Diego reach his goal of walking more that 70,000 steps this week?

   (From Unit 5)

10. A median of a triangle is a segment drawn from one vertex to the midpoint of the opposite side. For Triangle ABC, with $A(6,10)$, $B(-3,7)$, and $C(5,3)$, how long is the median drawn from A to the midpoint of BC?

    (From Unit 3)

Lesson 22: Rewriting Quadratic Expressions in Factored Form (Part Two)

PREPARATION

Lesson Goals	Learning Targets
• Given a quadratic expression of the form $x^2 + bx + c$, where c is negative, write an equivalent expression in factored form. • When multiplying a sum and a difference, explain (orally and in writing) how the numbers and signs in the factors relate to the numbers in the product.	• When given a quadratic expression given in standard form with a negative constant term, I can write an equivalent expression in factored form. • I can explain how the numbers and signs in a quadratic expression in factored form relate to the numbers and signs in an equivalent expression in standard form.

Lesson Narrative

In the previous lesson, students transformed quadratic expressions from standard form into factored form. There, the factored expressions are products of two sums, $(x+m)(x+n)$, or two differences, $(x-m)(x-n)$. Students continue that work in this lesson, extending it to include expressions that can be rewritten as products of a sum and a difference, $(x+m)(x-n)$.

Through repeated reasoning, students notice that when we apply the distributive property to multiply out a sum and a difference, the product has a negative constant term, but the linear term can be negative or positive (MP8). Students make use of structure as they use this insight to transform quadratic expressions into factored form (MP7). They see that if a quadratic expression in standard form (with coefficient 1 for x^2) has a negative constant term, one of its factors must have a negative constant term and the other must have a positive constant term.

What strategies or representations do you anticipate students might use in this lesson?

Focus and Coherence

Building On	Building Towards
NC.7.NS.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers, using the properties of operations, and describing real-world contexts using sums and differences. **NC.7.NS.2:** Apply and extend previous understandings of multiplication and division. **a.** Understand that a rational number is any number that can be written as a quotient of integers with a non-zero divisor. **b.** Apply properties of operations as strategies, including the standard algorithms, to multiply and divide rational numbers and describe the product and quotient in real-world contexts. **c.** Use division and previous understandings of fractions and decimals. • Convert a fraction to a decimal using long division. • Understand that the decimal form of a rational number terminates in 0s or eventually repeats.	**NC.M1.A-SSE.3:** Write an equivalent form of a quadratic expression $ax^2 + bx + c$, where a is an integer, by factoring to reveal the solutions of the equation or the zeros of the function the expression defines. **NC.M1.A-REI.4:** Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L22 Cool-down (print 1 copy per student)

LESSON

Warm-up: Sums and Products (*5 minutes*)

Building On: NC.7.NS.1; NC.7.NS.2

This warm-up serves two purposes. The first is to recall that if the product of two numbers is negative, then the two numbers must have opposite signs. The second is to review how to add two numbers with opposite signs.

Step 1

- Provide students with a few minutes to complete the task statement.

Student Task Statement

1. The product of the integers 2 and -6 is -12. List all the other pairs of integers whose product is -12.
2. Of the pairs of factors you found, list all pairs that have a positive sum. Explain why they all have a positive sum.
3. Of the pairs of factors you found, list all pairs that have a negative sum. Explain why they all have a negative sum.

Step 2

- Ask students to share their list of factors. As students list pairs, write them on the board. Once all the pairs are listed, highlight that each pair has a positive number and a negative number because the product we are after is a negative number, and the product of a positive and a negative number is negative.

- Invite students to share their responses for the next two questions. Consider displaying a number line for all to see and using arrows to visualize the additions of factors. Make sure students understand that when adding a positive number and a negative number, the result is the difference of the absolute values of the numbers, and that sum takes the sign of the number that is farther from zero.

DO THE MATH	PLANNING NOTES

Activity 1: Negative Constant Terms (*15 minutes*)

Instructional Routines: Notice and Wonder; Discussion Supports (MLR8) - Responsive Strategy	
Building Towards: NC.M1.A-SSE.3; NC.M1.A-REI.4	

In this activity, students encounter quadratic expressions that are in standard form and that have a negative constant term. They notice that, when such expressions are rewritten in factored form, one of the factors is a sum and the other is a difference. They connect this observation to the fact that the product of a positive number and a negative number is a negative number.

Students also recognize that the sum of the two factors of the constant term may be positive or negative, depending on which factor has a greater absolute value. This means that the sign of the coefficient of the linear term (which is the sum of the two factors) can reveal the signs of the factors.

Students use their observations about the structure of these expressions and of operations to help transform expressions in standard form into factored form (MP7).

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping. They will remain in these pairs for the remainder of the lesson.
- Give students a few minutes of quiet think time to work on the first question. Have students share with their partner.

RESPONSIVE STRATEGY

In pairs, ask students to take turns describing the differences they notice between the expressions in each table. Display the following sentence frames for all to see: "____ and ____ are different because . . .", "One thing that is different is . . ." and "I noticed ____, so . . . ." Encourage students to challenge each other when they disagree. This will help students clarify their reasoning about the relationship between quadratic expressions with a negative constant term written in factored and standard form.

Discussion Supports (MLR8)

RESPONSIVE STRATEGY

Create a display of important terms and vocabulary. During Step 1, take time to review terms students will need to access for this activity. Invite students to suggest language or diagrams to include that will support their understanding of factored form, standard form, linear term, constant term, coefficient, squared term, expression, and factors.

Supports accessibility for:
Conceptual processing; Language

Student Task Statement

1. These expressions are like the ones we have seen before. Each row has a pair of equivalent expressions.

   Complete the table. If you get stuck, consider drawing a diagram.

Factored form	Standard form
$(x+5)(x+6)$	
	$x^2+13x+30$
$(x-3)(x-6)$	
	$x^2-11x+18$

Step 2

- Before students begin the third question, display a completed table for the first question and the incomplete table for the third question for all to see. Engage students in the *Notice and Wonder* routine by asking students to talk to their partner about something they notice or wonder about the expressions in the table.
- Ask students to work quietly on the remaining two questions before conferring with their partner. Continue to display the two tables for the discussion in Step 2.

Student Task Statement

2. Looking at the completed table for question 1 and the table below in question 3, what do you notice and wonder?
3. These expressions are in some ways unlike the ones we have seen before. Each row has a pair of equivalent expressions.

   Complete the table. If you get stuck, consider drawing a diagram.

Factored form	Standard form
$(x+12)(x-3)$	
	$x^2-9x-36$
	$x^2-35x-36$
	$x^2+35x-36$

4. Name some ways that the expressions in the second table are different from those in the first table (aside from the fact that the expressions use different numbers).

Step 3

- Direct students' attention to the second table. Invite some students to complete the missing expressions and explain their reasoning. Discuss questions such as:
  - "How did you know what signs the numbers in the factored expressions would take?" (The two numbers must multiply to -36, which is a negative number, so one of the factors must be positive and the other must be negative.)
  - "How do you know which factor should be positive and which one negative?" (If the coefficient of the linear term in standard form is positive, the factor of 36 with the greater absolute value is positive. If the coefficient of the linear term is negative, the factor of 36 with the greater absolute value is negative. This is because the sum of a positive and a negative number takes the sign of the number with the greater absolute value.)
- If not mentioned in students' explanations, point out that all the factored expressions in the second table contain a sum and a difference. This can be attributed to the negative constant term in the equivalent standard form expression.

DO THE MATH	PLANNING NOTES

Activity 2: Factors of 100 and -100 (*15 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1) - Responsive Strategy	
Building Towards: NC.M1.A-SSE.3; NC.M1.A-REI.4	

This activity aims to solidify students' observations about the structure connecting the standard form and factored form. Students find all pairs of factors of a number that would lead to a positive sum, a negative sum, and a zero sum. They then look for patterns in the numbers and draw some general conclusions about what must be true about the numbers to produce a certain kind of sum. Along the way, students practice looking for regularity through repeated reasoning (MP8).

RESPONSIVE STRATEGY

Support effective and efficient use of tools and assistive technologies. Some students may benefit from a demonstration or access the step-by-step instructions for using Desmos to find pairs of factors for a given number.

Supports accessibility for: Organization; Memory; Attention

Step 1

- In the same pairs as Activity 1, have students complete the tables for the first two questions. Consider asking one partner to answer the first question, while the other partner answers the second question.
- Pause for class discussion.

***Advancing Student Thinking*:** When completing the tables to find b, some students may multiply the factors rather than add them. Remind them that what we are looking for is the coefficient of the linear term.

Consider completing one row of the table and displaying a rectangle diagram to remind students how the value of b is obtained when we rewrite an expression such as $(x+20)(x+5)$ in standard form. Applying the distributive property gives $x^2+20x+5x+100$ or $x^2+25x+100$. Point out that in standard form, the product of the factors,100, is the constant term. If the coefficient of the linear term is what we are after, we need to find the sum of the factors.

Student Task Statement

1. Consider the expression $x^2+bx+100$.

   Complete the first table with all pairs of factors of 100 that would give positive values of b and the second table with factors that would give negative values of b.

   For each pair, state the b value they produce. (Use as many rows as needed.)

   Positive value of b

Factor 1	Factor 2	b (positive)

   Negative value of b

Factor 1	Factor 2	b (negative)

2. Consider the expression $x^2+bx-100$.

   Complete the first table with all pairs of factors of -100 that would result in positive values of b, the second table with factors that would result in negative values of b, and the third table with factors that would result in a zero value of b.

   For each pair of factors, state the b value they produce. (Use as many rows as there are pairs of factors. You may not need all the rows.)

Positive value of b

Factor 1	Factor 2	b (positive)

Negative value of b

Factor 1	Factor 2	b (negative)

Zero value of b

Factor 1	Factor 2	b (zero)

Step 2

- Before students answer the last question, consider displaying the completed tables for all to see and inviting students to observe any patterns or structure in them. Discuss questions such as:
  - "How are the pairs of factors of 100 like or unlike those of -100?" (The numbers are the same, but the signs are different.)
  - "In the first two tables, what do you notice about factor pairs that give positive b values?" (They are both positive.) "What about factor pairs that give negative b values?" (They are negative.)
  - "In the next two tables, what do you notice about factor pairs that give positive b values?" (One factor is positive, and the other is negative. The positive number has a greater absolute value.)
  - "What about factor pairs that give negative b values?" (One factor is positive, and the other is negative. The positive number has a greater absolute value.)
  - "What do you notice about the pair of factors that give a b value of 0?" (They are opposites.)
- Encourage students to use these insights to answer the last question, working collaboratively with partners.

3. Write each expression in factored form:

   a. $x^2 - 25x + 100$ b. $x^2 + 15x - 100$

   c. $x^2 - 15x - 100$ d. $x^2 + 99x - 100$

Are You Ready For More?

How many different integers b can you find so that the expression $x^2 + 10x + b$ can be written in factored form?

Step 3

- Ask students to share their responses to the last question. Discuss how the work in the first two questions helped them rewrite the quadratic expressions in factored form.
- Highlight that the sign of the constant term can help us anticipate the signs of the numbers in the factors, making it a helpful first step in rewriting quadratic expressions in factored form. If the constant term is positive, the factors will have two negative numbers or two positive numbers. If the constant term is negative, the factors will have one positive number and one negative number. From there, we can determine which two factors give the specified value of b in $x^2 + bx + c$.

RESPONSIVE STRATEGY

As an alternative to Step 3, have students jot down some initial notes in response to the question, "How would you explain to a classmate who is absent today how to . . . ?" Then, arrange students in pairs and invite them to share their ideas and give and get feedback. Arrange students for a second paired conversation with a new partner to give and get feedback on their ideas. Lastly, give students time to revise their notes into an improved draft that is stronger and clearer than their initial writing.

Stronger and Clearer Each Time (MLR1)

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to encourage students to connect their number sense to the factoring process. Over time, students will develop intuition about which combination of numbers will or will not work to produce a factored form equivalent to the given standard form.

Choose whether students should have an opportunity to reflect in their workbooks or talk through these with a partner.

To help students consolidate the observations and insights from this lesson, consider asking them to describe to a partner or write down their responses to prompts such as:

- "How would you explain to a classmate who is absent today how to rewrite $x^2 + 16x - 36$ in factored form?"
- "How would you explain how to rewrite $x^2 - 5x - 24$ in factored form?"
- "Suppose you are rewriting the quadratic expression $x^2 + bx + c$ in factored form $(x+m)(x+n)$. How will the factors be different when the c is positive versus when c is negative?"

PLANNING NOTES

Student Lesson Summary and Glossary

When we rewrite expressions in factored form, it is helpful to remember that:

- Multiplying two positive numbers or two negative numbers results in a positive product.
- Multiplying a positive number and a negative number results in a negative product.

This means that if we want to find two factors whose product is 10, the factors must be both positive or both negative. If we want to find two factors whose product is -10, one of the factors must be positive and the other negative.

Suppose we wanted to rewrite $x^2 - 8x + 7$ in factored form. Recall that subtracting a number can be thought of as adding the opposite of that number, so that expression can also be written as $x^2 + -8x + 7$. We are looking for two numbers that:

- Have a product of 7. The candidates are 7 and 1, and -7 and -1.
- Have a sum of -8. Only -7 and -1 from the list of candidates meet this condition.

The factored form of $x^2 - 8x + 7$ is therefore $(x + -7)(x + -1)$ or, written another way, $(x-7)(x-1)$.

To write $x^2 + 6x - 7$ in factored form, we would need two numbers that:

- Multiply to make -7. The candidates are 7 and -1, and -7 and 1.
- Add up to 6. Only 7 and -1 from the list of candidates add up to 6.

The factored form of $x^2 + 6x - 7$ is $(x + 7)(x - 1)$.

Cool-down: The Missing Symbols (*5 minutes*)

Addressing: NC.M1.A-SSE.3, NC.M1.A-REI.4

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

Here are pairs of equivalent expressions in standard form and factored form. Find the missing symbols and numbers.

1. $x^2 \boxed{} 16x \boxed{} 17$ and $(x + 1)(x - 17)$
2. $x^2 \boxed{} 16x \boxed{} 17$ and $(x - 1)(x + 17)$
3. $x^2 + 3x - 28$ and $(x + \boxed{})(x - \boxed{})$
4. $x^2 - 12x - 28$ and $(x \boxed{} \boxed{})(x \boxed{} \boxed{})$

Student Reflection:

a. Today I struggled with…

b. To be successful with this, I need…

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Check in with your norms and routines. Are they promoting engagement from all of your students? Are there any adjustments you might make so that all students do math tomorrow?

Practice Problems

1. Find two numbers that do the following. If you get stuck, try listing all the factors of the first number.
   a. multiply to -40 and add to -6
   b. multiply to -40 and add to 6
   c. multiply to -36 and add to 9
   d. multiply to -36 and add to -5
2. Create a diagram to show that $(x-5)(x+8)$ is equivalent to $x^2+3x-40$.
3. Write a $+$ or a $-$ sign in each box so the expressions on each side of the equal sign are equivalent.
   a. $(x\boxed{}18)(x\boxed{}3)=x^2-15x-54$
   b. $(x\boxed{}18)(x\boxed{}3)=x^2+21x+54$
   c. $(x\boxed{}18)(x\boxed{}3)=x^2+15x-54$
   d. $(x\boxed{}18)(x\boxed{}3)=x^2-21x+54$
4. Match each quadratic expression in standard form with its equivalent expression in factored form.

a. $x^2-2x-35$	1. $(x+5)(x+7)$
b. $x^2+12x+35$	2. $(x-5)(x-7)$
c. $x^2+2x-35$	3. $(x+5)(x-7)$
d. $x^2-12x+35$	4. $(x-5)(x+7)$

5. Rewrite each expression in factored form. If you get stuck, try drawing a diagram.

   a. $x^2 - 3x - 28$

   b. $x^2 + 3x - 28$

   c. $x^2 + 12x - 28$

   d. $x^2 - 28x - 60$

6. Which equation has exactly one solution?

   a. $x^2 = -4$
   b. $(x+5)^2 = 0$
   c. $(x+5)(x-5) = 0$
   d. $(x+5)^2 = 36$

   (From Unit 7, Lesson 20)

7. Elena solves the equation $x^2 = 7x$ by dividing both sides by x to get $x = 7$. She says the solution is 7.

   Lin solves the equation $x^2 = 7x$ by rewriting the equation to get $x^2 - 7x = 0$. When she graphs the equation $y = x^2 - 7x$, the x-intercepts are $(0,0)$ and $(7,0)$. She says the solutions are 0 and 7.

   Do you agree with either of them? Explain or show how you know.

   (From Unit 7, Lesson 20)

8. Add or subtract:

   a. $(4x^2 + 3x + 7) + (8x^2 - 5x - 2)$

   b. $(m^2 - 9) + (3m^2 - 4m + 16)$

   c. $(7.2h^2 + 3h - 3.5) - (2.4h^2 - 5h + 1)$

   (From Unit 7, Lessons 14 and 15)

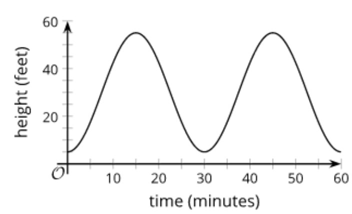
9. A bacteria population, p, can be represented by the equation $p = 100,000 \cdot (\frac{1}{4})^d$, where d is the number of days since it was measured.

   a. What was the population 3 days before it was measured? Explain how you know.

   b. What is the last day when the population was more than 1,000,000? Explain how you know.

   (From Unit 6)

10. The graph represents function $H(t)$, defined as the height of a passenger car on a ferris wheel, in feet, as a function of time, in seconds.



   Use the graph to help you:

   a. Find $H(0)$.

   b. Does $H(t) = 0$ have a solution? Explain how you know.

   c. Describe the domain of the function.

   d. Describe the range of the function.

   (From Unit 5)

Lesson 23: Rewriting Quadratic Expressions in Factored Form (Part Three)

PREPARATION

Lesson Goals	Learning Targets
• Understand that multiplying a sum and a difference, $(x+m)(x-m)$, results in a quadratic with no linear term and explain (orally) why this is the case. • When given quadratic expressions with no linear term, write equivalent expressions in factored form.	• I can describe and justify the structure of the results when multiplying a sum and a difference, $(x+m)(x-m)$. • When given quadratic expressions in the form of x^2+bx+c, I can rewrite them in factored form.

Lesson Narrative

So far, the quadratic expressions that students have transformed from standard form to factored form have at least a squared term and a linear term. In this lesson, students encounter quadratic expressions without a linear term and consider how to write them in factored form.

Students begin by studying numerical examples and noticing that expressions such as $(20+1)(20-1)$ and 20^2-1^2 (which is a difference of two squares) are equivalent. Through repeated reasoning, students are able to generalize the equivalence of these two forms as $(x+m)(x-m)=x^2-m^2$ (MP8). Then, they make use of the structure relating the two expressions to rewrite expressions (MP7) from one form to the other.

Along the way, they encounter a variety of quadratic expressions that can be seen as differences of two squares, including those in which the squared term has a coefficient other than 1, or expressions that involve fractions.

Students also consider why a difference of two squares (such as x^2-25) can be written in factored form, but a sum of two squares (such as x^2+25) cannot be, even though both are quadratic expressions with no linear term.

After this lesson, students will have the tools they need to solve factorable quadratic equations given in standard form by first rewriting them in factored form. That work begins in the next lesson.

What is the main purpose of this lesson? What is the one thing you want your students to take away from this lesson?

Focus and Coherence

Building On	Building Towards
NC.6.EE.3: Apply the properties of operations to generate equivalent expressions without exponents. **NC.7.EE.1:** Apply properties of operations as strategies to: • Add, subtract, and expand linear expressions with rational coefficients. • Factor linear expression with an integer GCF.	**NC.M1.A-SSE.3:** Write an equivalent form of a quadratic expression ax^2+bx+c, where a is an integer, by factoring to reveal the solutions of the equation or the zeros of the function the expression defines. **NC.M1.A-REI.4:** Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L23 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.7.EE.1

Students will discuss the addition of opposite terms in this bridge, leading into multiplying binomials that result in a difference of perfect squares. In those expressions, the opposite terms add to 0, and students will be able to develop that relationship in the bridge.

Student Task Statement

Diego and Noah are trying to remember how to simplify $7x+-7x$. Diego thinks the answer is x, and Noah thinks the answer is 0. Do you agree with Diego, Noah, both, or neither? Explain your answer.

DO THE MATH	PLANNING NOTES

Warm-up: Products of Large-ish Numbers (*5 minutes*)

Instructional Routine: Math Talk	
Building On: NC.6.EE.3	

This *Math Talk* prompts students to recall strategies for multiplying mentally, which encourages them to look for and use structure in the expressions (MP7).

Each expression can be evaluated in different ways. For example, $19 \cdot 21$ can be viewed as $19 \cdot (20 + 1)$, as $21 \cdot (20 - 1)$, or as $(20 - 1) \cdot (20 + 1)$, among other ways. Reasoning flexibly about the structure of numerical expressions encourages students to do the same when rewriting quadratic expressions in this lesson and beyond.

- Display one problem at a time.
- Give students quiet think time for each problem and ask them to give a signal when they have an answer and a strategy.
- Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:
  - "Who can restate ______'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone solve the problem in a different way?"
  - "Does anyone want to add on to ______'s strategy?"
  - "Do you agree or disagree? Why?"
- Keep all problems displayed throughout the talk.

RESPONSIVE STRATEGY

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Find each product mentally.

1. $9 \cdot 11$
2. $19 \cdot 21$
3. $99 \cdot 101$
4. $109 \cdot 101$

DO THE MATH

PLANNING NOTES

Activity 1: Can Products Be Written as Differences? (*15 minutes*)

Instructional Routine: Critique, Correct, Clarify (MLR3)	
Building Towards: NC.M1.A-SSE.3; NC.M1.A-REI.4	

In this activity, students multiply expressions of the form $(x+m)$ and $(x-m)$. Through repeated reasoning, they discover that the expanded product of such factors can be expressed as a difference of two square numbers: x^2-m^2 (MP8). Students use diagrams and the distributive property to make sense of their observations.

In the last question, they have an opportunity to notice that an expression in the form $(x+m)^2$ is not equivalent to x^2+m^2, to discourage overgeneralizing. Later in the lesson, they will use their understanding of the structure relating the equivalent expressions to transform quadratic expressions in standard form into factored form and vice versa (MP7).

To answer the first two questions, some students may simply evaluate the expressions rather than reasoning about their structure. For example, to see if $(10+3)(10-3)$ is equivalent to 10^2-3^2, they may calculate $13 \cdot 7$ and $100-9$ and see that both are 91. Ask students to see if they could show that this is not a coincidence. Could they show, for example, that $(50+4)(50-4)$ would also be equivalent to 50^2-4^2?

Step 1

- Ask students to arrange in pairs or use visibly random grouping. Students will remain in these groups for the remainder of the lesson.
- Provide quiet time for students to work on problems 1 and 2 independently. After independent work time, have students confer with their partner on their work so far and then complete the task collaboratively.

RESPONSIVE STRATEGY

Use color coding and annotations to highlight connections between representations in a problem. For example, use color coding to highlight the opposites that occur in the linear term after the products are distributed. Then, use annotations to illustrate the sum is zero, which results in no linear term. Encourage students to annotate their work while working independently.

Supports accessibility for: Visual-spatial processing

M **Monitoring Tip:** As students work, look for those who expand the factored expression using the distributive property and leave the linear terms in the partial products uncombined in order to show the equivalence of the two expressions.

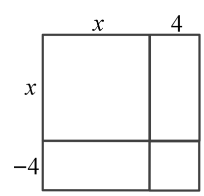
Advancing Student Thinking: Some students may struggle to generalize the pattern after just a few examples. Before starting the activity synthesis, provide additional factored expressions for students to expand, for instance: $(x+3)(x-3)$, $(2x+1)(2x-1)$, $(4x+5)(4x-5)$, and $(6-x)(6+x)$.

Student Task Statement

1. Clare claims that $(10+3)(10-3)$ is equivalent to 10^2-3^2 and $(20+1)(20-1)$ is equivalent to 20^2-1^2. Do you agree? Show your reasoning.
2.
   a. Use your observations from the first question and evaluate $(100+5)(100-5)$. Show your reasoning.
   b. Check your answer by computing $105 \cdot 95$.
3. Is $(x+4)(x-4)$ equivalent to x^2-4^2?

   Support your answer:

   a. With a diagram
   b. Without a diagram



4. Is $(x+4)^2$ equivalent to x^2+4^2? Support your answer, either with or without a diagram.

Are You Ready For More?

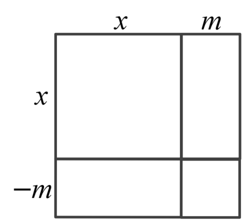
1. Explain how your work in the previous questions can help you mentally evaluate $22 \cdot 18$ and $45 \cdot 35$.

2. Here is a shortcut that can be used to mentally square any two-digit number. Let's take 83^2, for example.
   - 83 is $80 + 3$.
   - Compute 80^2 and 3^2, which give 6,400 and 9. Add these values to get 6,409.
   - Compute $80 \cdot 3$, which is 240. Double it to get 480.
   - Add 6,409 and 480 to get 6,889.

   Try using this method to find the squares of some other two-digit numbers. (With some practice, it is possible to get really fast at this!) Then, explain why this method works.

Step 2

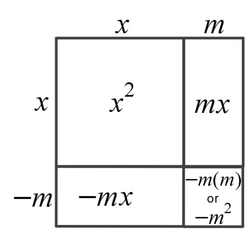
- Use the *Critique, Correct, Clarify* routine to help students evaluate and improve upon the mathematical argument of others.
  - Before students share their explanations for the last question, display a fictitious student's incorrect response. For example, "$(x+4)^2$ is equivalent to $x^2 + 4^2$ because when you square an expression you square each term." Invite two or three students to share ideas about what parts of the response are incorrect, incomplete, or unclear. As students share ideas, annotate the displayed response to indicate which parts need improvement.



  - Give students 2 minutes to work individually or in pairs to write a second draft of the response that is correct and clear. Listen for students who use diagrams or examples to illustrate the error in the author's reasoning. Listen also for the language students use to explain how to use the distributive property to square an expression.
  - Invite one student or pair to read their second draft aloud. Scribe as they share. Invite the rest of the class to help revise the response further with additional ideas and clearer wording. This process of whole-class collective editing generates a third draft response that can be used as a reference.

Step 3

- Invite previously identified students to share additional responses and reasoning.
- To help students generalize their observations, display the expression $(x+m)(x-m)$ and a blank diagram that can be used to visualize the expansion of the factors. Ask students what expressions go in each rectangle.



- Illustrate that when the terms in each factor are multiplied out, the resulting expression has two squares, one with a positive coefficient and the other with a negative coefficient (x^2 and m^2) and two linear terms that are opposites (mx and $-mx$). Because the sum of mx and $-mx$ is 0, what remains is the difference of x^2 and m^2, or $x^2 - m^2$. There is now no linear term.
- Emphasize that knowing this structure allows us to rewrite into factored form any quadratic expression that has no linear term and is a difference of a squared variable and a squared constant. For example, we can write $x^2 - 9$ as $(x+3)(x-3)$ because we know that when the latter is expanded, the result is $x^2 - 9$.

- Use the last question to point out the importance of paying attention to the particulars of the structure of these expressions (the subtraction in the first expression, the presence of both addition and subtraction in the second). For example, we can't use any patterns observed in this activity to rewrite $x^2 + 9$ in factored form.

DO THE MATH	PLANNING NOTES

Activity 2: What If There Is No Linear Term? (*10 minutes*)

Instructional Routine: Discussion Supports (MLR8) - Responsive Strategy
Building Towards: NC.M1.A-SSE.3; NC.M1.A-REI.4

In this activity, students use the insights from the previous activity to write equivalent quadratic expressions in standard form and factored form. They see that when a quadratic expression in standard form is a difference of two squares (a squared variable with a coefficient that is also a perfect square, a^2x^2, and a squared constant, m^2) and has no linear term, the factored form is $(ax + m)(ax - m)$.

Students also notice that when a quadratic expression is a sum (instead of a difference) of a squared variable and a squared constant, it cannot be written in factored form.

Step 1

- In the same partner groups, have students select who will be Partner A and who will be Partner B.
- Ask students to write as many equivalent expressions as time permits while aiming to complete at least the first six rows and the last row of the table. Make sure there will be 4–5 minutes for the discussion in Step 2.

***Advancing Student Thinking*:** Some students may struggle to see the numbers in the expressions in standard form as perfect squares. Prompt them to create a list or table of square numbers ($1^2 = 1$, $2^2 = 4$, $3^2 = 9$, and so on) to have as a handy reference. Others may benefit by rewriting both terms as squares before writing the factored form. Demonstrate how to rewrite $49x^2 - 81$ as $(7x)^2 - (9)^2$ and $\frac{1}{4}x^2 - 25$ as $(\frac{1}{2}x)^2 - (5)^2$.

Student Task Statement

Each row has a pair of equivalent expressions.

Complete the table.

If you get stuck, consider drawing a diagram. (Heads up: one of them is impossible.)

Partner	Factored form	Standard form
Partner A	$(x-10)(x+10)$	
Partner B	$(2x+1)(2x-1)$	
Partner A	$(4-x)(4+x)$	
Partner B		x^2-81
Partner A		$49-y^2$
Partner B		$9z^2-16$
Partner A		$25t^2-81$
Partner B	$(c+\frac{2}{5})(c-\frac{2}{5})$	
Partner A		$\frac{49}{16}-d^2$
Partner B	$(x+5)(x+5)$	
Partner A		x^2-6
Partner B		x^2+100

Step 2

- Consider displaying the incomplete table for all to see and fill in student responses that align to the questions below. Give the class time to examine the responses and to bring up any disagreements or questions. Discuss with students:
  - “How can we check if the expression in factored form is indeed equivalent to the given expression in standard form?” (We can expand the factored expression by applying the distributive property and see if it gives the expression in standard form.)
  - “Some of the expressions show a squared variable subtracted from a number instead of the other way around. Can we still write an equivalent expression in factored form?” (Yes. As long as the expression in standard form can be written as a difference of two squares, it can be written in factored form.)

RESPONSIVE STRATEGY

Use this routine to support whole-class discussion. After each student shares, provide the class with the following sentence frames to help them respond: “I agree because . . .” or “I disagree because . . . .” If necessary, revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.

Discussion Supports (MLR8)

- "What if the number is not a perfect square, for example: $x^2 - 5$?" (We can still write it in factored form by thinking about what number can be squared to get 5. Both $\sqrt{5}$ and $-\sqrt{5}$ can be squared to get 5. Regardless of which number we use, the factored form is $(x+\sqrt{5})(x-\sqrt{5})$.)
- "Why can $x^2 - 100$ be written in factored form but $x^2 + 100$ cannot?" (One possible approach is to rewrite the former as $x^2 + 0x - 100$. We learned previously that to write this expression in factored form, we would need to look for two numbers whose product is -100 and whose sum is 0. The numbers 10 and -10 meet this requirement. For $x^2 + 0x - 100$, however, we need two numbers whose product is 100 and whose sum is 0. No such numbers exist. To have a sum of 0, one number has to be positive and the other negative, so their product can't be positive 100.)

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to use patterns in numerical differences of perfect squares to generalize and extend those patterns to differences of perfect squares containing variables.

Choose whether students should have an opportunity to reflect in their workbooks or talk through these with a partner.

To help students consolidate and articulate their understanding of the relationship between $(ax+m)(ax-m)$ and $a^2x^2 - m^2$, ask them to reflect, in writing or by talking to a partner, on questions such as:

- "The expression $(x+4)(x-4)$ has a sum and a difference, and so does $(x+8)(x-2)$. When expanded into standard form, why does one have a linear term but not the other?" (Multiplying out $(x+4)(x-4)$ gives two linear terms that are opposites, $4x$ and $-4x$, which add up to 0, so the linear term disappears. Multiplying out $(x+8)(x-2)$ also gives two linear terms, $8x$ and $-2x$. Because they are not opposites, their sum is not 0, so the linear term remains.)
- "Can $\frac{1}{4} - 100m^2$ be seen as a difference of two squares? Can it be written in factored form? If so, what would it be?" (Yes. $\frac{1}{2}$ squared is $\frac{1}{4}$ and $(10m)^2$ is $100m^2$. The factored form is $(\frac{1}{2}+10m)(\frac{1}{2}-10m)$.)
- "Think of another example of a quadratic expression in factored form that, when rewritten in standard form, is a difference of two squares and does not have a linear term. What is the expression in standard form?"

PLANNING NOTES

Student Lesson Summary and Glossary

Sometimes expressions in standard form don't have a linear term. Can they still be written in factored form?

Let's take $x^2 - 9$ as an example. To help us write it in factored form, we can think of it as having a linear term with a coefficient of 0: $x^2 + 0x - 9$. (The expression $x^2 - 0x - 9$ is equivalent to $x^2 - 9$ because 0 times any number is 0, so $0x$ is 0.)

We know that we need to find two numbers that multiply to make -9 and add up to 0. The numbers 3 and -3 meet both requirements, so the factored form is $(x+3)(x-3)$.

To check that this expression is indeed equivalent to $x^2 - 9$, we can expand the factored expression by applying the distributive property: $(x+3)(x-3) = x^2 - 3x + 3x + (-9)$. Adding $-3x$ and $3x$ gives 0, so the expanded expression is $x^2 - 9$.

In general, a quadratic expression that is a difference of two squares and has the form $a^2 - b^2$ can be rewritten as $(a+b)(a-b)$.

Here is a more complicated example: $49 - 16y^2$. This expression can be written as $7^2 - (4y)^2$, so an equivalent expression in factored form is $(7+4y)(7-4y)$.

What about $x^2 + 9$? Can it be written in factored form?

Let's think about this expression as $x^2 + 0x + 9$. Can we find two numbers that multiply to make 9 but add up to 0? Here are factors of 9 and their sums:

- 9 and 1, sum: 10
- -9 and -1, sum: -10
- 3 and 3, sum: 6
- -3 and -3, sum: -6

For two numbers to add up to 0, they need to be opposites (a negative and a positive), but a pair of opposites cannot multiply to make positive 9, because multiplying a negative number and a positive number always gives a negative product.

Because there are no numbers that multiply to make 9 and also add up to 0, it is not possible to write $x^2 + 9$ in factored form using the kinds of numbers that we know about.

Cool-down: Can These Be Rewritten in Factored Form? (*5 minutes*)

Building Towards: NC.M1.A-SSE.3; NC.M1.A-REI.4

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

Write each expression in factored form. If it is not possible, write "not possible."

1. $a^2 - 36$
2. $49 - 25b^2$
3. $c^2 + 9$
4. $\frac{100}{81} - 16d^2$

Student Reflection:

When working with a sum and a difference, I am more comfortable converting (*standard to factored form* or *factored form to standard form*) ____________ because…

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Think about a time you recently made a mistake during math class. How did you leverage your mistake to show students that mistakes are just learning in process?

Practice Problems

1. Match each quadratic expression given in factored form with an equivalent expression in standard form. One expression in standard form has no match.

   a. $(y+x)(y-x)$
   b. $(11+x)(11-x)$
   c. $(x-11)(x+11)$
   d. $(x-y)(x-y)$

   1. $121-x^2$
   2. $x^2+2xy-y^2$
   3. y^2-x^2
   4. $x^2-2xy+y^2$
   5. x^2-121

2. Both $(x-3)(x+3)$ and $(3-x)(3+x)$ contain a sum and a difference and have only 3 and x in each factor.

   If each expression is rewritten in standard form, will the two expressions be the same? Explain or show your reasoning.

3.
   a. Show that the expressions $(5+1)(5-1)$ and 5^2-1^2 are equivalent.

   b. The expressions $(30-2)(30+2)$ and 30^2-2^2 are equivalent and can help us find the product of two numbers. Which two numbers are they?

   c. Write $94 \cdot 106$ as a product of a sum and a difference and then as a difference of two squares. What is the value of $94 \cdot 106$?

4. Write each expression in factored form. If not possible, write “not possible.”

   a. $x^2 - 144$
   b. $x^2 + 16$
   c. $25 - x^2$
   d. $b^2 - a^2$
   e. $100 + y^2$

5. Create a diagram to show that $(x-3)(x-7)$ is equivalent to $x^2 - 10x + 21$.

   (From Unit 7, Lesson 21)

6. Select **all** the expressions that are equivalent to $8 - x$.

   a. $x - 8$
   b. $8 + (-x)$
   c. $-x - (-8)$
   d. $-8 + x$
   e. $x - (-8)$
   f. $x + (-8)$
   g. $-x + 8$

   (From Unit 7, Lesson 21)

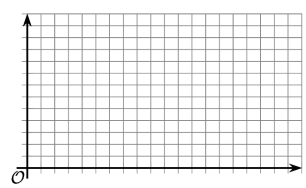
7. What are the solutions to the equation $(x-a)(x+b) = 0$?

   a. a and b
   b. $-a$ and $-b$
   c. a and $-b$
   d. $-a$ and b

   (From Unit 7, Lesson 19)

8. The function f, defined by $f(t) = 1{,}000 \cdot (1.07)^t$, represents the amount of money in a bank account t years after it was opened.

   a. How much money was in the account when it was opened?
   b. Sketch a graph of f.
   c. When does the account value reach \$2,000?

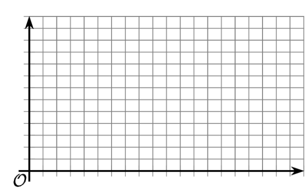


   (From Unit 6)

9. Mai fills a tall cup with hot cocoa, 12 centimeters in height. She waits 5 minutes for it to cool. Then, she starts drinking in sips, at an average rate of 2 centimeters of height every 2 minutes, until the cup is empty.

   The function C gives the height of hot cocoa in Mai’s cup, in centimeters, as a function of time, in minutes.

   a. Sketch a possible graph of C. Be sure to include a label and a scale for each axis.
   b. What quantities do the domain and range represent in this situation?
   c. Describe the domain and range of C.



   (From Unit 5)

10. Select all expressions equivalent to $11 - 5x$.

   a. $6x + 11 - x$
   b. $6 - 6x + 5 + x$
   c. $-2(3x-5) + (x+1)$
   d. $2(3x+5) - (11x-1)$

   (Addressing NC.7.EE.1)

Lesson 24: Solving Quadratic Equations by Using Factored Form

PREPARATION

Lesson Goals	Learning Targets
• Recognize that the number of solutions to a quadratic equation can be revealed when the equation is written as an $expression\ in\ factored\ form = 0$. • Use factored form and the zero product property to solve quadratic equations.	• I can recognize quadratic equations that have 0, 1, or 2 real solutions when they are written in factored form. • I can solve a quadratic equation using factored form.

Lesson Narrative

In this lesson, students apply what they learned about transforming expressions into factored form to make sense of quadratic equations and persevere in solving them (MP1). They see that rearranging equations so that one side of the equal sign is 0, rewriting the expression in factored form, and then using the zero product property make it possible to solve equations that they previously could only solve by graphing. These steps also allow them to easily see—without graphing and without necessarily completing the solving process—the number of solutions that the equations have.

Share some ways you see this lesson connecting to previous lessons in this unit. What connections will you want to make explicit?

Focus and Coherence

Building On	Addressing
NC.8.EE.7: Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable. • Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions. • Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides. **NC.M1.A-APR.3:** Understand the relationships among the factors of a quadratic expression, the solutions of a quadratic equation, and the zeros of a quadratic function.	**NC.M1.A-SSE.3:** Write an equivalent form of a quadratic expression $ax^2 + bx + c$, where a is an integer, by factoring to reveal the solutions of the equation or the zeros of the function the expression defines. **NC.M1.A-REI.1:** Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning. **NC.M1.A-REI.4:** Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*10 minutes*)
  - Graphing technology is required in this activity. Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L24 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.EE.7

The purpose of this bridge is to consider the number of solutions of a linear equation. This idea will be useful when students explore the number of possible solutions to a quadratic equation.

Student Task Statement

For each equation, identify if there are no solutions, one solution, or infinitely many solutions.

1. $5x+7=5x+7$
2. $3(x+4)=3x+11$
3. $2x-6=8x$

DO THE MATH

PLANNING NOTES

Warm-up: Why Would You Do That? (*10 minutes*)

Building Towards: NC.M1.A-REI.4

In this activity, students find at least one solution of $x^2-2x-35=0$ by substituting different values of x, evaluating the expression, and checking if it has a value of 0. Experiencing this inefficient method puts students in a better position to appreciate why it may be desirable to write $x^2-2x-35$ in factored form and use the zero product property.

Step 1

- Provide students with a minute of quiet work time to complete problems 1 and 2.
- Ask students if anyone found a value that made the expression equal 0.
  - If no one has found a solution, record their responses in a table of values and provide additional time for them to keep looking.
  - If a solution was found, ask students to share the value and how they know it is a solution.
- Provide students a couple of minutes to look for the other value that makes the expression equal to 0.

Student Task Statement

Let's try to find at least one solution to $x^2 - 2x - 35 = 0$.

1. Choose a whole number between 0 and 10.
2. Evaluate the expression $x^2 - 2x - 35$, using your number for x.
3. If your number doesn't give a value of 0, look for someone in your class who may have chosen a number that does make the expression equal 0. Which number is it?
4. There is another number that would make the expression $x^2 - 2x - 35$ equal 0. Can you find it?

Step 2

- If a student found that -5 makes the expression equal 0, ask them to demonstrate that $(-5)^2 - 2(-5) - 35$ equals 0.
- Discuss with students:
  - "Can $x^2 - 2x - 35$ be written in factored form? What are the factors?" (Yes: $(x-7)$ and $(x+5)$.)
  - "If $x^2 - 2x - 35$ can be written as $(x-7)(x+5)$, can we solve $(x-7)(x+5) = 0$ instead?" (Yes.) "Will the solutions change if we use this equation?" (No. The equations are equivalent, so they have the same solutions.)
  - "Why might someone choose to rewrite $x^2 - 2x - 35 = 0$ and solve $(x-7)(x+5) = 0$ instead? (Because the expression is equal to 0, rewriting it in factored form allows us to use the zero product property to find both solutions. It may be more efficient than substituting and evaluating many values for x. It also makes it possible to see how many solutions there are, which is not always easy to tell when the quadratic expression is in standard form.)

DO THE MATH	PLANNING NOTES

Activity 1: Let's Solve Some Equations! (*10 minutes*)

Instructional Routines: Notice and Wonder; Discussion Supports (MLR8) - Responsive Strategy	
Addressing: NC.M1.A-REI.1; NC.M1.A-REI.4	

In this activity, students solve a variety of quadratic equations by integrating what they learned about rewriting quadratic expressions in factored form and their understanding of the zero product property. They begin by analyzing and explaining the steps in a solution strategy, and then applying their observations to solve other equations, both of which require sense making and perseverance (MP1). Students practice attending to precision (MP6) as they study solution steps and communicate what each step does or means.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Display the worked example in the activity statement for all to see.
- Provide students 1 minute of quiet think time to study the example. Then, using the *Notice and Wonder* routine, ask students to be prepared to share something they notice or wonder.
- Give students another minute to discuss their observations and questions with a partner.

Step 2

- Provide students time to work quietly on the first question. Encourage students to use relevant mathematical vocabulary in their explanations (such as "constant term," "squared term," "factored form," and "zero product property"). After a couple of minutes, tell students to check in with their partner and discuss their responses to question 1.
- Before students proceed to the second set of questions, pause for a class discussion. Ask students to explain each step of Tyler's solving process and record their explanations for all to see. Encourage students to use reasoning language as opposed to position language: for instance, say "they subtracted 99 from both sides" rather than "they moved the 99 over to the other side."
- Make sure students recognize that going from the second line to the third line in Tyler's work involves finding two factors of -99 that add to -2. Also discuss why there are two equations at the end. Students should recall that if two numbers multiply to equal 0, then one of the factors must be 0.
- Ask students to solve as many equations in the second question as time permits.

RESPONSIVE STRATEGY

Activate or supply background knowledge. Demonstrate how students can continue to use diagrams to rewrite equations in factored form by first rewriting each equation so that one side is equal to 0.

Supports accessibility for: Visual-spatial processing; Organization

Monitoring Tip: As students work, notice the equations many students find challenging and those on which errors are commonly made. Discuss these challenges and errors during Step 3.

Student Task Statement

1. To solve the equation $n^2 - 2n = 99$, Tyler wrote out the following steps. Analyze Tyler's work. Write down what they did in each step.

$n^2 - 2n = 99$	Original Equation
$n^2 - 2n - 99 = 0$	Step 1
$(n-11)(n+9) = 0$	Step 2
$n - 11 = 0$ or $n + 9 = 0$	Step 3
$n = 11$ or $n = -9$	Step 4

2. Solve each equation by rewriting it in factored form and using the zero product property. Show your reasoning.
   a. $x^2 + 8x + 15 = 0$
   b. $x^2 - 8x + 12 = 5$
   c. $x^2 - 10x - 11 = 0$
   d. $49 - x^2 = 0$
   e. $(x+4)(x+5) - 30 = 0$

Are You Ready For More?

Solve this equation and explain or show your reasoning.

$(x^2 - x - 20)(x^2 + 2x - 3) = (x^2 + 2x - 8)(x^2 - 8x + 15)$

Step 3

- Consider displaying the solutions for all to see and discussing only the equations that students found challenging and any common errors.
- The last equation is unlike most equations students have seen. Invite students to share how they solved that equation. Discuss questions such as:
  - "Was Tyler's first step necessary? Why or why not?" (Yes. The zero product property can not be used unless the product equals 0.)
  - "Can we use the zero product property to write $x + 4 = 0$ and $x + 5 = 0$? Why or why not?" (No. The zero product property can be applied only to products that equal 0. The expression $(x+4)(x+5) - 30$ has a product for one of its terms, but the expression itself is a difference.)
  - "How can it be solved, other than by graphing?" (We can expand the factored expressions using the distributive property and write an equivalent equation: $x^2 + 9x + 20 - 30 = 0$ or $x^2 + 9x - 10 = 0$. This last equation can then be written as $(x+10)(x-1) = 0$, which allows it to be solved.)

RESPONSIVE STRATEGY

Use this routine to support whole-class discussion. For each solution strategy that is shared about the last equation, ask students to restate what they heard using their own everyday language. Consider providing students time to restate what they hear to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others.

Discussion Supports (MLR8)

DO THE MATH	PLANNING NOTES

Activity 2: Revisiting Quadratic Equations with Only One Solution (*10 minutes*)

Instructional Routines: Graph It; Compare and Connect (MLR7) - Responsive Strategy	
Building On: NC.M1.A-APR.3	**Addressing:** NC.M1.A-SSE.3

This *Graph It* activity reinforces what students learned earlier about the connections between the solutions of a quadratic equation and the zeros of a quadratic function. Previously, students were given equations and asked to graph them to determine the number of solutions and their values. Here, they are prompted to work the other way around: to write an equation to represent a quadratic function with only one solution. To do so, students need to make use of the structure of the factored form and their knowledge of the zero product property (MP7).

Step 1

- Keep students in pairs.
- Provide students access to graphing technology such as Desmos.
- Provide time for students to work with their partner to complete problems 1 and 2.
- Ask students to compare their responses to problems 1 and 2 with another pair of students. If there is any disagreement, tell students to discuss their reasoning and try to come to an agreement.
- As groups finish comparing, ask students to resume working with their original partner and complete problems 3 and 4.

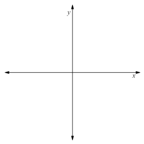
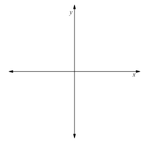
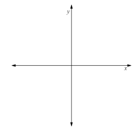
RESPONSIVE STRATEGY

Support effective and efficient use of tools and assistive technologies. Some students may benefit from a demonstration or access to step-by-step instructions for using Desmos.

Supports accessibility for: Organization; Memory; Attention

Student Task Statement

1. The other day, we saw that a quadratic equation can have 0, 1, or 2 solutions. Sketch graphs that represent three quadratic functions: one that has no zeros, one with 1 zero, and one with 2 zeros.

No zeros	One zero	Two zeros
		

2. Use graphing technology to graph the function defined by $f(x) = x^2 - 2x + 1$. What do you notice about the x-intercepts of the graph? What do the x-intercepts reveal about the function?

3. Solve $x^2 - 2x + 1 = 0$ by using the factored form and zero product property. Show your reasoning. What solutions do you get?

4. Write an equation to represent another quadratic function that you think will only have one zero. Graph it to check your prediction.

Step 2

- Invite students to share how they solved the equation algebraically. Next, invite students to share the equations they generated. Record and display them for all to see.
- Students most likely have written equations in the form of $g(x) = (x + m)(x + m)$. Ask students why the factored form, rather than the standard form, might have been preferred. Highlight that by using the same expression for the two factors, we know that the solution to $(x + m)(x + m) = 0$ will be a single number.
- Ask students to describe the graph of a quadratic function with one solution. Point out that this means that the function will have only one zero, and the graph of the function will have a single horizontal intercept.

RESPONSIVE STRATEGY

Use this routine to prepare students for the whole-class discussion. At the appropriate time, invite student pairs to create a visual display of their equation and graph of a quadratic function with only one zero for problem 4. Allow students time to quietly circulate and analyze at least two other visual displays in the room. Give students quiet think time to consider how the zero is represented in the equation and graph of the quadratic function. Next, ask students to return to their partner and discuss what they noticed. Listen for and amplify observations that connect the zero of the function with the x-intercept of the graph. This will help students make connections between the algebraic and graphical representation of quadratic functions.

Compare and Connect (MLR7)

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to understand how factoring a quadratic expression equal to zero can be used to solve quadratic equations. Also, students see how the factored form of the quadratic expression equal to zero can be used to identify the number of real solutions to the equation.

Choose whether students should first have an opportunity to reflect in their workbooks before talking these through with a partner.

Display a series of equations that, prior to this lesson, students could only solve by graphing. For instance:

- $x^2 + 3x - 18 = 0$
- $x^2 - 4 = 5x$
- $x(x - 7) = -6$
- $2x^2 - 9x + 10 = 0$
- $(x + 6)(x - 6) = 11$

Ask students to choose an equation that they think they could solve without graphing. Then, ask them to explain to a partner why they believe they could solve the equation.

Consider displaying $(x + 9)(x - 9) = 19$ for all to see and using it as an example: "I think I can solve $(x + 9)(x - 9) = 19$ because I know the expression on the left is equivalent to $x^2 - 9^2$, and I can rewrite the equation as $x^2 - 81 = 19$, which I can solve by rearranging the terms."

Then, ask if any of the equations appear to be unsolvable other than by graphing and why. Of the equations shown here, $2x^2 - 9x + 10 = 0$ is the only one that students aren't yet equipped to solve because the coefficient of x^2 is not 1. Students will begin looking at such equations in an upcoming lesson.

PLANNING NOTES

Student Lesson Summary and Glossary

Recently, we learned strategies for transforming expressions from standard form to factored form. In earlier lessons, we have also seen that when a quadratic expression is in factored form, it is pretty easy to find values of the variable that make the expression equal zero. Suppose we are solving the equation $x(x + 4) = 0$, which says that the product of x and $x + 4$ is 0. By the zero product property, we know this means that either $x = 0$ or $x + 4 = 0$, which then tells us that 0 and -4 are solutions.

Together, these two skills—writing quadratic expressions in factored form and using the zero product property when a factored expression equals 0—allow us to solve quadratic equations given in other forms. Here is an example:

$n^2 - 4n = 140$	Original equation
$n^2 - 4n - 140 = 0$	Subtract 140 from each side so the right side is 0
$(n - 14)(n + 10) = 0$	Rewrite in factored form
$n - 14 = 0$ or $n + 10 = 0$	Apply the zero product property
$n = 14$ or $n = -10$	Solve each equation

When a quadratic equation is written as $expression\ in\ factored\ form = 0$, we can also see the number of solutions the equation has.

In the example earlier, it was not obvious how many solutions there would be when the equation was $n^2-4n-140=0$. When the equation was rewritten as $(n-14)(n+10)=0$, we could see that there were two numbers that could make the expression equal 0: 14 and -10.

How many solutions does the equation $x^2-20x+100=0$ have?

Let's rewrite it in factored form: $(x-10)(x-10)=0$. The two factors are identical, which means that there is only one value of x that makes the expression $(x-10)(x-10)$ equal 0. The equation has only one solution: 10.

Cool-down: Conquering More Equations (*5 minutes*)

Addressing: NC.M1.A-REI.4

Cool-down Guidance: Press Pause
If students are still making substantial errors with factoring, spend some time reviewing student work from cool-downs and providing additional practice opportunities through practice problems.

Cool-down

Solve each equation by rewriting it in factored form and using the zero product property. Show your reasoning.

1. $x^2+12x+11=0$
2. $x^2-3=1$
3. $x^2-6x+7=-2$

Student Reflection:

In what ways have you encouraged others recently in math class? Who have you helped? How did that feel? In what ways can you support others?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

How did the work of the previous lessons lay the foundation for students to be successful in this lesson?

Practice Problems

1. Find **all** the solutions to each equation.

   a. $x(x-1)=0$
   b. $(5-x)(5+x)=0$
   c. $(2x+1)(x+8)=0$
   d. $(3x-3)(3x-3)=0$
   e. $(7-x)(x+4)=0$

2. Rewrite each equation in factored form and solve using the zero product property.

   a. $d^2-7d+6=0$
   b. $x^2+18x+81=0$
   c. $u^2+7u-60=0$
   d. $x^2+0.2x+0.01=0$

3. Here is how Elena solves the quadratic equation $x^2-3x-18=0$.

$$x^2-3x-18=0$$
$$(x-3)(x+6)=0$$
$$x-3=0 \text{ or } x+6=0$$
$$x=3 \text{ or } x=-6$$

   Is her work correct? If you think there is an error, explain the error and correct it.

   Otherwise, check her solutions by substituting them into the original equation and showing that the equation remains true.

4. Tyler is working on solving a quadratic equation, as shown here.

$$p^2-5p=0$$
$$p(p-5)=0$$
$$p-5=0$$
$$p=5$$

   They think that their solution is correct because substituting 5 for p in the original expression p^2-5p gives $5^2-5(5)$, which is $25-25$ or 0.

   Explain the mistake that Tyler made and show the correct solutions.

5. Which expression is equivalent to $x^2 - 7x + 12$?

   a. $(x+3)(x+4)$
   b. $(x-3)(x-4)$
   c. $(x+2)(x+6)$
   d. $(x-2)(x-6)$

   (From Unit 7, Lesson 21)

6. These quadratic expressions are given in standard form. Rewrite each expression in factored form. If you get stuck, try drawing a diagram.

   a. $x^2 + 7x + 6$ b. $x^2 - 7x + 6$ c. $x^2 - 5x + 6$ d. $x^2 + 5x + 6$

   (From Unit 7, Lesson 21)

7. Choose a statement to correctly describe the zero product property.

   If a and b are numbers, and $a \cdot b = 0$, then:

   a. Both a and b must equal 0.
   b. Neither a nor b can equal 0.
   c. Either $a = 0$ or $b = 0$.
   d. $a + b$ must equal 0.

   (From Unit 7, Lesson 19)

8. Select **all** the functions whose output values will eventually overtake the output values of function f defined by $f(x) = 25x^2$.

   a. $g(x) = 5(2)^x$
   b. $h(x) = 5^x$
   c. $j(x) = x^2 + 5$
   d. $k(x) = (\frac{5}{2})^x$
   e. $n(x) = 2x^2 + 5$

   (From Unit 7, Lesson 3)

9. (*Technology required*.) A moth population, p, is modeled by the equation $p = 500,000 \cdot (\frac{1}{2})^w$, where w is the number of weeks since the population was first measured.

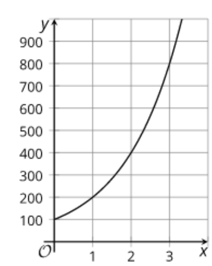
   a. What was the moth population when it was first measured?

   b. What was the moth population after 1 week? What about 1.5 weeks?

   c. Use technology to graph the population and find out when it falls below 10,000.

   (From Unit 6)

10. Here is a graph of the function f given by $f(x) = 100 \cdot 2^x$.



   Suppose g is the function given by $g(x) = 50 \cdot (1.5)^x$.

   Will the graph of g meet the graph of f for any positive value of x? Explain how you know.

   (From Unit 6)

11. For each equation, identify if there are no solutions, one solution, or infinitely many solutions.

   a. $3x + 9 = 3(x+9)$ b. $2(x+4) = 2(x+2) + 4$ c. $5x - 4 = 8x - 4$

   (Addressing NC.8.EE.7)

Lesson 25: Rewriting Quadratic Expressions in Factored Form (Part Four)

PREPARATION

Lesson Goals	Learning Targets
• Given a quadratic expression of the form $ax^2 + bx + c$, where a is not 1, write an equivalent expression in factored form. • Write a quadratic equation that represents a context and solve by factoring.	• When given quadratic expressions of the form $ax^2 + bx + c$, and a is not 1, I can write equivalent expressions in factored form. • I can use the factored form of a quadratic expression to answer questions about a situation.

Lesson Narrative

Up to this point, most quadratic expressions that students have transformed from standard form to factored form had a leading coefficient of 1: that is, they were in the form of $x^2 + bx + c$ because the squared term had a coefficient of 1. There were a few instances in which students rewrote expressions in standard form with a leading coefficient other than 1. Those expressions were differences of two squares, where there were no linear terms (for instance, $9x^2 - 64$ or $25x^2 - 9$). Students learned to rewrite these as $(3x + 8)(3x - 8)$ or $(5x + 3)(5x - 3)$, respectively.

In this lesson, students consider how to rewrite expressions in standard form where the leading coefficient is not 1 and the expression is not a difference of two squares. They notice that the same structure used to rewrite $x^2 + 5x + 4$ as $(x + 4)(x + 1)$ can be used to rewrite expressions such as $3x^2 + 8x + 4$, but the process is now a little more involved because the coefficient of x^2 has to be taken into account when finding the right pair of factors. The work here gives students many opportunities to look for and make use of structure (MP7).

In what ways will you encourage students to persevere in this lesson?

Focus and Coherence

Building On	Addressing
NC.4.OA.4: Find all factor pairs for whole numbers up to and including 50 to: ● Recognize that a whole number is a multiple of each of its factors. ● Determine whether a given number is a multiple of a given one-digit number. ● Determine if the number is prime or composite. **NC.M1.A-SSE.1a:** Interpret expressions that represent a quantity in terms of its context. **a.** Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.	**NC.M1.A-SSE.3:** Write an equivalent form of a quadratic expression $ax^2 + bx + c$, where a is an integer, by factoring to reveal the solutions of the equation or the zeros of the function the expression defines. **NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems. **NC.M1.A-REI.4:** Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
  - Timing A Blob of Water visual
- **Activity 3** (*Optional, 20 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U7.L25 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.4.OA.4

The purpose of this bridge is for students to practice finding factor pairs. In this lesson, a strategy for factoring quadratic expressions is to guess and check possible factors. Students will need to be able to identify the factor pairs of the coefficients to generate the possible factors.

Student Task Statement

A "factor pair" for a given number is a pair of whole numbers that, when multiplied, result in that number. For example, 5 and 4 are a factor pair of 20 because $5 \cdot 4 = 20$.

Find all factor pairs of each of the following numbers.

1. 12
2. 36
3. 5

DO THE MATH	PLANNING NOTES

Warm-up: Quadratic Expressions (*5 minutes*)

Instructional Routine: Which One Doesn't Belong?	
Building On: NC.M1.A-SSE.1a	

This *Which One Doesn't Belong?* warm-up prompts students to carefully analyze and compare quadratic expressions. In making comparisons, students need to look for common structure and have a reason to use language precisely (MP7, MP6). The activity also enables the teacher to hear the terminology students know and how they talk about characteristics of the different forms of expressions.

As students discuss in groups, listen for rationales that are based on the structure of the expressions. Select those students or groups to share their thinking during class discussion.

Step 1

- Ask students to arrange themselves into small groups or use visibly random grouping.
- Display the expressions for all to see.
- Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, ask each student to share their reasoning about why a particular item does not belong, and together find at least one reason each item doesn't belong.

Student Task Statement

Which one doesn't belong? Explain your reasoning.

a. $(x+4)(x-3)$	b. $3x^2-8x+5$
c. x^2-25	d. x^2+2x+3

Step 2

- Ask each group to share one reason why a particular expression doesn't belong.
- Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.
- During the discussion, ask students to explain the meaning of any terminology they use, such as standard form, factored form, linear term, or coefficient. Also, press students on unsubstantiated claims. For example, if they claim that "b" is the only one that cannot be written in factored form, ask them to show how they know.

DO THE MATH

PLANNING NOTES

Activity 1: A Little More Advanced (*15 minutes*)

Instructional Routines: Notice and Wonder; Discussion Supports (MLR8) - Responsive Strategy	
Building Towards: NC.M1.A-SSE.3; NC.M1.A-REI.4	

Most of the factored expressions students saw were of the form $(x+m)(x+n)$ or $x(x+m)$. In this activity, students work with expressions of the form $(px+m)(qx+n)$. They expand expressions such as $(2x+1)(x+10)$ into standard form and look for structure that would allow them to go in reverse (MP7): that is, to transform expressions of the form ax^2+bx+c, where a is not 1 (such as $2x^2+12x+10$), into factored form.

Going from factored form to standard form is fairly straightforward given students' experience with the distributive property. Going in reverse, however, is a bit more challenging when the coefficient of x^2 is not 1. With some guessing and checking, students should be able to find the factored form of the expressions in the second question, but they should also notice that this process is not straightforward.

Step 1

- Remind students that they have seen quadratic expressions such as $16t^2+800t+400$ and $5x^2+21x-20$, where the coefficient of the squared term is not 1. Solicit some ideas from students on how to write the factored form for expressions such as these.
- Keep students in their small groups and ask them to split up the work for completing the first table, with each group member rewriting one expression into standard form. Once all equivalent expressions have been written, ask students to discuss in their group what they *Notice and Wonder* about the expressions.

RESPONSIVE STRATEGY

Activate or supply background knowledge. Demonstrate how students can continue to use diagrams to rewrite expressions in which the coefficient of the squared term is not 1. Invite students to begin by generating a list of factors and to test them using the diagram. Encourage students to persist with this method, reiterating the fact that they are not necessarily expected to immediately recognize which factors will work without testing them. Allow students to use calculators to ensure inclusive participation.

Supports accessibility for: Visual-spatial processing; Organization

RESPONSIVE STRATEGY

Give students time to make sure that everyone in the group can describe something they noticed or wondered. Invite groups to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to speak and clarify their thinking, and will improve the quality of explanations shared during the whole-class discussion. Then make sure to vary who is selected to represent the work of the group, so that students get accustomed to preparing each other to fill that role.

Discussion Supports (MLR8)

- Display the incomplete table in the first question for all to see, and invite students to share the expanded expressions in standard form. Record the expressions in the right column.
- Ask students to share the things they noticed and wondered. Record and display their responses for all to see.
- If not mentioned by students, point out that each pair of factors starts with $3x$ and x, which multiply to make $3x^2$. Each pair of factors also has constant terms that multiply to make 4. The resulting expressions in standard form are all different, however, because using different pairs of factors of 4 and arranging them in different orders produce different expanded expressions.
- Ask students to keep these observations in mind as they work collaboratively to complete the second question.

M **Monitoring Tip:** As students use guess and check to factor the expressions, look for ways they organize their work and keep track of what they have tried and need to try. This may include writing out the factor pairs for a and c and/or writing out the different combinations of factors. Let them know that they may be asked to share their strategy later.

Advancing Student Thinking: Some students may not think to check their answers to the second question and stop as soon as they think of a pair of factors that give the correct squared term and constant term. Encourage them to check their answers with a partner by giving time to do so. Consider providing a non-permanent writing surface or extra paper so students could try out their guesses and check their work without worrying about having to erase if they make a mistake the first time or two.

Student Task Statement

Complete the tables so that each row has a pair of equivalent expressions. If you get stuck, try drawing a diagram.

1.

Factored form	Standard form
$(3x+1)(x+4)$	
$(3x+2)(x+2)$	
$(3x+4)(x+1)$	

2.

Factored form	Standard form
	$5x^2+21x+4$
	$3x^2+15x+12$
	$6x^2+19x+10$

Are You Ready For More?

Here are three quadratic equations, each with two solutions. Find both solutions to each equation, using the zero product property somewhere along the way. Show each step in your reasoning.

1. $x^2=6x$
2. $x(x+4)=x+4$
3. $2x(x-1)+3x-3=0$

Step 2

- Display for all to see the incomplete table in the second question. Select students to complete the missing expressions in standard form and to briefly explain their strategy. To rewrite $ax^2 + bx + c$, students are likely to have tried putting different factors of a and of c in the factored expression such that when the factors are expanded, they yield a linear term with the coefficient b.
- Facilitate a whole-group discussion that focuses on reasoning about the factors more generally. Discuss questions such as:
  - "To rewrite expressions such as $x^2 + bx + c$, we looked for two numbers that multiply to make c and add up to b. The expressions here are of the form $ax^2 + bx + c$. Are we still looking for two numbers that multiply to make c? Why or why not?" (Yes. The constant terms in the factored expression must multiply to make c.)
  - "Do we need to look for factors of a? Why or why not?" (Yes. Those factors will be the coefficient of x in the factored expressions. They must multiply to make a.)
  - "Are we still looking for two factors of c that add up to b? Why or why not?" (No. The value of b is no longer just the sum of the two factors of c because the two factors of a are now involved.) "How does this affect the rewriting process?" (It makes it more complicated, because now there are four numbers to contend with, and there are many more possibilities to consider.)

DO THE MATH	PLANNING NOTES

Activity 2: Timing A Blob of Water (*10 minutes*)

Instructional Routine: Three Reads (MLR6)	
Addressing: NC.A-SSE.3; NC.M1.A-CED.1; NC.M1.A-REI.4	

In this activity, students encounter a real-world situation modeled by a quadratic function. Students will use factoring to solve an equation and relate the solution to the situation. It is important to note that many quadratic functions that model real-world situations cannot be written in factored form. These often have rational or irrational solutions. In Math 1, students solve these equations by graphing. In future courses, they will explore more productive techniques for solving.

Step 1

- Use the *Three Reads* routine to support reading comprehension, without solving, for students.
  - First Read: Without displaying the problem, read the problem aloud to the class.
    - Ask students: “What is this situation about? What is going on here?” (An engineer is designing a water fountain.)
    - Spend less than a minute scribing their understanding of the situation in a place where all can see. Do not correct students, but do clarify any unfamiliar words. Visuals often help. In this situation, students may think the water fountain is referring to the fountains found inside buildings that provide drinking water. Display the Timing A Blob of Water visual of a decorative water fountain to help clarify.
  - Second Read: Display the problem and ask a student volunteer to read it aloud to the class again.
    - Ask: “What are the quantities and relationships in this situation?”
    - Again, spend less than a minute scribing student responses. Listen for, and amplify, the important quantities that vary in relation to each other in this situation: height of a drop of water, in meters, and time, in seconds.
  - Third Read: Invite students to read the problem again to themselves, or ask another student volunteer to read aloud.
    - Ask: “How might we approach the question being asked? What is the first thing you will do?
    - Spend 1–2 minutes scribing student ideas as they brainstorm possible starting points. Be sure to stop students who begin to share a complete solution; the goal is to crowdsource some starting points.
- Offer students the opportunity to work independently or with a partner to solve the problem.

Advancing Student Thinking: Students may not have factored an expression where the coefficient of the squared term is negative before. Encourage students to start by listing factor pairs of -5. Proceeding this way, some students may factor the expression as $(-5t-1)(t-2)$, and others may have come up with $(5t+1)(-t+2)$. If students are concerned about the discrepancy, ask them to expand each of the factored forms and verify that they are equivalent to the original expression, $-5t^2+9t+2$. This is a case where there are multiple equivalent factored forms.

Student Task Statement

An engineer is designing a fountain that shoots out drops of water. The nozzle from which the water is launched is 2 meters above the ground. It shoots out a drop of water at a vertical velocity of 9 meters per second.

Function h models the height in meters, h, of a drop of water t seconds after it is shot out from the nozzle. The function is defined by the equation $h(t)=-5t^2+9t+2$.

How many seconds until the drop of water hits the ground?

1. Write an equation that we could solve to answer the question.
2. Solve the equation by writing the expression in factored form and using the zero product property.
3. What is the answer to the original question? Explain how it is related to the solutions to the equation.

Step 2

- Ask students to share the factored form of the expression and briefly explain their strategy.
- Ask for students to share their answer to the question. Highlight responses that include why the answer to the question is the positive solution to the equation.

DO THE MATH	PLANNING NOTES

Activity 3: Making It Simpler (*Optional, 20 minutes*)

Instructional Routine: Discussion Supports (MLR8) - Responsive Strategy	
Addressing: NC.M1.A-SSE.3	

This activity is optional. It allows students to investigate another way (besides guessing and checking) to find the factors of quadratic expressions in standard form where the leading coefficient is not 1.

Earlier in this lesson, students discussed the challenge of factoring expressions in the form of ax^2+bx+c when $a \neq 1$. In this case, the value of b cannot be found by using the sum of the two factors of c because the factors of a are also involved. Students will explore a relationship between the standard form and its factored form to identify a new strategy that will help them to rewrite in factored form more efficiently.

In general, there is a relationship between ax^2+bx+c and its factored form.

- We need to find two factors of a. Let's call them p and q.
- We need to find two factors of c. Let's call them m and n.
- The factored form can be written as $(px+m)(qx+n)$.
- To see what p, q, m, and n should be, we apply the distributive property to the factors to get: $(px)(qx)+(np)x+(mq)x+mn$ or $(pq)x^2+(np+mq)x+mn$.
- We see that pq must multiply to make a and mn must multiply to make c. Notice that $a \cdot c = pqmn$.
- We can also see that $np+mq$ is equal to b.
- Thus, an important observation is that there are factors of $a \cdot c$ (np and mq) that add to b.

It is this relationship that students will explore in the activity as an additional strategy for factoring quadratics.

Step 1

- Display the expression $8x^2 + 22x + 15$.
- Ask students to factor the expression and keep track of the different trials before finding the correct two factors.
- Ask for volunteers to share the factors that did not work. Record these for all to see.

$$(8x+1)(x+15) \qquad (2x+1)(4x+15)$$
$$(8x+15)(x+1) \qquad (2x+15)(4x+1)$$
$$(8x+3)(x+5) \qquad (2x+5)(4x+3)$$
$$(8x+5)(x+3)$$

- Ask students for the factors of the expression. ($(2x+3)(4x+5)$)
- Tell students that the strategy of guessing and checking is not very efficient when there are several factors of a and c. In this activity, they will study another strategy that can simplify the process of rewriting quadratics into factored form.
- Ask students to arrange themselves into pairs or use visibly random grouping.
- Provide students time to complete questions 1 and 2.

Student Task Statement

1. Here are two strategies for expanding $(2x+3)(4x+5)$ to write the equivalent expression $8x^2 + 22x + 15$.

Expanding using the distributive property	Expanding using a diagram
$2x(4x+5) + 3(4x+5)$ $8x^2 + 10x + 12x + 15$ $8x^2 + 22x + 15$	$\begin{array}{c|c|c} & 2x & 3 \\ \hline 4x & 8x^2 & 12x \\ \hline 5 & 10x & 15 \end{array}$ $8x^2 + 22x + 15$

   a. Identify the two terms that add to $22x$ and find the product of the coefficients.

   b. Find the product of the coefficients of the first term $8x^2$ and the last term 15.

   c. What do you notice?

2. Expand each of the following and check if what you noticed in question 1 holds true.

   a. $(3x-4)(x+7)$ b. $(5x-1)(2x-3)$ c. $(px+m)(qx+n)$

Step 2

- Facilitate a whole-class discussion using the following questions:
  - Display the expression $8x^2 + 10x + 12x + 15$ for all to see and ask, "What did you notice about the coefficients of the terms in the expression $8x^2 + 10x + 12x + 15$?" (The coefficients of the two middle terms have the same product as the coefficients of the first and last term.) As students respond, annotate the expression to identify the coefficients they are referencing.
  - "How can this help in factoring the expression $8x^2 + 22x + 15$?" (We can work backwards. Find the factors of $a \cdot c$ that, when added, will be equal to b.) Ask students to try this reasoning and record for all to see.
    - $8 \cdot 15 = 120$; $12 \cdot 10 = 120$ and $12 + 10 = 22$
  - "Let's look at how we can use this information to factor the expression."
    - "We can begin by rewriting the expression replacing $22x$ with $10x$ and $12x$." Write $8x^2 + 10x + 12x + 15$ for all to see.
    - "How is this related to the process of expanding $(2x+3)(4x+5)$ as shown in problem 1?" (It's the next-to-last step when expanding using the distributive property. These are the terms in the diagram that fill in the empty spaces.)
    - "Explain how you would reason backwards to get the factors." (The first two terms are rewritten using the distributive property as $2x(4x+5)$, and the last two terms are written as $3(4x+5)$. From there you use the distributive property again to write it as $(2x+3)(4x+5)$. In the diagram, the first row has a $4x$ in common. This means that the height of the first row is $4x$, and the width of the two sections must be $2x$ and 3 since $4x \cdot 2x = 8x^2$ and $4x \cdot 3 = 12x$. The second row has a 5 in common. This means that the second row has a height of 5, while the widths of the sections are still the same. So the height and width of the rectangle diagram are $(4x+5)$ and $(2x+3)$. Students may ask if it matters which term is written first. If time allows, explore reversing the placement of the two terms, $10x$ and $12x$, and seeing that while the individual steps may look different, the expression in factored form is the same.)
- Ask students to continue working on questions 3 and 4.

> M **Monitoring Tip:** Identify students who use the distributive property and those who use the diagram to complete the factoring. Let them know that they may be asked to share their strategy later.

Student Task Statement

3. Consider how this pattern can help in factoring the expression $6x^2 + 19x + 10$.

    a. Find the product of $a \cdot c$.

    b. Find the factors of $a \cdot c$ that add to make b.

    c. Use the factors to fill in the missing information and finish writing the expression in factored form.

Using the distributive property	**Using a diagram**
$6x^2 + \_\_\_\_\_ x + \_\_\_\_\_ x + 10$	<table><tr><td>$6x^2$</td><td>$\_\_\_\_ x$</td></tr><tr><td>$\_\_\_\_ x$</td><td>10</td></tr></table>

4. Try this method to write each of these in factored form.

    a. $6x^2 + 17x + 12$

    b. $5x^2 - 17x + 6$

Step 3

- Invite previously selected students to share how they factored the expressions using the method they just learned.
- Discuss questions such as:
    - "Do you think this method is simpler or harder than guessing and checking? Is it quicker or slower?"
    - "In what ways is this method simpler? In what ways is it harder?"

RESPONSIVE STRATEGY

Use this routine to support whole-class discussion. After each student shares, provide the class with the following sentence frames to help them clarify their understanding of the speaker's ideas and to press for further details: "How did you get . . . ?", "Why did you . . . ?", "How do you know . . . ?" If necessary, revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.

Discussion Supports (MLR8)

DO THE MATH

PLANNING NOTES

Lesson Debrief

The purpose of this lesson is to extend what students know about rewriting quadratic expressions in factored form to include expressions where the leading coefficient is not 1. Highlight that for some quadratic expressions, the right pairs of factors to use might be easily spotted, but for others, students may need to use the process of guessing and checking several combinations until they find the correct factors.

Choose whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

Display the two expressions: $x^2 - 9x + 14$ and $2x^2 + x - 28$.

- “For which expression might you need to guess and check to rewrite in factored form? Explain why.” (The expression $2x^2 + x - 28$ because it has a $2x^2$ instead of an x^2.)
- Ask students to factor the expression.
- If time permits, have students share with one another and then invite a student or two to share with the class an explanation of how they or their partner factored the expression.

PLANNING NOTES

Student Lesson Summary and Glossary

In some cases, rewriting an expression of the form $ax^2 + bx + c = 0$ in factored form is challenging.

For example, what is the factored form of $6x^2 + 11x - 35$?

We know that it could be $(3x + \square)(2x + \square)$, or $(6x + \square)(x + \square)$, but will the second number in each factor be -5 and 7, 5 and -7, 35 and -1, or -35 and 1? And in which order?

We have to do some guessing and checking before finding the equivalent expression that would allow us to solve the equation $6x^2 + 11x - 35 = 0$.

Once we find the right factors, we can proceed to solving using the zero product property, as shown here:

$$6x^2 + 11x - 35 = 0$$
$$(3x - 5)(2x + 7) = 0$$
$$3x - 5 = 0 \text{ or } 2x + 7 = 0$$
$$n = \frac{5}{3} \text{ or } x = -\frac{7}{2}$$

Rewriting quadratics into factored form and using the zero product property has its limitations. Sometimes it can be challenging to factor, especially when a is not 1. Other times the quadratic cannot be factored. In those cases we can use a graph to approximate solutions. Other methods for solving quadratics will be explored in future courses.

Cool-down: How Would You Solve This Equation? *(5 minutes)*

Addressing: NC.M1.A-REI.4
Cool-down Guidance: More Chances Lesson 26 offers another opportunity to factor and solve when a is not 1.

Cool-down

Solve the equation $2x^2 - 7x + 5 = 0$ by factoring and using the zero product property. Show your work.

Student Reflection:

What is at least one thing you did differently within the past week or so to help yourself in doing, enjoying, or understanding mathematics?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Think about a recent time from class when your students were confused. What did you do to support them in reasoning about their confusion together as a community of learners?

Practice Problems

1. To write $11x^2 + 17x - 10$ in factored form, Diego first listed pairs of factors of -10.

$$(\_\_ + 5)(\_\_ + -2)$$
$$(\_\_ + 2)(\_\_ + -5)$$
$$(\_\_ + 10)(\_\_ + -1)$$
$$(\_\_ + 1)(\_\_ + -10)$$

   a. Use what Diego started to complete the rewriting.

   b. How did you know you'd found the right pair of expressions? What did you look for when trying out different possibilities?

2. To rewrite $4x^2 - 12x - 7$ in factored form, Jada listed some pairs of factors of $4x^2$:

$$(2x + \_\_)(2x + \_\_)$$
$$(4x + \_\_)(1x + \_\_)$$

   Use what Jada started to rewrite $4x^2 - 12x - 7$ in factored form.

3. Rewrite each quadratic expression in factored form. Then, use the zero product property to solve the equation.

   a. $7x^2 - 22x + 3 = 0$ b. $4x^2 + x - 5 = 0$ c. $9x^2 - 25 = 0$

4. Han is solving the equation $5x^2 + 13x - 6 = 0$.

   Here is his work:

$$5x^2 + 13x - 6 = 0$$
$$(5x - 2)(x + 3) = 0$$
$$x = 2 \text{ or } x = -3$$

   Describe Han's mistake. Then, find the correct solutions to the equation.

5. Which equation shows a next step in solving $9(x - 1)^2 = 36$ that will lead to the correct solutions?

   a. $9(x - 1) = 6$ *or* $9(x - 1) = -6$
   b. $3(x - 1) = 6$
   c. $(x - 1)^2 = 4$
   d. $(9x - 9)^2 = 36$

   (From Unit 7, Lesson 18)

6. To solve the equation $0 = 4x^2 - 28x + 39$, Elena uses technology to graph the function $f(x) = 4x^2 - 28x + 39$. She finds that the graph crosses the x-axis at $(1.919, 0)$ and $(5.081, 0)$.

   a. What is the name for the points where the graph of a function crosses the x-axis?

   b. Use a calculator to compute $f(1.919)$ and $f(5.081)$.

   c. Explain why 1.919 and 5.081 are approximate solutions to the equation $0 = 4x^2 - 28x + 39$ and are not exact solutions.

   (From Unit 7, Lesson 17)

7. A picture is 10 inches wide by 15 inches long. The area of the picture, including a frame that is x inch(es) thick, can be modeled by the function $A(x) = (2x + 10)(2x + 15)$.

   a. Use function notation to write a statement that means: the area of the picture, including a frame that is 2 inches thick, is 266 square inches.

   b. What is the total area if the picture has a frame that is 4 inches thick?

   (From Unit 7, Lesson 16)

8. Add or subtract:

   a. $(5x^2 - 4x + 3) - (2x^2 - 4x - 7)$ b. $(9 - 7x^2) + (3x^2 - 3.4x + 5.2)$ c. $(\frac{3}{4}x^2 - 4x) + (\frac{3}{4}x^2 - 4x + \frac{5}{8})$

   (From Unit 7, Lessons 14 and 15)

9. (*Technology required*.) The number of people, p, who watch a weekly TV show is modeled by the equation $p = 100,000 \cdot (1.1)^w$, where w is the number of weeks since the show first aired.

   a. How many people watched the show the first time it aired? Explain how you know.

   b. Use technology to graph the equation.

   c. In which week does the show first get an audience of more than 500,000 people?

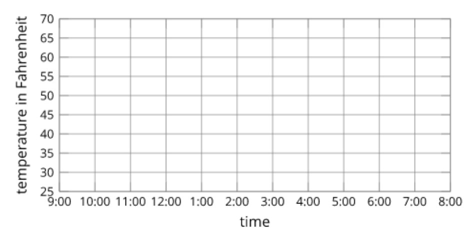
   (From Unit 6)

10. Here is a description of the temperature at a certain location yesterday.

    "It started out cool in the morning, but then the temperature increased until noon. It stayed the same for a while until it suddenly dropped quickly! It got colder than it was in the morning, and after that, it was cold for the rest of the day."

    Sketch a graph of the temperature as a function of time.

    (From Unit 5)



Lesson 26: Factor to Identify Key Features and Solve Equations

PREPARATION

Lesson Goals	Learning Targets
• Given a quadratic function of the form $f(x) = ax^2 + bx + c$ with $a \neq 1$, factor to identify key features. • Given a quadratic equation of the form $ax^2 + bx + c = 0$ with $a \neq 1$, solve by factoring.	• I can factor a quadratic function of the form $f(x) = ax^2 + bx + c$ in order to identify and explain key features of the function. • I can factor and solve a quadratic equation of the form $ax^2 + bx + c = 0$.

Lesson Narrative

Students have been building an understanding of the factored form of quadratic expressions and its different uses. They have transformed expressions from standard form into factored form. They have used factored form to solve quadratic equations and identify key features of a quadratic function. In the last lesson, students factored quadratic expressions when the leading coefficient was not 1.

In this lesson, students revisit identifying key features of quadratic functions and solving quadratic equations given quadratic expressions of the form of $ax^2 + bx + c$ with $a \neq 1$.

What are the big ideas around quadratic functions and solving quadratic equations that you want to come across in this lesson?

Focus and Coherence

Addressing
NC.M1.A-SSE.3: Write an equivalent form of a quadratic expression $ax^2 + bx + c$, where a is an integer, by factoring to reveal the solutions of the equation or the zeros of the function the expression defines.
NC.M1.A-REI.4: Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.
NC.M1.F-IF.7: Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.
NC.M1.F-IF.8a: Use equivalent expressions to reveal and explain different properties of a function. **a.** Rewrite a quadratic function to reveal and explain different key features of the function.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*10 minutes*)
- **Cool-down** (*5 minutes*)
    - M1.U7.L26 Cool-down (print 1 copy per student)

LESSON

Warm-up: Sketching the Graph of a Quadratic Function (*5 minutes*)

Addressing: NC.M1.F-IF.7

The purpose of this warm-up is to revisit how to use factored form to identify key features of a function and sketch a graph. Students can use a four-function calculator or a scientific calculator but should not use graphing technology.

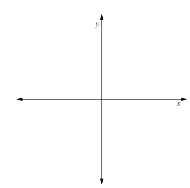
Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.
- Give students 1 minute of quiet think time and ask them to work with their partner.

Student Task Statement

Without using technology, sketch the graph of $f(x) = (2x+3)(x-4)$ by finding the:

1. x-intercepts
2. y- intercept
3. vertex



Step 2

- Display a set of axes for all to see.
- Ask for student volunteers to present how they found each of the key features and graph the key feature for all to see.
- If time permits, graph the function using Desmos and compare with the sketch.

DO THE MATH	PLANNING NOTES

Activity 1: The Grasshopper's Jump (*10 minutes*)

Instructional Routines: Co-Craft Questions (MLR5) - Responsive Strategy; Compare and Connect (MLR7)	
Addressing: NC.M1.F-IF.8a	

In previous lessons students have analyzed the factored and standard forms of a quadratic expression to identify and describe the key features of a quadratic function. This activity builds on that prior knowledge to include a function model where $a \neq 1$. Students factor the quadratic and use the factored and/or standard form to answer questions.

Step 1

- Ask students to arrange themselves into groups of three or use visibly random grouping.
- Allow students to use a four-function or scientific calculator but not graphing technology. The purpose of the task is to rewrite the function into factored form and analyze the forms to identify key features.

RESPONSIVE STRATEGY

Before asking students to answer the given questions, display only the description and ask students to write down at least one mathematical question that could be asked about the situation. Keep in mind students do not need to answer their created questions. Invite students to share and refine their questions with a partner, and then with the whole class. Record questions shared with the class in a public space. This helps students produce the language of mathematical questions as they reason about the relationship between time and height above the ground.

Co-Craft Questions (MLR5)

Monitoring Tip: As students work, look for the different ways students approach each question. This may include:

- using different but equivalent factored forms
- identifying the initial height by using the c value of the standard form or by substituting $t = 0$ into the factored or standard form
- identifying the maximum height by determining the midpoint between the two horizontal intercepts or using the formula $t = \frac{-b}{2a}$ then substituting into the factored or standard form

Let students know that they may be asked to share their thinking during the whole-class discussion.

Student Task Statement

A grasshopper is sitting on a flower and jumps to the ground. The quadratic function $h(t) = -5t^2 + t + 6$ models the grasshopper's height above the ground, in inches, as a function of time, t, in seconds.

1. Write the function in factored form.
2. Use the factored form or the standard form to answer each of the following questions.
   a. At what height was the grasshopper when it was sitting on the flower? Explain how you know.
   b. How many seconds after jumping off the flower did the grasshopper land on the ground? Explain how you know.
   c. What was the maximum height above the ground of the grasshopper?

Step 2

- Prepare for the *Compare and Connect* routine by asking student groups to create a visual display of their work with enough detail so that other students can interpret their reasoning.

- Create a display of the context together with the function in both standard form and factored form, with space to annotate during the whole-class discussion.
- For each question, invite one or two groups to use their visual displays to present different solution pathways and highlight connections between the context, the question, and the different forms of the function. Invite students to share any differences or connections between their own strategies and those of their classmates. Ask students:
  - "What are the advantages of factoring to identify key features of quadratic functions?" (You can use the zero product property to solve equations or find the zeros of the function if you have the factored form.)
  - "What are the challenges of using factoring to identify key features of quadratic functions?" (Not all quadratics can be factored and sometimes guessing and checking is inefficient when there are multiple factors of a and c.)

DO THE MATH	PLANNING NOTES

Activity 2: Solve by Factoring (*15 minutes*)

Instructional Routine: Round Robin
Addressing: NC.M1.A-SSE.3; NC.M1.A-REI.4

In previous lessons, students factored quadratic expressions to solve equations. In most cases, the expressions were limited to a leading coefficient of 1. In the last lesson, students factored quadratic expressions when the leading coefficient was not 1. The purpose of this activity is for students to strengthen their skill at solving equations of the form $ax^2 + bx + c = 0$ when $a \neq 1$. Students analyze the work of others that includes common errors often made when solving by factoring. As students construct viable arguments and critique the reasoning of others (MP3), they increase their ability to identify errors in their own reasoning.

Step 1

- Keep students in their small groups.
- Provide students a few minutes of individual quiet think time to consider each problem.
- Use the *Round Robin* routine to structure a discussion about the errors made by each student.
- Once students have come to agreement on the errors and how to proceed, ask them to solve the equation independently and then compare their solutions within their groups.
- Have groups repeat this process for each problem they have time for, making sure to have a few minutes for the discussion after group collaboration.

Student Task Statement

1. Han solved the equation $6x^2 + 11x - 10 = 0$ by factoring. His work is shown below.

   $6x^2 + 11x - 10 = 0$
   $(2x - 5)(3x + 2) = 0$
   $2x - 5 = 0$ or $3x + 2 = 0$
   $x = \frac{5}{2}$ or $x = \frac{-2}{3}$

   Are Han's solutions correct? If the solutions are correct, explain how you know. If the solutions are incorrect, identify the error.

2. Mai is solving the equation $4x^2 + 12x + 5 = 0$. She is using guess and check to factor the expression $4x^2 + 12x + 5$. So far she has tried $(4x + 1)(x + 5)$ and $(4x + 5)(x + 1)$, and neither of them has worked. She is not sure what to do now. What suggestion would you give Mai that would help her factor the expression?

3. Kiran is solving the equation $5x^2 - 12x + 11 = 4$. He is using guess and check to factor the expression $5x^2 - 12x + 11$. So far he has tried $(5x - 1)(x - 11)$ and $(5x - 11)(x - 1)$, and neither of them has worked. What suggestion would you give Kiran that would help him understand the error he has made?

Step 2

- Facilitate a whole-class discussion about common errors, ways to identify them, and how to correct them. Consider asking some of the following questions to further the discussion:
  - "What were some of the errors you found in this activity?" (the wrong signs in the factors, not trying all of the factors of the coefficient of a, not having the quadratic expression equal to zero)
  - "What are the ways you can check the factoring of a quadratic expression?" (write it in standard form by multiplying and see if it is the same, graph the equation in factored form and standard form and see if they have the same graph)
  - "What are ways you can check your solutions to equations?" (substitute the values in for x and evaluate to see if the two sides of the equation are equal, graph the equation and look at the x values of the x intercepts)

DO THE MATH	PLANNING NOTES

Lesson Debrief (*10 minutes*)

The purpose of this lesson is for students to practice factoring a quadratic expressions of the form $ax^2 + bx + c$ with $a \neq 1$ and use the factored form to identify key features of a quadratic function and solve quadratic equations.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion. • Display the expressions $x^2 + 4x - 12$ and $8x^2 + 10x - 3$ and facilitate a whole-class discussion focused on how the expressions are similar and how they are different. – "How are the two expressions similar?" (They are both quadratics.) – "How are they different?" (The coefficients and constants are different.) – "What considerations need to be made when factoring either expression?" (For the first one, you can use the factors of -12 that add to be 4. For the second, you have factors of 8 to consider as well as the factors of -3. Use guess and check to factor that expression.) – "What is the advantage of factored form?" (Using the zero product property, you can find the zeros of the function, solutions to equations, or the x-intercepts of the graph.)	**PLANNING NOTES**

Student Lesson Summary and Glossary

We have seen that when a quadratic expression is expressed in standard form, it can be beneficial to rewrite it in factored form. The factored form, along with the zero product property, allows us to solve quadratic equations and identify key features of quadratic functions.

Recently, we saw that factoring an expression of the form $ax^2 + bx + c$ when a is not 1 can be challenging; however, when factoring is possible, it gives us another way to reason with quadratics.

Here is an example of factoring a quadratic expression to identify key features of a function:

- A diver's height above the water, in meters, as a function of time t, in seconds, since jumping from the diving board can be modeled by the function $h(t) = -5t^2 + 2t + 3$.
- The coefficient a is -5. We will need to try several possibilities to factor the expression. These include:
  - $(-5t + 3)(t + 1) = -5t^2 - 2t + 3$
  - $(-5t - 3)(t - 1) = -5t^2 + 2t + 3$ $\leftarrow$ This is the equivalent factored form.
- Using the factored form and the zero product property we can determine that:
  - The zeros of the function are $t = \frac{-3}{5}$ and $t = 1$.
  - The horizontal intercepts are $(\frac{-3}{5}, 0)$ and $(1, 0)$.
- This means that the diver hits the water 1 second after jumping from the diving board.
- Using the horizontal intercepts, we can determine that the maximum height occurs when $t = 0.2$. Since $h(0.2) = 3.2$, the maximum height of the diver is 3.2 meters. This occurs 0.2 seconds after jumping from the diving board.

Here is an example of factoring a quadratic expression to solve a quadratic equation.

- Given the equation $8x^2 - 2x - 21 = 0$. we notice that the quadratic expression is equal to zero and that the coefficient of the squared term is 8. To factor the expression we will need to try some possibilities. These include:
  - $(8x-7)(x+3) = 8x^2 + 17x - 21$
  - $(8x-3)(x+7) = 8x^2 + 53x - 21$
  - $(4x-3)(2x+7) = 8x^2 + 22x - 21$
  - $(4x-7)(2x+3) = 8x^2 - 2x - 21$ ← This is the equivalent factored form.
- We can apply the zero product property to solve $(4x-7)(2x+3) = 0$. Thus, the solutions are $x = \frac{7}{3}$ and $x = \frac{-3}{2}$.

Rewriting a quadratic expression to identify key features of a quadratic function or to solve a quadratic equation can be useful especially when used with the zero product property. It is limited, however, to only those expressions that can be factored.

Cool-down: Maximum Area of a Rectangle (*5 minutes*)

Addressing: NC.M1.F-IF.8a

Cool-down Guidance: Press Pause
If possible, review the cool-down and the different student responses prior to the End-of-Unit assessment.

Cool-down

The function $A(w) = -2w^2 + 11w - 5$ represents the area of a rectangle, in square feet, as a function of the width w, in feet. Factor the quadratic expression and use the factored form to determine the maximum area of the rectangle and the width of that rectangle.

Student Reflection:

What has been one of your favorite topics or activities in math over the last few weeks? What about it makes it one of your favorites?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

With which math ideas from today's lesson did students grapple most? Did this surprise you, or was this what you expected?

Practice Problems

1. Jada is solving the equation $9x^2 - 34x - 8 = 0$. She is using guess and check to factor the expression $9x^2 - 34x - 8$. She found that $(9x - 2)(x + 4) = 9x^2 + 34x - 8$, but she doesn't have it yet because the expression should have a $-34x$.
   a. What suggestion would you give Jada that would help her factor the expression?
   b. Find the solutions to the equation.

2. The function $A(w) = -2w^2 + 25w - 33$ represents the area of a rectangle, in square feet, as a function of the width, w, in feet. Factor the quadratic expression and use the factored form to determine the maximum area of the rectangle and the width of that rectangle.

3. Rewrite each quadratic expression in standard form.

   a. $(x+1)(7x+2)$ b. $(8x+1)(x-5)$ c. $(2x+1)(2x-1)$ d. $(4+x)(3x-2)$

   (From Unit 7, Lesson 25)

4. Find the missing expression in parentheses so that each pair of quadratic expressions is equivalent. Show that your expression meets this requirement.
   a. $(4x-1)(\underline{\quad\quad})$ and $16x^2 - 8x + 1$
   b. $(9x+2)(\underline{\quad\quad})$ and $9x^2 - 16x - 4$
   c. $(\underline{\quad\quad})(-x+5)$ and $-7x^2 + 36x - 5$

   (From Unit 7, Lesson 25)

5. (*Technology required.*) For each equation, find the approximate solutions by graphing.

   a. $x^2 + 10x + 8 = 0$ b. $x^2 - 4x - 11 = 0$

   (From Unit 7, Lesson 25)

6. Match an equation in the first column with an equivalent equation in the second column.

a. $y = 3x - 5$
b. $(x+1)(x-1) = 0$
c. $4t(t-2) = 10$
d. $x^2 + 2x + 1 = 0$
e. $4t + 16 = 10$
f. $3x = 5y$

1. $4(t+4) = 10$
2. $y - 3x + 5 = 0$
3. $y = \frac{3x}{5}$
4. $x^2 - 1 = 0$
5. $4t^2 - 8t = 10$
6. $(x+1)(x+1) = 0$

(From Unit 7, Lesson 25)

7. Solve each equation.

a. $p^2 + 10 = 7p$

b. $x^2 + 11x + 27 = 3$

c. $(y+2)(y+6) = -3$

(From Unit 7, Lesson 24)

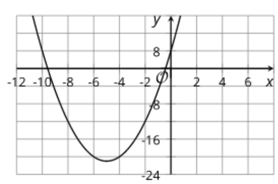
8. Match the expressions in factored form with the expressions in standard form.

Expressions in factored form	**Functions in standard form**
a. $(2a+5)(a+4)$	1. $f(x) = 2a^2 + 13a + 20$
b. $(3a-1)(a-10)$	2. $g(x) = 16a^2 - 25$
c. $(a+7)(5a-2)$	3. $h(x) = 5a^2 + 33a - 14$
d. $(4a-5)(4a-5)$	4. $j(x) = 16a^2 - 40a + 25$
e. $(4a-5)(4a+5)$	5. $k(x) = 18a^2 + 71a + 28$
f. $(2a+7)(9a+4)$	6. $m(x) = 3a^2 - 31a + 10$

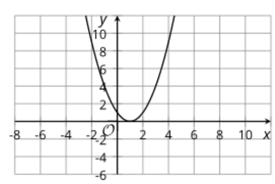
(From Unit 7, Lesson 21)

9. For each function f, decide if the equation $f(x) = 0$ has 0, 1, or 2 solutions. Explain how you know.

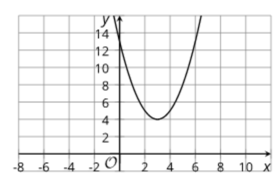
a.



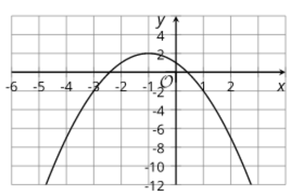
b.



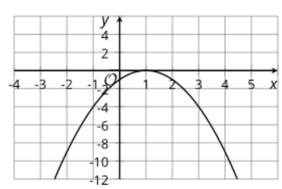
c.



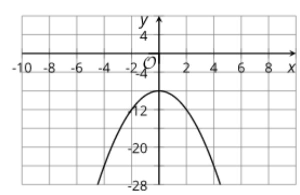
d.



e.



f.



(From Unit 7, Lesson 20)

Lesson 27: Post-Test Activities

PREPARATION

Lesson Goal	Learning Targets
• Provide students the opportunity to reflect and share feedback on their own progress and on the culture and instruction happening in the class.	• I can reflect on my progress in mathematics. • I can share feedback that can help make me and my teacher grow.

Lesson Narrative

This lesson, which should occur after the Unit 7 End-of-Unit Assessment, allows for students to reflect on the unit, share feedback, conference with the teacher, and engage in activities that support the work of the upcoming unit.

Gathering student feedback is a powerful and strategic way to learn about students and improve instructional practices. It also creates student and family buy-in and centers students as decision makers and problem solvers in their own learning.

What do you hope to learn about your students during this lesson?

Agenda, Materials, and Preparation

- **Activity 1** (*20 minutes*)
  – End-of-Unit 7 Student Survey (print 1 copy per student)
- **Activity 2** (*25 minutes*)
  – Towering Sequence Hanoi (Applet preferred): https://bit.ly/HanoiSequence
  – Coins: If not using the digital applet, each student could use a quarter, nickel, penny, and dime, and a piece of paper with 3 circles drawn on it.

LESSON

Activity 1: End-of-Unit 7 Student Survey (*20 minutes*)

The End-of-Unit 7 Student Survey is a critical opportunity for teachers to gather low-stakes, non-evaluative feedback to support equitable instruction. The survey is also highly beneficial for students as it is designed to encourage self-awareness, self-management, social awareness, relationship skills, and responsible decision making. Provide students a chance to quietly and independently complete this survey after they complete their testing.

One-on-One Conferences

Conducting one-on-one conferences with students, using the surveys as a data point, is encouraged. These conferences can be done as students complete their surveys and are engaging in Activity 2. Potential conference topics include:

- student responses to the daily student reflections
- student response to the end-of-unit student survey (as students finish them)
- executive functioning skills
- student learning contracts
- goal setting and self-evaluation

Activity 2: The Tower of Hanoi[1] (*25 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1) - Responsive Strategy	
Building Towards: NC.M1.F-BF.1a	

In this activity, students experiment with solving the Tower of Hanoi puzzle for different numbers of starting discs. They look for a pattern in the sequence generated by listing the number of moves needed to solve the puzzle for different numbers of discs and then describe the pattern informally (MP8). The other purpose of this activity is to establish that students are expected to try things out, look for patterns, explain their thinking, justify their responses, and listen respectfully to their classmates in preparation for the sequence work in Unit 8. In Lesson 1 of that unit, students will investigate another puzzle in which the number of moves to solve each successive version forms a sequence with a recognizable rule.

This activity works best when each group has access to manipulatives or devices that can run the GeoGebra applet because students will benefit from seeing the relationship in a dynamic way.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Set up either a physical puzzle with 2 discs or display or share the digital version (preferred) with 2 discs: https://bit.ly/HanoiSequence.
- Ask students to read the two rules to the puzzle and then give them time to solve the puzzle with 2 discs. Let them know that they need to keep track of the number of moves needed and that the goal is to find the smallest number of moves possible.
  - If necessary, ask a student to demonstrate why it takes 3 moves to solve the puzzle with 2 discs.

RESPONSIVE STRATEGY

Use this routine to give students a structured opportunity to revise and refine their explanations for how Jada used the solution for 3 discs to help her solve the puzzle for 4 discs. As students share their responses with their partner, listeners should press for details and clarity as appropriate based on what each speaker produces. Provide students with prompts for feedback that will help individuals strengthen their ideas and clarify their language. For example, "What pattern did Jada notice?" or "How do you know your answer is the smallest number of moves?" Students can borrow ideas and language from each partner to strengthen their final product.

Stronger and Clearer Each Time (MLR1)

Step 2

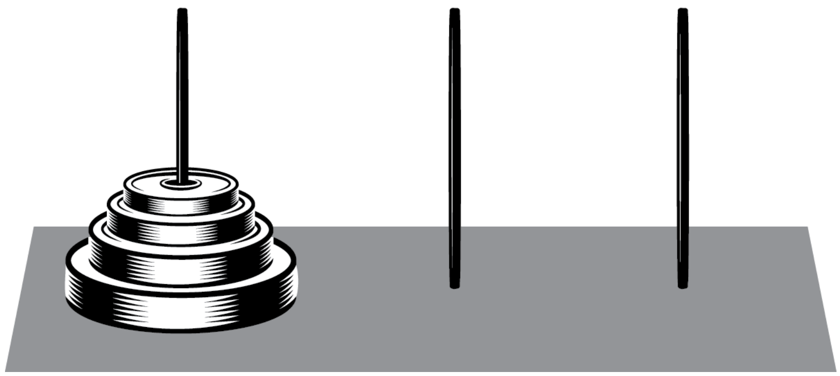
- Before students continue to work through the puzzle and task, remind them they need to keep track of the number of moves needed for each new number of discs, and that they need to try to find the smallest number of moves.
- Encourage students to check in with those around them to see if anyone found a solution with fewer moves and update their table as they do so.

[1] Adapted from IM 9–12 Math Algebra 2, Unit 1, Lesson 1 https://curriculum.illustrativemathematics.org/HS/teachers/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

***Advancing Student Thinking*:** Students may incorrectly interpret the rules or presume there is no better solution once they find one solution that works. Encourage groups to regularly switch who is in charge of moving the discs and to check with other groups around them once they think they have found a solution with the fewest number of moves.

Student Task Statement

In the Tower of Hanoi puzzle, a set of discs sits on a peg, while there are 2 other empty pegs.



A "move" in the Tower of Hanoi puzzle involves taking a disc and moving it to another peg. There are two rules:

- Only move 1 disc at a time.
- Never put a larger disc on top of a smaller one.

You complete the puzzle by building the complete tower on a different peg than the starting peg. As you answer the following questions, note the number of moves you've made in the table below with the goal of taking the smallest number of moves possible.

1. Using 3 discs, complete the puzzle. What is the smallest number of moves you can find?
2. Using 4 discs, complete the puzzle. What is the smallest number of moves you can find?
3. Jada says she used the solution for 3 discs to help her solve the puzzle for 4 discs. Describe how this might happen.
4. How many moves do you think it will take to complete a puzzle with 5 discs? Explain or show your reasoning.
5. How many moves do you think it will take to complete a puzzle with 7 discs?

Number of discs	1	2	3	4	5	6	7
Number of moves							

Are You Ready For More?

A legend says that a Tower of Hanoi puzzle with 64 discs is being solved, one move per second. How long will it take to solve this puzzle? Explain how you know.

TEACHER REFLECTION

As you finish up this unit, reflect on the norms and activities that have supported each student in learning math. List ways you have seen students grow as mathematicians throughout this work.

List ways you have seen yourself grow as a teacher.

What will you continue to do and what will you improve upon in Unit 8?